	Prelim Revision 1 – Answers	60
1.	$U_n = a + d(n-1),$ $U_{20} = 5 + d(20) \rightarrow \quad 62 = 5 + 19d, \;\; d = 3$ $S_n = \frac{1}{2}n\big(2a + d(n-1)\big) \qquad S_{40} = \frac{1}{2} \times 40\big(10 + 3(39)\big) = 2540$	
2.	$\left(\frac{1}{2}x\right)^4 + 4\left(\frac{1}{2}x\right)^3 \times (-3)^1 + 6\left(\frac{1}{2}x\right)^2 \times (-3)^2 + 4\left(\frac{1}{2}x\right) \times (-3)^3 + (-3)^4$ $= \frac{1}{16}x^4 - \frac{3}{2}x^3 + \frac{27}{2}x^2 - 54x + 81$	
3.	$f'(x) = (2+x)^5 + 5x(2+x)^4$ $= (2+x)^4\big((2+x) + 5x\big)$ $= (2+x)^4(2+6x) \;\; or \; 2(1+3x)(2+x)^4$	
4.	$Inverse \; = \frac{1}{10+2\alpha}\begin{pmatrix} 2 & -\alpha \\ 2 & 5 \end{pmatrix}, \qquad 10 + 2\alpha = 0, \;\; x = \; -5$	
5.	$\frac{x^3+3x^2-8x+2}{x^2-2x+1} = (x+5) + \frac{x-3}{x^2-2x+1}$ $\frac{x-3}{(x-1)^2} = \frac{A}{x-1} + \frac{B}{(x-1)^2}$ $\frac{x-3}{(x-1)^2} = \frac{1}{x-1} - \frac{2}{(x-1)^2}$ $\frac{x^3+3x^2-8x+2}{x^2-2x+1} = (x+5) + \frac{1}{x-1} - \frac{2}{(x-1)^2}$	

6.	$$\begin{pmatrix} 1 & -1 & 1 & : & 1 \\ 2 & 1 & -3 & : & -6 \\ 1 & -1 & 2 & : & 2 \end{pmatrix}$$ $$\begin{pmatrix} 1 & -1 & 1 & : & 1 \\ 0 & 3 & -5 & : & -8 \\ 0 & 0 & 1 & : & 1 \end{pmatrix} \quad R_2 - 2R_1, \quad R_3 - R_1$$ $$\boldsymbol{c = 1}, \quad 3b - 5c = -8 \quad 3b = -3, \quad \boldsymbol{b = -1}$$ $$a - b + c = 1, \; a + 2 = 1, \; \boldsymbol{a = -1}$$	
7.	For $u = x^4$, $du = 4x^3dx$ and $\frac{1}{2}du = 2x^3dx$ $$\int \frac{2x^3}{1+x^8}dx = \frac{1}{2}\int \frac{du}{1+u^2} = \frac{1}{2}\tan^{-1}u + C = \frac{1}{2}\tan^{-1}(x^4) + C$$	
8.	$$\begin{aligned} A^4 &= (4A-3I)(4A-3I) \\ &= 16A^2 - 24AI + 9I \\ &= 16A^2 - 24A + 9I \\ &= 16(4A-3I) - 24A + 9I \\ &= 64A - 48I - 24A + 9I \\ &= 40A - 39I \end{aligned}$$ $$P = 40, \quad q = -39$$	
9.	$$\frac{dx}{dt} = 27t^2, \quad \frac{dy}{dt} = 6t, \quad \frac{dy}{dx} = \frac{6t}{27t^2} = \frac{2}{9t}$$ When $t = 1$, $x = 9$, $y = 2$ and $m = \frac{2}{9}$ equation of the tangent is $y = \frac{2}{9}x$	
10.	$$\begin{aligned} \int x^2e^{-x}\,dx &= -x^2e^{-x} - \int -e^{-x} \times 2x\,dx \\ &= -x^2e^{-x} + \int 2xe^{-x}\,dx \\ &= -x^2e^{-x} + [-2xe^{-x} - \int -2e^{-x}\,dx] \\ &= -x^2e^{-x} + [-2xe^{-x} + 2\int e^{-x}\,dx] \\ &= -x^2e^{-x} - 2xe^{-x} - 2e^{-x} + C \end{aligned}$$	

11.

