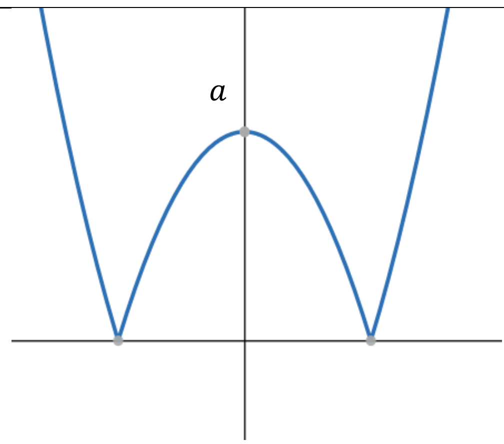
	Revision 1 for the A/B October Test	**40**
1	Express the binomial expansion of $\left(3 + \frac{1}{2}x\right)^4$	**3**
2	Differentiate (a) $\cos^{-1} 4x$ (b) xe^{x^2}	**2** **3**
3	The first term of an arithmetic sequence is 2 and the 20th term is 97. Obtain the sum of the first 50 terms	**4**
4	The function $f(x)$ is defined by $f(x) = x^2 - a^2$ (a) State whether $f(x)$ is odd, even or neither. Give a reason for your answer (b) Sketch the graph of $y = \|f(x)\|$	 **2** **1**
5	Find the equation of the tangent to the curve $x^2 + 3xy + y^2 = -1$ at the point $(-1,1)$	**5**
6	Given that $u_k = 11 - 2k, (k \geq 1)$. Obtain a formula for $$S_n = \sum_{r=1}^{n} u_k$$ Find the values of n for which $S_n = 21$	 **4**
7	Prove by induction that $\frac{d^n}{dx^n}(xe^x) = (x+n)e^x$ for all integers $n \geq 1$	**5**

8	(a) Express $\frac{x^2}{(x+1)^2}$ in the form $A + \frac{B}{x+1} + \frac{C}{(x+1)^2}$, $x \neq -1$ stating the values of the constants A, B and C. (b) A curve is defined by $y = \frac{x^2}{(x+1)^2}$ (i) write down the equations of the asymptotes (ii) Find any stationary points or points of inflexion and justify their nature (iii) Sketch the curve clearly marking the features found above	**3** **2** **4** **2**

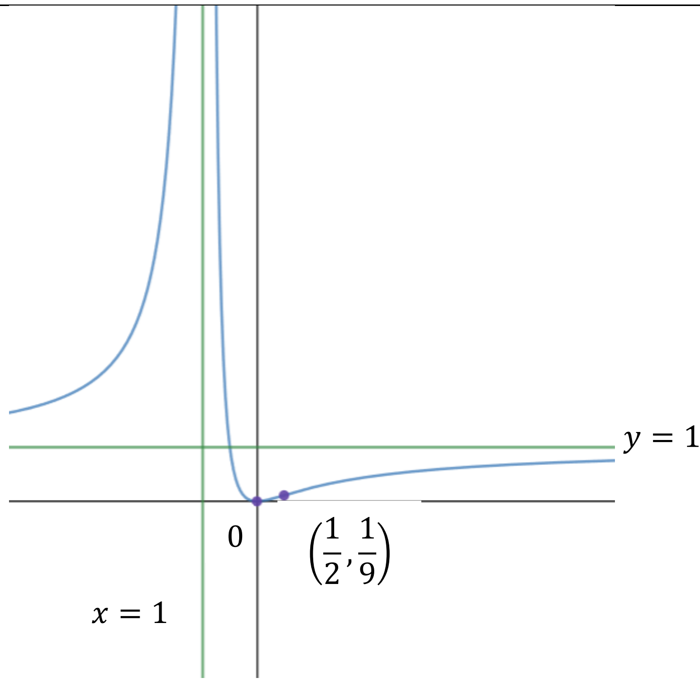
	Revision 1 – Answers	**40**
1	$3^4+4\cdot3^3\cdot\left(\frac{1}{2}x\right)+6\cdot3^2\cdot\left(\frac{1}{2}x\right)^2+4\cdot3^1\cdot\left(\frac{1}{2}x\right)^3+\left(\frac{1}{2}x\right)^4$ $=81+54x+\frac{27}{2}x^2+\frac{3}{2}x^3+\frac{1}{16}x^4$	
2(a) (b)	$y=\cos^{-1}u,\ \ u=4x \qquad \frac{dy}{dx}=-\frac{1}{\sqrt{1-u^2}}\times4=-\frac{4}{\sqrt{1-16x^2}}$ $\frac{dy}{dx}=(1)\times e^{x^2}+x\times2xe^{x^2}=e^{x^2}+2x^2e^{x^2}$	
3	$97=2+d(20-1),\ 95=19d,\ d=5$ $\sum_{k=0}^{50-1}(2+5k)=25(4+49\times5)=6225$	
4	$f(x)=x^2-a^2,$ $f(-x)=(-x)^2-a^2$ $=x^2-a^2=f(x)$ $f(-x)=f(x)$, thus the graph is even (reflects in the y-axis) 	
5	$\frac{d}{dx}(x^2)+\frac{d}{dx}(3xy)+\frac{d}{dx}(y^2)=\frac{d}{dx}(-1)$ $2x+3y+(3x)\frac{dy}{dx}+(2y)\frac{dy}{dx}=0$ $2x+3y=(-3x-2y)\frac{dy}{dx}$ $\frac{dy}{dx}=-\frac{2x+3y}{3x+2y}$ The gradient of the tangent is $-\frac{-2+3}{-3+2}=1,$ the equation of the tangent is $y=x+2$	

