	Prelim Revision 2 – Answers	
1.	$f'(x) = \cos x \cos x - \sin x \sin x$ $f'(x) = \cos^{2x} - \sin^2 x \quad or \quad \cos 2x$	
2.	$\dfrac{dy}{dx} = \dfrac{3e^{3x}(2x+1) - e^{3x}(2)}{(2x+1)^2}$ $\dfrac{dy}{dx} = \dfrac{e^{3x}(6x+1)}{(2x+1)^2}$	
3.	$ar^2 = 24$ and $ar^4 = 6$, $\dfrac{6}{24} = \dfrac{r^4}{r^2}$ so $\dfrac{1}{4} = r^2$ the common ratio is $\dfrac{1}{2}$ this series has a sum to infinity as $\left\lvert\dfrac{1}{2}\right\rvert < 1$ first term is 96 sun to infinity is $S_\infty = \frac{96}{1-\frac{1}{2}} = 192$	
4.	$C^T = \begin{pmatrix} -2 & 3 & 2 \\ -1 & -1 & 0 \\ 1 & 1 & -1 \end{pmatrix}$ $\det D = 1 \times \det\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + 4 \times \det\begin{pmatrix} k+5 & 1 \\ 2 & 0 \end{pmatrix} + 1 \times \det\begin{pmatrix} k+5 & 0 \\ 2 & 1 \end{pmatrix}$ $= -1 - 8 + k + 5 = k - 4$ For D^{-1} to not exist $k - 4 = 0,\ k = 4$	

5.	$1-3i$ | 1 -4 11 -14 -30
	  | 0 $1-3i$ $-12+6i$ $17+9i$ 30
	    1 $-3-3i$ $-1+6i$ $3+9i$ [0]
	
	No remainder so $i - 3i$ is a solution to this polynomial. As is $z = 1 + 3i$
	$1+3i$ | 1 $-3-3i$ $-1+6i$ $3+9i$
	  | 0 $1+3i$ $-2-6i$ $-3-9i$
	    1 -2 -3 [0]
	$z^4 - 4z^3 + 11z^2 - 14z - 30 = (z-1-3i)(z-1+3i)(z^2-2z-3)$
	$= (z-1-3i)(z-1+3i)(z-3)(z+1)$
	Thus the solutions are $z = 1 \pm 3i,\ z = 3, z = -1$	

6.	$\frac{9}{(x+3)(x-3)} = \frac{A}{x+3} + \frac{B}{x-3} \quad A = -\frac{3}{2}, \quad B = \frac{3}{2}$ $\int \frac{9}{x^2-9}\,dx = \frac{3}{2}\int \frac{1}{x-3}\,dx - \frac{3}{2}\int \frac{19}{x+3}\,dx$ $= \frac{3}{2}\ln(x-3) - \frac{3}{2}\ln(x+3) + C$ $= \frac{3}{2}\ln\left(\frac{x-3}{x+3}\right) + C$	
7.	$\begin{pmatrix} 1 & -1 & 2 & : & -3 \\ -1 & 2 & -3 & : & 2 \\ 2 & -1 & \alpha & : & 1 \end{pmatrix}$ $\begin{pmatrix} 1 & -1 & 2 & : & -3 \\ 0 & 1 & -1 & : & -1 \\ 0 & 1 & \alpha-4 & : & 7 \end{pmatrix} \quad R_2 + R_1, \quad R_3 - 2R_1$ $\begin{pmatrix} 1 & -1 & 2 & : & -3 \\ 0 & 1 & -1 & : & -1 \\ 0 & 0 & \alpha-3 & : & 8 \end{pmatrix} \quad R_3 - R_2$ $(\alpha-3)z = 8, \quad z = \frac{8}{\alpha-3}$ When $\alpha = 3$ the final row is inconsistent since $0 \neq 8$ so there are no solutions When $\alpha = -13$, $\quad z = \frac{8}{-16} z = -\frac{1}{2},$ $y - z = -1, \quad y = -\frac{3}{2},$ $x + \frac{3}{2} - 1 = -3, \quad x = -\frac{7}{2}$	
8.	When $x = 1$, $\quad y^2 - 2y - 3 = 0 \quad (y-3)(y+1) = 0, \; y > 0 \; so \; y = 3$ Implicit differentiation $\frac{d}{du}(xy^2) + \frac{d}{du}(-2x^2y) = \frac{d}{du}(3)$ $y^2 + 2xy\frac{dy}{dx} - 4xy - 2x^2\frac{dy}{dx} = 0$ $(2xy - 2x^2)\frac{dy}{dx} = 4xy - y^2$ $\frac{dy}{dx} = \frac{4xy - y^2}{2xy - 2x^2}$ For $x = 1, y = 3$ gradient is $\frac{12-9}{6-2} = \frac{3}{4}$ Equation of the straight line is $y = \frac{3}{4}x + \frac{9}{4} \quad or \quad 4y = 3x + 9$	

