HIGHER MATHS COURSE

JUNE 2016

OUTLINE OF COURSE

JUNE -					
EXPRESSIONS & FUNCTIONS	Periods	RELATIONSHIPS & CALCULUS		APPLICATIONS	
1.1 Logs and Exponentials Simple Log & Exp equations Laws of logs and exp Applications	8	**1.1 Solving Algebraic Equations** Factorising polynomials Remainder Theorem Applications **(October test)**	4 Oct Hol 6	**(Relationships and Calculus)** **1.4 Integration** Integrating polynomials Integrating $(px + q)^n$ Integrate psin(qx + r) Differential equations Definite Integrals for polynomials & trig functions **REVISION & PRELIMS**	9
1.2 Trig Expressions Exact values, Radians Addition & Double Angle Form Wave Function	6 Summer Holidays 4	**1.2 Solving Trig Equations** Equations - degrees and radians Compound angle equations Equations involving identities Equations involving wave function	9		
1.3 Related Functions Graphs of related functions Composite Functions Inverse Functions	11	**1.3 Differentiation** Gradient function Differentiation of polynomials Differentiation of trig functions Chain Rule Equation of tangents Stationary points Curve sketching ** **Graphs of f ' (x) E&F 1.3**	16	**1.2Circles** Circle equation $(x-a)^2+(y-b)^2=r^2$ General equation of circle Tangency Intersecting circles	6
1.4 Vectors Unit vectors I, j, k Position vectors Internal division of line Collinearity Scalar Product & properties Perpendicular vectors	12	**(Applications)** **1.1 Equations of lines** Parallel and Perpendicular lines Collinearity Gradients and Angles Median, Altitude, Perpendicular Bisector and Angle bisector	6	**1.3 Sequences** Nth term formulae Recurrence Relations Limits of a sequence	4

The first column refers to broad skills areas.

The second column is the mandatory skills, knowledge and understanding given in the *Course Assessment Specification*. This includes a description of the Unit standard and the added value for the Course assessment. Skills which could be sampled to confirm that learners meet the minimum competence of the Assessment Standards are indicated by a diamond bullet point. Those skills marked by an arrow bullet point are considered to be beyond minimum competence for the Units, but are part of the added value for the Course Assessment.

The third column gives suggested learning and teaching contexts to exemplify possible approaches to learning and teaching. These also provide examples of where the skills could be used in activities.

Mathematics (Higher) Expressions and Functions Operational skills UNIT 1

Applying algebraic skills to logarithms and exponentials

Skill- 1.1 Logs and Exponential	**Description of Unit standard and added value**	**Learning and teaching contexts**
Manipulating algebraic expressions **L&L HIGHER Chapter 1** **Page 2 - 22** **PERIODS = 8**	• Simplifying an expression, using the laws of logarithms and exponents • Solving logarithmic and exponential equations • Using the laws of logarithms and exponents ➢ Solve for a and b equations of the following forms, given two pairs of corresponding values of x and y: ➢ $\log y = b\log x + \log a \quad y = ax^b$ and, $\log y = x\log b + \log a \quad y = ab^x$ ➢ Use a straight line graph to confirm relationships of the form $y = ax^b \quad y = ab^x$ ➢ Model mathematically situations involving the logarithmic or exponential function	Link logarithmic scale to science applications, eg decibel scale for sound, Richter scale of earthquake magnitude, astronomical scale of stellar brightness, acidity and pH in chemistry and biology. Note link between scientific notation and logs to base 10. Real-life contexts involving logarithmic and exponential characteristics, eg rate of growth of bacteria, calculations of money earned at various interest rates over time, decay rates of radioactive materials.