6	$$S_n = \sum_{r=1}^{n} 11 - 2k = \sum_{r=1}^{n} 11 + 2\sum_{r=1}^{k} k$$ $$= 11n - 2\left(\frac{n}{2}(n+1)\right)$$ $$= 11n - n^2 - n = 10n - n^2$$ $$10n - n^2 = 21, \quad n^2 - 10n + 21 = 0, \qquad (n-3)(n-7) = 0$$ For $S_n = 21 \quad n = 3$ and 7	
7	For $n = 1$, LHS $\frac{d}{dx}(xe^x) = e^x + xe^x = (1+x)e^x = (x+1)e^x$ RHS $(x+1)e^x$ so true for $n = 1$ Assume that the statement is true for $n = k\, \frac{d^k}{dx^k}(xe^x) = (x+k)e^x$ Now consider $n = k + 1$ $$\frac{d^{k-1}}{dx^{k+1}}(xe^x) = \frac{d}{dx}\left(\frac{d^k}{dx^k}(xe^x)\right)$$ $$= \frac{d}{dx}(x+k)e^x$$ $$= 1(e^x) + e^x(x+k)$$ $$= e^x(1+x+k)$$ $$= \big(x + (k+1)\big)e^x$$ So statement is true for $n = k + 1$. Since true for $n = 1, n = k$ and $n = k + 1$, then by induction this is true for all positive integers n	

8	(a) By algebraic long division $\frac{x^2}{(x+1)^2} = 1 - \frac{2x+1}{(x+1)^2} = 1 - \frac{2}{x+1} + \frac{1}{(x+1)^2}$ $A = 1, B = -2$ and $C = 1$ (b) (i) $y = \frac{x^2}{(x+1)^2}$ is undefined when $x = -1$, so has a vertical asymptote at $x = -1$ From part (a) so there is a horizontal asymptote at $y = 1$. Or $f(x) = 1 - \frac{2}{x+1} + \frac{1}{(x+1)^2}$ $tends\ to \to 1\ as\ x \to \infty$ (ii) $f'(x) = \frac{2x(x+1)^2 - 2x^2(x+1)}{(x+1)^4} = \frac{2x}{(x+1)^3}$ $f''(x) = \frac{2(x+1)^3 - 6x(x+1)^2}{(x+1)^6} = \frac{2-4x}{(x+1)^4}$ $f'(x) = 0$ when $2x = 0, x = 0$ thus there is a stationary point at $(0,0)$ $f''(0) > 0$ so this is a minimum stationary point. $f''(x) = 0$ when $2 - 4x = 0,\ x = \frac{1}{2}$, possible point of inflexion at $\left(\frac{1}{2}, \frac{1}{9}\right)$	

x	$\rightarrow$	½	$\rightarrow$
$f''(x)$	Positive	0	negative
concavity	up		down

Change of concavity so $\left(\frac{1}{2}, \frac{1}{9}\right)$ is a point of inflexion

	$y = 1$ 0 $\left(\frac{1}{2}, \frac{1}{9}\right)$ $x = 1$	