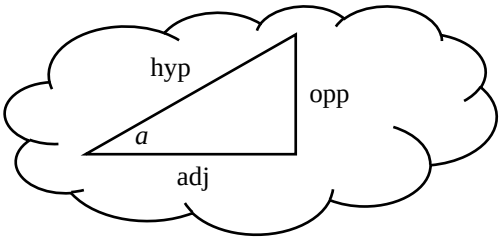
New Trig Functions

- Know the definitions of sin, cos, tan, sec, cosec and cot
- Recognise graphs of functions
- Know the derivative of tan
- Find the derivatives of sec, cosec and cot

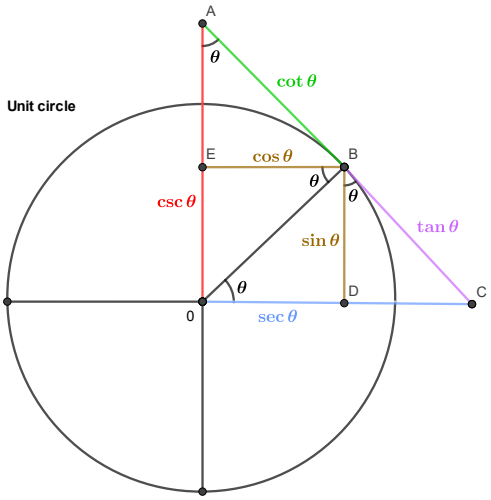


Write down **all** the possible ratios of opp, adj and hyp.

You should have $\frac{opp}{hyp}$ $\frac{adj}{hyp}$ $\frac{opp}{adj}$ $\frac{adj}{opp}$ $\frac{hyp}{opp}$ $\frac{hyp}{adj}$

As we know $\sin a = \frac{opp}{hyp}$, $\cos a = \frac{adj}{hyp}$ and $\tan a = \frac{opp}{adj}$.
We also now have $\mathbf{sec}\, \boldsymbol{a} = \frac{\boldsymbol{hyp}}{\boldsymbol{adj}}$ (the **secant** function) which is the reciprocal of cos, $\mathbf{cosec}\, \boldsymbol{a} = \frac{\boldsymbol{hyp}}{\boldsymbol{opp}}$ (the **cosecant**) which is the reciprocal of sin and $\mathbf{cot}\, \boldsymbol{a} = \frac{\boldsymbol{adj}}{\boldsymbol{opp}}$ (the **cotangent**) which is the reciprocal of tan.

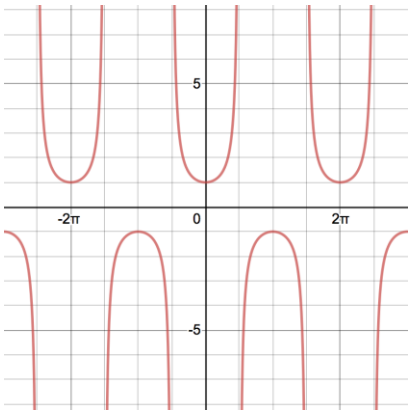
Formal definitions of these new trigonometric functions are on the unit circle as we have seen for sin, cos and tan.



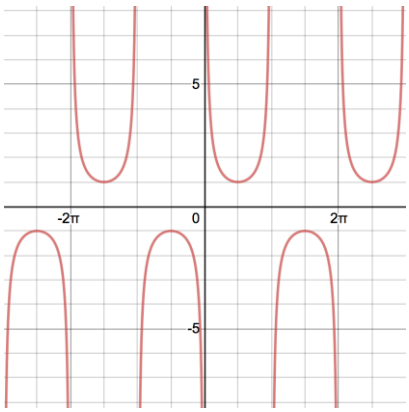
Look for right-angled triangles. You should be able to see that:
$\sin^2\theta + \cos^2\theta = 1$ $\tan^2\theta + 1 = \sec^2\theta$ $\cot^2\theta + 1 = \mathrm{cosec}^2\theta$

Calculus: New Trigonometric Functions

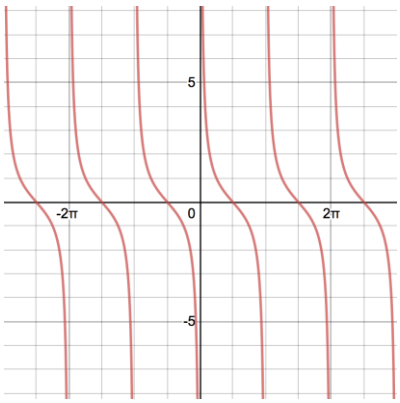
Graphs



$y = \sec x$



$y = \operatorname{cosec} x$



$y = \cot x$

Derivatives of trig functions

1. Find the derivative of tan x

$y = \tan x = \frac{\sin x}{\cos x}$ Treat this as a quotient with $f(x) = \sin x$ and $g(x) = \cos x$.

$f'(x) = \cos x \qquad g'(x) = -sinx$

$$\frac{dy}{dx} = \frac{\cos x.\cos x - (\sin x.-sinx)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x}$$

$$= \sec^2 x$$

Learn

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

2. Find the derivative of sec x

$y = \sec x = \frac{1}{\cos x}$ or $(\cos x)^{-1}$

$$\frac{dy}{dx} = -(\cos x)^{-2} - \sin x$$

$$= \frac{sinx}{\cos^2 x}$$

$$= \frac{1}{\cos x}.\frac{\sin x}{\cos x}$$

$$= \sec x \tan x$$

Calculus: New Trigonometric Functions

3. Prove that $\frac{d}{dx}\text{cosec } x = -\text{cosec } x \cot x$

4. Prove that $\frac{d}{dx}\cot x = -\text{cosec}^2 x$