Applying trigonometric skills to manipulating expressions		
Skill- 1.2 Trigonometric Expressions	**Description of Unit standard and added value**	**Learning and teaching contexts**
Manipulating trigonometric expressions **L&L HIGHER Chapter 2** **Page 23 - 52** **PERIODS = 6 + 4 = 10**	Application of: • the addition or double angle formulae • trigonometric identities Convert $a\cos x\ b + \sin x$ to $k\cos(x \pm \alpha)$ or $k\sin(x \pm \alpha)$, $k>0$ • α in 1st quadrant ➢ α in any quadrant	Learners can be shown how formulae for $\cos(\alpha+\beta)$ and $\sin(\alpha+\beta)$ can be used to prove formulae for $\sin 2\alpha$, $\cos 2\alpha$, $\tan(\alpha + \beta)$. Emphasise the distinction between $\sin x°$ and $\sin x$ (degrees and radians). Learners should be given practice in applying the standard formulae, eg expand $\sin 3x$ or $\cos 4x$. Learners should be exposed to geometric problems which require the use of addition or double angle formulae. Example of use in science: a train of moving water waves of wavelength λ has a profile $y = H \sin\{2\pi[t/T - x/\lambda]\}$
TEXTBOOK ERRORS ***Chapter 2***	*<u>Where?</u>* *Page 51 Example 2.33*	*<u>What?</u>* *Write in what's missing* $\sin x - \sqrt{3}\cos x$ *and answer should read* $2\sin\left(x + \frac{5\pi}{3}\right)$

Applying algebraic and trigonometric skills to functions		
Skill- 1.3 Related Functions	**Description of Unit standard and added value**	**Learning and teaching contexts**
Identifying and sketching related functions **L&L HIGHER Chapter 3 p67 ex3.14 should be (3,3)** **Page 53 - 82** * After Ch 10 on Differentiation	• Identify and sketch a function after a transformation of the form $kf(x)$, $f(kx)$, $f(x) + k$, $f(x+k)$ or a combination of these ➢ Sketch $y = f'(x)$ given the graph of $y = f(x)$ * ➢ Sketch the inverse of a logarithmic or an exponential function ➢ Completing the square in a quadratic expression where the coefficient of x^2 is non-unitary	Use of graphic calculators here to explore various transformations. Learners should be able to recognise a function from its graph. Interpret formulae/equations for maximum/minimum values and when they occur.
TEXTBOOK ERRORS ***Chapter 3***	<u>*Where?*</u> *Page 58 Exercise 3A, Question 1(h)* *Page 65 Exercise 3C* *Page 69 Exercise 3D, Question 3(f)* *Page 73 Exercise 3E, Question 3(d)* *Page 78 Exercise 3F, Question 10(b)* *Page 78 Exercise 3F, Question 12(b)*	<u>*What?*</u> *Should read* ***y = 2 f (x) – 3*** *Q10 Should read* ***greater than 5*** *Q11 Should read* ***less than 16*** *Should read* $\boldsymbol{y = 3 + 6^{(x+1)}}$ *Should read* $\boldsymbol{y = \log_2 (x+3)}$ *Should read* ***y = 4cos x + 3sin x + 2*** *Should read* ***value of x between 0 and 180***
Determining composite and inverse functions **L&L HIGHER Chapter 4** **Page 83 - 91** **TOTAL PERIODS = 11**	• Determining a composite function given $f(x)$ and $g(x)$, where $f(x)$, $g(x)$ can be trigonometric, logarithmic, exponential or algebraic functions — including basic knowledge of domain and range • $f^{-1}(x)$ of functions ➢ Know and use the terms domain and range	$f(g(x))$ where $f(x)$ is a trigonometric function/logarithmic function and $g(x)$ is a polynomial. Learners should be aware that $f(g(x)) = x$ implies $f(x)$ and $g(x)$ are inverses.

Applying geometric skills to vectors		
Skill- 1.4 Vectors	**Description of Unit standard and added value**	**Learning and teaching contexts**
Determining vector connections **L&L HIGHER Chapter 5** **Page 92 - 113**	• Determining the resultant of vector pathways in three dimensions • Working with collinearity • Determining the coordinates of an internal division point of a line	Learners should work with vectors in both two and three dimensions. In order to 'show' collinearity, communication should include mention of parallel vectors and 'common point'. Distinction made between writing in coordinate and component form.
TEXTBOOK ERRORS ***Chapter 5***	*Where?* *Page 98 Exercise 3A, Question 4 (i) (b)* *Page 104 Exercise 3C, Question 3* *Page 111 Example 5.19a* *Page 112 Exercise 3A, Question 4 (b)*	*What?* *Should read* ***2p – 5q + r*** *Should read* ***The point Z divides the line XY in…*** *The points* ***A and C should inter change*** *Vector should be* $\begin{pmatrix} 6 \\ -4 \\ 2 \end{pmatrix}$

