Advanced Higher Mathematics

The knowledge and skills covered by the course is detailed below. *Grade A/B material in italic type.*

Calculus

Differentiation	**VG**	**OT**	**NS**
Differentiate functions including polynomials, sin$x$, cos$x$, tan$x$, sec$x$, cosec$x$, cot$x$, e^x and ln$x$.			
Differentiate functions using the chain rule			
Differentiate functions using the product rule			
Differentiate functions using the quotient rule			
Use above rules to differentiate functions requiring more than one application.			
Use logarithmic differentiation.			
Differentiate an inverse trig function.			
Use implicit differentiation to find first and *second derivatives.*			
Use parametric differentiation to find first and *second derivatives.*			

Properties of Functions			
Use vocabulary **function, domain, range, inverse function, critical point, local minimum/maximum, global minimum/maximum, continuous, discontinuous and asymptote**.			
Determine the domain and range of a function.			
Use derivative tests for locating and identifying maxima and minima.			
Use derivative tests to locate points of inflexion.			
Sketch related graphs $y = kf(x)$, $y = f(x) + k$, $y = f(x+k)$, $y = f(kx)$ and $y = \|f(x)\|$.			
Determine whether a function is even, odd or neither and use properties in graph sketching.			
Find vertical and non-vertical asymptotes.			
Sketch graphs of real rational functions showing zeros, asymptotes, critical points and symmetry.			

Integration	**VG**	**OT**	**NS**
Integrate expressions using standard results: $\int x^n dx = \frac{x^{n+1}}{n+1} + c, n \neq -1$ $\int \sin x\, dx = -\cos x + c$ $\int \cos x\, dx = \sin x + c$ $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + c$ $\int \sec^2 x\, dx = \tan x + c$ $\int e^x dx = e^x + c$ $\int \frac{1}{x} dx = \ln\lvert x\rvert + c$ $\int \frac{1}{1+x^2} dx = \tan^{-1} x + c$			
Integrate functions using a substitution.			
Prove standard integrals by substitution: $\int f(ax+b)\,dx = \frac{1}{a}F(ax+b)+c$ where F is the antiderivative of f, and $\int \frac{f'(x)}{f(x)} dx = \ln\lvert f(x)\rvert + c$.			
Integrate rational functions by writing in partial fractions.			
Integrate by parts *including more than one application*.			

Solving Differential Equations	**VG**	**OT**	**NS**
Recognise differential equations and understand the terms **linear, order, general solution, arbitrary constant, particular solution and initial conditions.**			
Solve first order differential equations with separable variables.			
Solve first order linear differential equations using the integrating factor method and find particular solutions.			
Solve second order homogenous ODEs with constant coefficients, finding the general solution in each of the *three cases.*			
Solve second order non-homogeneous ODEs with constant coefficients using the auxiliary equation and particular integral method.			

Applications of Calculus	**VG**	**OT**	**NS**
Apply differentiation to rectilinear motion.			
Apply parametric equations to rectilinear motion.			
Solve practical problems involving rates of change.			
Apply differentiation to optimisation problems.			
Apply integration to the evaluation of areas and volumes of revolution *including integration w.r.t. y.*			
Find the Maclaurin expansion for simple functions and composites and their *range of validity*.			
Use the Maclaurin expansion to find a power series for a simple function to a stated number of terms.			

Algebra

Binomial theorem	**VG**	**OT**	**NS**
Calculate permutations and combinations.			
Understand the notation $n!$ ${}^nC_r$ and $\binom{n}{r}$			
Generate Pascal's triangle up to $n = 7$.			
Expand brackets using the binomial theorem.			
Find specific terms in the expansion.			

Partial fractions	**VG**	**OT**	**NS**
Express a proper rational function as a sum of partial fractions.			
Reduce an improper rational function to a polynomial and a proper rational function.			

