Solutions to Applications of Algebra and Calculus 1.3

1.
$$\sum_{r=1}^{20} 3r = 3\sum_{r=1}^{20} r = 3 \times \frac{20}{2}(20+1) = 630$$

$$\sum_{r=1}^{15}(4r-3) = 4\sum_{r=1}^{15} r - \sum_{r=1}^{15} 3 = 4 \times \frac{15}{2}(15+1) - 3 \times 15 = 435$$

$$\sum_{r=5}^{15}(2r+1) = \sum_{r=1}^{15}(2r+1) - \sum_{r=1}^{4}(2r+1)$$
$$= \left(2 \times \frac{15}{2}(15+1) + 15\right) - \left(2 \times \frac{4}{2}(4+1) + 4\right) = 231$$

2. $2 \times 1 - 1 = 1$
$1^2 = 1$ so true for *n* = 1.
Assume true for *n* = *k*, *i.e.*
$$\sum_{r=1}^{k}(2r-1) = k^2$$
Consider *n* = *k* + 1
$$\sum_{r=1}^{k+1}(2r-1) = \sum_{r=1}^{k}(2r-1) + 2(k+1) - 1$$
$$= k^2 + 2(k+1) - 1$$
$$= k^2 + 2k + 2 - 1$$
$$= k^2 + 2k + 1$$
$$= (k+1)^2$$
True for *n* = *k* + 1.

The formula is true for *n* = 1 and <u>if true</u> for *n* = *k*, <u>then true</u> for *n* = *k* + 1. Hence by induction, the formula is true $\forall\, n \in \boldsymbol{N}$.

(Note the <u>underlined</u> wording. This is essential to achieve full marks!)

3. $3 \times 1 - 1 = 2$
$\frac{1(3\times 1+1)}{2} = 2$ so true for *n* = 1.
Assume true for *n* = *k*, *i.e.*
$$\sum_{r=1}^{k}(3r-1) = \frac{k(3K+1)}{2}$$
Consider *n* = *k* + 1
$$\sum_{r=1}^{k+1}(3r-1) = \sum_{r=1}^{k}(3r-1) + 3(k+1) - 1$$
$$= \frac{k(3K+1)}{2} + 3(k+1) - 1$$

$$= \frac{k(3K+1)}{2} + 3k + 2$$
$$= \frac{3k^2+k}{2} + \frac{6k+4}{2}$$
$$= \frac{3k^2+7k+4}{2}$$
$$= \frac{(3k+4)(k+1)}{2}$$
$$= \frac{(k+1)(3(k+1)+1)}{2}$$

True for $n = k + 1$.

The formula is true for $n = 1$ and if true for $n = k$, then true for $n = k + 1$. Hence by induction, the formula is true $\forall\, n \in \boldsymbol{N}$.