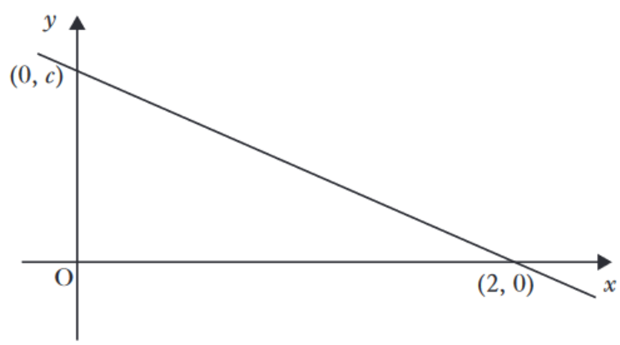
	Revision 2 for the A/B October Test	**40**
1	Differentiate (a) $x^3 \ln(4x+1)$ (b) $\frac{\cos x}{x}, \quad x \neq 0$	**3** **3**
2	Write down and simplify the general term in the expansion of $\left(x + \frac{1}{x}\right)^{10}$ Hence or otherwise obtain the term independent of x	**3** **2**
3	A geometric sequence has first term 80 and common ratio $\frac{1}{3}$ Calculate the sum to infinity of the associated geometric series	**2**
4	Express $\frac{3x^2-5x+2}{x^3+x}$ in partial fractions	**3**
5	Given that $y = 3^{2x}$, use logarithmic differentiation to obtain $\frac{dy}{dx}$ in terms of x	**3**
6	Part of the straight-line graph of a function $f(x)$ is shown  (a) Sketch the graph of the inverse function $f^{-1}(x)$, showing points of intersection with the axes. (b) Sketch the graph of the modulus function $\lvert f(x) \rvert$	 **4**

7	For the curves with the parametric equations $$x = 2 + t^2, \;\; y = t^3 - 2t$$ Obtain $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ as functions of t	**5**
8	Prove by induction that, for all positive integers n, $$\sum_{r=1}^{n} r(3r-1) = n^2(n+1)$$	**5**
9	The function f is defined by $f(x) = \frac{x-3}{x+2}, x \neq -2$ (a) Obtain algebraically the asymptotes of the graph of f (b) Prove that f has no stationary points (c) Does the graph of f have any points of inflexion? Justify your answer	 **3** **2** **2**

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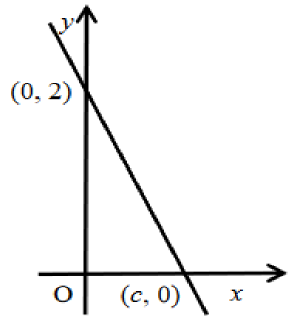
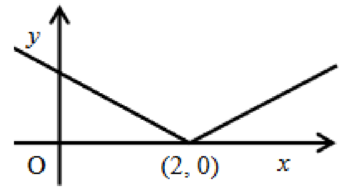
1(a) (b)	$\frac{dy}{dx} = 3x^2\ln(4x+1) + x^3 \times \frac{4}{4x+1} = 3x^2\ln(4x+1) + \frac{4x^3}{4x+1}$ $\frac{dy}{dx} = \frac{(-\sin x)x - 1(\cos x)}{x^2} = \frac{-x\sin x - \cos x}{x^2} = -\frac{x\sin x + \cos x}{x^2}$	
2	The general term is $\binom{10}{r}(x)^{10-r}\left(\frac{1}{x}\right)^r = \binom{10}{r}\frac{x^{10-r}(1)^r}{x^r} = \binom{10}{r}(1)^r x^{10-2r}$ The independent term occurs when $x^{10-2r} = x^0,\ r = 5 \quad \binom{10}{5}(1)^5 x^0 = 252$	
3	The sum to infinity is $\frac{80}{1-\frac{1}{3}} = 120$	
4	$\frac{3x^2-5x+2}{x^3+x} = \frac{3x^2-5x+2}{x(x^2+1)}$ $\frac{3x^2-5x+2}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$ $\frac{3x^2-5x+2}{x(x^2+1)} = \frac{A(x^2+1)}{x} + \frac{x(Bx+C)}{x^2+1}$ Using $x = 0$, $\quad \boldsymbol{2 = A}$, Using $x = 1$, $\quad 0 = A + B + C$, $\quad B + C = -4$ Using $x = -1$, $\quad 10 = 2A + B - C$, $\quad B - C = 6$, $\quad \boldsymbol{B = 1},\ \boldsymbol{C = -5}$ $\frac{3x^2-5x+2}{x(x^2+1)} = \frac{2}{x} + \frac{x-5}{x^2+1}$	
5	$y = 3^{2x}$, $\quad \ln y = \ln 3^{2x}$, $\quad \ln y = 2x\ln 3$ $\frac{d}{dx}(\ln y) = \frac{d}{dx}(2x\ln 3)$ $\frac{1}{y}\frac{dy}{dx} = 2\ln 3$ $\frac{dy}{dx} = (2\ln 3)y = 2\ln 3.3^{2x}$	

6	(a) Inverse function $y = f^{-1}(x)$ (b) Modulus function $y = \lvert f(x) \rvert$  	
7	$\frac{dy}{dt} = 3t^2 - 2, \quad \frac{dx}{dt} = 2t \qquad \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3t^2-2}{2t}$ $\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{3t^2-2}{2t}\right) \times \frac{1}{2t} = \left(\frac{6t\cdot 2t-2\cdot(3t^2-2)}{4t^2}\right) \times \frac{1}{2t} = \frac{3t^2+2}{4t^3}$	
8	For $n = 1$, LHS 2 RHS $1^2(1+1) = 2 = LHS$ so true for $n = 1$ Assume that the statement is true for $n = k$ $\sum_{r=1}^{k} r(3r-1) = k^2(k+1)$ Now consider $n = k + 1$ $\sum_{r=1}^{k+1} r(3r-1) = \sum_{r=1}^{k} r(3r-1) + f(k+1)$ $= k^2(k+1) + (k+1)(3(k+1)-1)$ $= (k+1)(k^2+3k+2)$ $= (k+1)(k+1)(k+2)$ $= (k+1)^2((k+1)+1)$ So statement is true for $n = k+1$. Since true for $n = 1, n = k$ and $n = k+1$, then by induction this is true for all positive integers n	

9(a)	A vertical asymptote exists at $x = -2$ Using algebraic long division $\frac{x-3}{x+2} = 1 - \frac{5}{x+2}$, so there is a horizontal asymptote at $y = 1$. Or $f(x) = \frac{x-3}{x+2} = \frac{1-\frac{3}{x}}{1+\frac{2}{x}}$ $this \rightarrow 1\ and\ x \rightarrow \infty$ showing a horizontal asymptote at $y = 1$	
(b)	$$f'(x) = \frac{(x+2)-(x-3)}{(x+2)^2} = \frac{5}{(x+2)^2}$$ $f'(x) > 0$ for all values of x, function is always increasing so there are no stationary values for $f(x)$	
(c)	$$f''(x) = -\frac{10}{(x+2)^3},\quad f''(x) \text{ is undefined } at\ x = -2$$ $f''(x) > 0$ for all values of x so there are no solutions to $f'(x) \neq 0$, $x = -2$ is an asymptote for $f(x)$ so there are no points of inflexion for $f(x)$ In the sketch below you can see that there are no changes in concavity