Sequences and Series	**VG**	**OT**	**NS**
Understand terms: **infinite sequence, infinite series, nth term, sum to n terms, sum to infinity, common difference, arithmetic sequence, common ratio, geometric sequence, recurrence relation.**			
Know how to find n^{th} terms: $U_n = a + (n-1)d$ for arithmetic sequences and $U_n = ar^{n-1}$ and for geometric sequences.			
Use summation formulae (given): $S_n = \frac{1}{2}n[2a + (n-1)d]$ for arithmetic sequences and $S_n = \frac{a(1-r^n)}{1-r}\ r \neq 1$ for geometric sequences.			
Know that $S_\infty = \frac{a}{1-r}\ \lvert r \rvert < 1$ for geometric sequences and expand $\frac{1}{1-r}$ and $\frac{1}{a+b}$ as geometric sequences.			

Summations and Mathematical Proof	**VG**	**OT**	**NS**
Understand sigma notation			
Use formulae for $\sum r$, $\sum r^2$ and $\sum r^3$ and prove related results			
Prove results by mathematical induction			

Complex numbers	**VG**	**OT**	**NS**
Identify the real and imaginary parts of a complex number.			
Perform operations +, -, × and ÷ on complex numbers and equate real and imaginary parts.			
Plot a complex number on an argand diagram.			
Know that a polynomial of degree n has n roots and that they occur in conjugate pairs.			
Factorise polynomials with real coefficients.			
Evaluate the modulus and argument of a complex number.			
Convert between polar and Cartesian form of a complex number.			
Use De Moivre's theorem to expand powers of complex numbers.			
Apply De Moivre's theorem to multiple angle formulae.			
Use De Moivre's theorem to find the nth roots of unity.			
Find the locus of a point in the complex plane.			

Direct and Indirect Proof	**VG**	**OT**	**NS**
Recognise the need for proof in mathematics.			
Understand the terms **implies** $(\Rightarrow)$, **is implied by** $(\Leftarrow)$ and **equivalence** $(\Leftrightarrow)$.			
Directly prove simple results.			
Disprove a conjecture by providing a counter example.			
Prove a result using the contrapositive.			
Know and use the fundamental theorem of arithmetic.			
Use proof by contradiction.			

Number Theory	**VG**	**OT**	**NS**
Know the division algorithm.			
Use the Euclidian Algorithm to find the greatest common divisor of two positive integers.			
Express the gcd as a linear combination of two integers.			
Use the division algorithm to write integers in bases other than 10.			

Matrices and Systems of Equations	**VG**	**OT**	**NS**
Understand terms: **matrix, element, row, column, order, identity matrix, inverse, determinant, singular, non-singular and transpose.**			
Add, subtract, multiply and equate matrices.			
Know properties: $A+B=B+A$ but $A\times B \neq B\times A$ $(AB)C=A(BC)$ $A(B+C)=AB+AC$ $(A')'=A$ $(A+B)'=A'+B'$ $(AB)'=B'A'$ $AB^{-1}=B^{-1}A^{-1}$ $Det(AB)=detAdetB$			
Calculate the determinant of 2×2 and a 3×3 matrices.			
Find the inverse of a 2×2 and a 3×3 matrix, where these exist.			
Apply the inverse matrix to the solution of a system of equations.			
Use 2×2 matrices to represent geometrical transformations in the (x, y) plane.			
Use a matrix to organise a system of equations.			
Perform elementary row operations.			
Reduce a matrix to an upper triangular form.			
Solve a 3×3 system of equations using Gaussian Elimination on an augmented matrix.			
Find the solution to a system of linear equations given as $A\boldsymbol{x}=b$ where there is a unique solution, *no solution and an infinite family of solutions.*			
Identify an ill-conditioned matrix.			

Vectors	**VG**	**OT**	**NS**
Calculate scalar and vector products in three dimensions.			
Know that $\boldsymbol{a}\times\boldsymbol{b} = -\boldsymbol{b}\times\boldsymbol{a}$.			
Know the equation of a line in vector, parametric and symmetric forms.			
Know the equation of a plane in vector, parametric and Cartesian forms.			
Find equations of lines and planes.			
Find the angle between two lines, between two planes and between a line and a plane.			
Find the intersection of two lines, a line and a plane and two or three planes.			