| | **Revision 1 Answers** |
|---|---|
| 1 | $3^4 + 4\cdot 3^3\cdot\left(\frac{1}{2}x\right) + 6\cdot 3^2\cdot\left(\frac{1}{2}x\right)^2 + 4\cdot 3^1\cdot\left(\frac{1}{2}x\right)^3 + \left(\frac{1}{2}x\right)^4 = 81 + 54x + \frac{27}{2}x^2 + \frac{3}{2}x^3 + \frac{1}{16}x^4$ |
| 2 | $\frac{2x^3+5x^2-6x+11}{(x^2+2x-3)} = 2x+1+\frac{14-2x}{x^2+2x-3} = 2x+1+\frac{A}{x-1}+\frac{B}{x+3}$<br>Using $x = -3,\ 20 = -4B, B = -5$ Using $x = 1,\ 12 = 4A,\ A = 3$<br>$\frac{2x^3+5x^2-6x+11}{(x^2+2x-3)} = 2x+1+\frac{3}{x-1}-\frac{5}{x+3}$ |
| 3(a) | $y = \cos^{-1}u,\ u = 4x$ $\frac{dy}{dx} = -\frac{1}{\sqrt{1-u^2}}\times 4 = -\frac{4}{\sqrt{1-16x^2}}$ |
| (b) | $y = e^u, u = x^2-1$ $\frac{dy}{dx} = e^u\times 2x = 2x\,e^{x^2-1}$ |
| (c) | $\frac{dy}{dx} = \frac{1(x^2+1)-2x(2+x)}{(x^2+1)^2} = \frac{x^2+1-4x-2x^2}{(x^2+1)^2} = \frac{1-x^2-4x}{(x^2+1)^2}$ |
| 4 | $y = 3^{2x},\ \ln y = \ln 3^{2x},\ \ln y = 2x\ln 3$<br>$\frac{d}{dx}(\ln y) = \frac{d}{dx}(2x\ln 3)$<br>$\frac{1}{y}\frac{dy}{dx} = 2\ln 3$<br>$\frac{dy}{dx} = (2\ln 3)y = 2\ln 3.3^{2x}$ |
| 5 | $\frac{dy}{dt} = 3t^2-2,\ \frac{dx}{dt} = 2t$ $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{3t^2-2}{2t}$<br>$\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{3t^2-2}{2t}\right)\times\frac{1}{2t} = \left(\frac{6t\cdot 2t-2\cdot(3t^2-2)}{4t^2}\right)\times\frac{1}{2t} = \frac{3t^2+2}{4t^3}$ |
| 6 | $\frac{6}{1+i} = \frac{6}{1+i}\times\frac{1-i}{1-i} = \frac{6-6i}{1-i^2} = \frac{6-6i}{2} = 3-3i$<br>$\lvert z\rvert = \sqrt{(3)^2+(-3)^2} = 3\sqrt{2}$ $\arg(z) = \tan^{-1}\left(-\frac{3}{3}\right) = \left(-\frac{\pi}{4}\right) or -45°$ |
| 7 | $97 = 2+d(20-1),\ 95 = 19d,\ d = 5$ $\sum_{k=0}^{50-1}(2+5k) = 25(4+49\times 5) = 6225$ |

| | |
|---|---|
| 8 | $f(x) = e^{2x}$, $f(0) = 1$<br>$f'(x) = 2e^{2x}$ $f'(0) = 2$<br>$f''(x) = 4e^{2x}$ $f''(0) = 4$<br>$f'''(x) = 8e^{2x}$ $f'''(0) = 8$<br><br>$f(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$ and so on, hence $f(x) = 1 + 2x + 2x^2 + \frac{4}{3}x^3 + \cdots$ |
| 9 | $\int_1^3 \frac{1}{1+2x}\,dx = \left[\frac{1}{2}\ln(1+2x)\right]_1^3 = \frac{1}{2}\ln(\frac{7}{3}) = 0.4236\ldots$ |
| 10 | If $u = x^2 - 1 \rightarrow du = 2x\,dx$,<br><br>$\int 2x\cos(x^2-1)\,dx = \int \cos u\,du = \sin u + C = \sin(x^2-1) + C$ |