9.	The general term is $\binom{4}{r}(3p^3)^{4-r}\left(\frac{-2}{p}\right)^r =$ $\binom{4}{r}(3^{4-r})(-2)^r(p^{12-3r})(p^{-r}) = \binom{4}{r}(3^{4-r})(-2)^r(p^{12-4r})$ The independent term happens when, $12-4r=0, r=3$ $\binom{4}{3}(3^1)(-2)^3(p^{12-12}) = 4\times 3\times -8 = -96$	
10.	$u = 1+\tan x\ ,\ \ du = \sec^2 x\ dx \quad x=\frac{\pi}{4}\ \ u=2,\ \ x=0\ \ u=1$ $\int_1^2 \frac{1}{u}du \ = [\ln u]_1^2 = \ln 2 - \ln 1 = \ln 2$ as required	
11.	For $f(x) = \sin^2 x\ ,\ f'(x) = 2\sin x\cos x = \sin 2x\ ,\ f''(x) = 2\cos 2x$ $f'''(x) = -4\sin 2x\ ,\ f^4(x) = -8\cos 2x$ $\sin^2 x = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \frac{f^4(0)}{4!}x^4$ $= 0 + 0 + \frac{2}{2!}x^2 + 0 - \frac{8}{4!}x^4 = x^2 - \frac{1}{3}x^4$ For $f(x) = \cos^2 x\ ,\ f'(x) = -\sin 2x\ ,$ and so on... $\cos^2 x = 1 - x^2 + \frac{1}{3}x^4$	
12.	For the complimentary function $4m^2+4m+1=0,$ $(2m+1)^2 = 0\ \ m = -\frac{1}{2}$ $\boldsymbol{y = Ae^{-\frac{1}{2}x} + Bxe^{-\frac{1}{2}x}}$ For the particular integral $y = mx + c,\ \frac{dy}{dx} = m,\ \frac{d^2y}{dx^2} = 0$ $4\times 0 + 4m + mx + c = 3x + 10$ $mx = 3x\ \ so\ m = 3,\quad 4m + c = 10\ \ so\ c = -2$ $\boldsymbol{y = 3x - 2}$ **Thus the general solution is** $\boldsymbol{y = Ae^{-\frac{1}{2}x} + Bxe^{-\frac{1}{2}x} + 3x - 2}$ Given $y = 2$ and $\frac{dy}{dx} = -3$ when $x = 0 \quad 2 = A - 2,\ A = 4$ $\boldsymbol{\frac{dy}{dx} = -\frac{1}{2}Ae^{-\frac{1}{2}x} + Be^{-\frac{1}{2}x} - \frac{1}{2}Bxe^{-\frac{1}{2}x} + 3}$ $-3 = -\frac{1}{2}A + B + 3,\quad -6 = -2 + B, -4 = B$ **Thus the particular solution is** $\boldsymbol{y = 4e^{-\frac{1}{2}x} - 4xe^{-\frac{1}{2}x} + 3x - 2}$	