Working with vectors
L&L HIGHER Chapter 6
Page 114 - 131
 	
TOTAL PERIODS = 12
	• Evaluate a scalar product given suitable information and determining the angle between 2 vectors
	  ➢ Apply properties of the scalar product
	  ➢ Using unit vectors **i**, **j**, **k** as a basis
		Also, introduce the zero vector.
		Perpendicular and distributive properties of vectors should be investigated, eg If |***a***|, |***b***| ≠ 0 then ***a . b*** = 0 if and only if the directions of ***a*** and ***b*** are at right angles.
		Example of broader application: sketch a vector diagram of the three forces on a kite, when stationary: its weight, force from the wind (assume normal to centre of kite inclined facing the breeze) and its tethering string. These must sum to zero.

TEXTBOOK ERRORS ***Chapter 6***	*Where?* *Page 118 Example 6.3 Part (b)* *Page 129 Exercise 6E, Question 1*	*What?* *Should read* ***q = j + 6k*** *Include vector* ***c into the question!***

Mathematics (Higher) Relationships and Calculus Operational skills UNIT 2		
Applying algebraic skills to solve equations		
Skill- 1.1 Solving Algebraic Equations	**Description of Unit standard and added value**	**Learning and teaching contexts**
Solving algebraic equations **L&L HIGHER Chapter 7** **Page 136 - 169** **PERIODS = 4 + 6 = 10**	• Factorising a cubic polynomial expression with unitary x^3 coefficient ➢ Factorising a cubic or quartic polynomial expression with non-unitary coefficient of the highest power Solving polynomial equations: • Cubic with unitary x^3 coefficient ➢ Cubic or quartic with non-unitary coefficient of the highest power Discriminant: • Given the nature of the roots of an equation, use the discriminant to find an unknown ➢ Solve quadratic inequalities, $ax^2 + bx + c \geq 0$ (or ≤ 0) Intersection: ➢ Finding the coordinates of the point(s) of intersection of a straight line and a curve or of two curves	Strategies for factorising polynomials, ie synthetic division, inspection, algebraic long division. Factorising quadratic at National 5 or in previous learning led to solution(s) which learners can link to graph of function. Factorising polynomials beyond degree 2 allows extension of this concept. Identifying when an expression is not a polynomial (negative/fractional powers). Recognise repeated root is also a stationary point. Emphasise meaning of solving $f(x) = g(x)$. Learners should encounter the Remainder Theorem and how this leads to the fact that for a polynomial equation, $f(x) = 0$, if $(x - h)$ is a factor of $f(x)$, h is a root of the equation and vice versa. Learners' communication should include a statement such as 'since $f(h) = 0$' or 'since remainder is 0'. Learners should also experience divisors/factors of the form $(ax - b)$. As far as possible, solutions of algebraic equations should be linked to a graph of function(s), with learners encouraged to make such connections. (Use of graphic calculators/refer to diagram in question/ sketch diagrams to check solutions.)
TEXTBOOK ERRORS ***Chapter 7***	*Where?* *Page 142 Example 7.9*	*What?* *Change* ***(x - 1) to (x - 2)***