(a) Vertical asymptote at $x = -2$

By polynomial division $g(x) = x - 2 + \frac{9}{x+2}$, there is an oblique asymptote at $y = x - 2$

(b) $\left(0, \frac{5}{2}\right)$

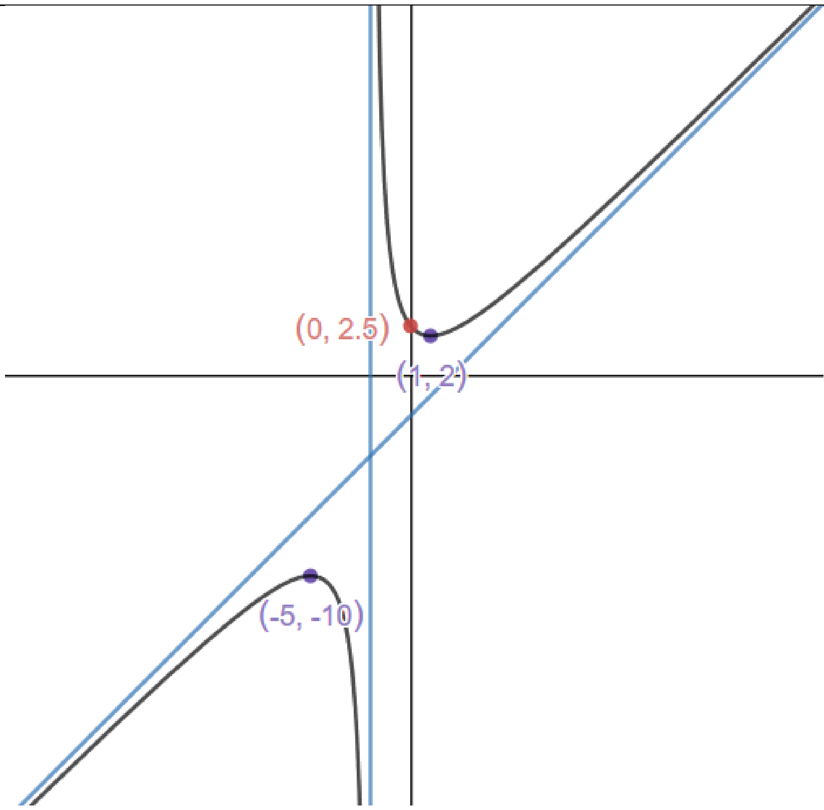
(c) $g'(x) = \frac{2x(x+2)-(x^2+5)}{(x+2)} = \frac{x^2+4x-5}{(x+2)^2} = \frac{(x+5)(x-1)}{(x+2)^2}$,

$\frac{(x+5)(x-1)}{(x+2)^2} = 0, \quad x = -5, 1$ Turning points at $(-5, -10), (1,2)$

$$g''(x) = \frac{18x+36}{(x+2)^4} = \frac{18}{(x+2)^3}, \quad g''(-5) < 0 \text{ and } g'(1) > 0$$

Maximum turning point at $(-5, -10)$ minimum turning point at $(1, 2)$

11.



12.

The auxiliary equation is $m^2 - 2m + 10 = 0, \quad m = \frac{2 \pm \sqrt{-36}}{2} = 1 \pm 3i$

The general solution is

$$y = e^x\,(A\cos 3x + B\sin 3x)$$

$y = 2$ when $x = 0$ $\quad 2 = 1(A + 0), \quad A = 2$

$$\frac{dy}{dx} = e^x(A\cos 3x + B\sin 3x) + e^x\,(3B\cos 3x - 3A\sin 3x)$$

$\frac{dy}{dx} = 5$ when $x = 0$

$$5 = 1(A + 0) + 1\,(0 + 3B), \quad 5 = 2 + 3B, \; B = 1$$

$$y = e^x\,(2\cos 3x + \sin 3x)$$