Applying trigonometric skills to solve equations		
Skill- 1.2 Solving Trig Equations	**Description of Unit standard and added value**	**Learning and teaching contexts**
Solving trigonometric equations **L&L HIGHER Chapter 8** **Page 170 - 201** **PERIODS = 9**	• Solve trigonometric equations in degrees, involving trigonometric formulae, in a given interval ➢ Solving trigonometric equations in degrees or radians, including those involving the wave function or trigonometric formulae or identities, in a given interval	Link to trigonometry of Expressions and Functions Unit. Real-life contexts should be used whenever possible. Solution of trigonometric equations could be introduced graphically. Recognise when a solution should be given in radians (eg $0 \le x \le \pi$). In the absence of a degree symbol, radians should be used. A possible application is the refraction of a thin light beam passing from air into glass. Its direction of travel is bent towards the line normal to the surface, according to Snell's law.
TEXTBOOK ERRORS ***Chapter 8***	*Where?* *Page 184 Exercise 8D Question 7* *Page 193 Exercise 8G Question 3 (b)*	*What?* *Should read* $t = \frac{1}{60}\left[394 + 128\cos\frac{2\pi(d+10)}{365}\right]$ *Should read* $\cos 4x + \cos 2x = 0$

Applying calculus skills of differentiation		
Skill- 1.3 Differentiation	**Description of Unit standard and added value**	**Learning and teaching contexts**
Differentiating functions **L&L HIGHER Chapter 9** **Page 202 - 237** Using differentiation to investigate the nature and properties of functions **L&L HIGHER Chapter 10** **Page 238 - 264** **PERIODS = 16**	• Differentiating an algebraic function which is, or can be simplified to, an expression in powers of x • Differentiating $k\sin x$, $k\cos x$ ➢ Differentiating a composite function using the chain rule • Determining the equation of a tangent to a curve at a given point by differentiation ➢ Determining where a function is strictly increasing/decreasing ➢ Sketching the graph of an algebraic function by determining stationary points and their nature as well as intersections with the axes and behaviour of $f(x)$ for large positive and negative values of x	Examples from science using the terms associated with rates of change, e.g acceleration, velocity. Learners should know that the gradient of a curve at a point is defined to be the gradient of the tangent to the curve at that point. Learners should know when a function is either strictly increasing, decreasing or has a stationary value, and the conditions for these. The second derivative or a detailed nature table can be used. Stationary points should include horizontal points of inflexion.
TEXTBOOK ERRORS ***Chapter 9***	*Where?* *Page 219 Exercise 9C Question 6c* *Page 229 Exercise 9F Question 1*	*What?* *Should read* $f'(x) = x^2$ *Should read* $f'(x) = \lim_{h \to 0} \frac{\sin x(\cos h - 1) + \sin h \cos x}{h}$
TEXTBOOK ERRORS ***Chapter 10***	*Where?* *Page 242 Exercise 10A Question 11* *Page 256 Exercise 10C Question 1 (b) and (c)* *Page 257 Exercise 10C Question 7*	*What?* *Should read* $f(x) = 2\cos(2x + \frac{\pi}{2}),\ 0 \le x \le \frac{\pi}{2}$ **Both questions are the same** *Should read* $y = 1 + 3x + \frac{12}{x^3}$

Applying calculus skills of integration		
Skill- 1.4 Integration	**Description of Unit standard and added value**	**Learning and teaching contexts**
Integrating functions **L&L HIGHER Chapter 11** **Page 265 - 282**	• Integrating an algebraic function which is, or can be, simplified to an expression of powers of x • Integrating functions of the form $f(x)$ $(x+q)^n$ n not equal to –1 • Integrating functions of the form $f(x) = p\cos x$ and $f(x) = p\sin x$ ➢ Integrating functions of the form $f(x) = (px+q)^n$ n not equal to –1 ➢ Integrating functions of the form ➢ $f(x) = p\cos(qx+r)$ and $p\sin(qx+r)$ ➢ Solving differential equations of the form $dy/dx = f(x)$	Know the meaning of the terms integral, integrate, constant of integration, definite integral, limits of integration, indefinite integral, area under a curve. Know that if $f(x) = F'(x)$ then $\int_a^b f(x)\,dx = F(b) - F(a)$ and $\int f(x)\,dx = F(x) + C$ where C is the constant of integration. Could be introduced by anti-differentiation. Learners should experience integration of $\cos^2 x$ and $\sin^2 x$ using $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$ and $\sin^2 x$
Using integration to calculate definite integrals **L&L HIGHER Chapter 12** **Page 283 – 290** **TOTAL PERIODS = 9**	• Calculating definite integrals of polynomial functions with integer limits ➢ Calculating definite integrals of functions with limits which are integers, radians, surds or fractions	Extend to area beneath the curve between the limits.

Mathematics (Higher) Applications Operational skills UNIT 3		
Applying algebraic skills to rectilinear shapes		
Skill- 1.1 Equations of Lines	**Description of Unit standard and added value**	**Learning and teaching contexts**
Applying algebraic skills to rectilinear shapes **L&L HIGHER Chapter 13** **Page 293 - 305** **PERIODS = 6**	• Finding the equation of a line parallel to and a line perpendicular to a given line • Using $m = \tan\theta$ to calculate a gradient or angle ➢ Using properties of medians, altitudes and perpendicular bisectors in problems involving the equation of a line and intersection of lines ➢ Determine whether or not two lines are perpendicular	Emphasise the 'gradient properties' of $m_1 = m_2$ and $m_1 m_2 = -1$. Use practical contexts for triangle work where possible. Emphasise differences in median, altitude etc. Perhaps investigate properties and intersections. Avoid approximating gradients to decimals. Knowledge of the basic properties of triangles and quadrilaterals would be useful. In order to 'show' collinearity, statement should include mention of 'common point', eg since $m_{AB} = m_{BC}$ and B is a common point. Understanding of terms such as orthocentre, circumcentre and concurrency.
Applying algebraic skills to circles		
Skill- 1.2 Circles	**Description of Unit standard and added value**	**Learning and teaching contexts**
Applying algebraic skills to circles **L&L HIGHER Chapter 14** **Page 306 - 317** **PERIODS = 6**	• Determining and using the equation of a circle • Using properties of tangency in the solution of a problem ➢ Determining the intersection of circles or a line and a circle	Link to work on discriminant (one point of contact). Develop equation of circle (centre the origin) from Pythagoras, and extend this to circle with centre (a,b) or relate to transformations. Demonstrate application of discriminant. Learners made aware of different ways in which more than one circle can be positioned, eg intersecting at one/two/no points, sharing same centre (concentric), one circle inside another. Practice in applying knowledge of geometric properties of circles in finding related points (eg stepping out method). Solutions should not be obtained from scale drawings.

Applying algebraic skills to sequences		
Skill- 1.3 Sequences	**Description of Unit standard and added value**	**Learning and teaching contexts**
Modelling situations using sequences **L&L HIGHER Chapter 15** **Page 318 - 327** **PERIODS = 4**	• Determining a recurrence relation from given information and using it to calculate a required term • Finding and interpreting the limit of a sequence, where it exists	Where possible, use examples from real-life situations such as where concentrations of chemicals/medicines are important.

Applying calculus skills to optimisation and area		
Skill- 1.4 Application of Calculus	**Description of Unit standard and added value**	**Learning and teaching contexts**
Applying differential calculus **L&L HIGHER Chapter 16** **Page 328 – 345** **P333 watch for Ex 16.2 as the function is times by -1 so the nature table is not what pupils get on working.**	• Determining the optimal solution for a given problem ➢ Determine the greatest/least values of a function on a closed interval ➢ Solving problems using rate of change	Max/min problems applied in context, eg minimum amount of card for creating a box, maximum output from machines. Rate of change linked to science. Optimisation in science, eg an aeroplane cruising at speed v at a steady height, has to use power to push air downwards to counter the force of gravity, and to overcome air resistance to sustain its speed. The energy cost per km of travel is given approximately by: $E = Av^2 + Bv^2$ (A and B depend on the size and weight of the plane). At the optimum speed dE/dv$=0$, thus get an expression for dv v_{opt} in terms of A and B.
Applying integral calculus **L&L HIGHER Chapter 17** **Page 346 - 369** **TOTAL PERIODS = 6**	• Finding the area between a curve and the x-axis • Finding the area between a straight line and a curve or two curves ➢ Determine and use a function from a given rate of change and initial conditions $r = 6.4 \text{ x} 10^6 \text{m}$ (the radius of the earth) to infinity.	Develop from Relationships and Calculus Unit. Use of graphical calculators for an investigative approach. Area between curves by subtraction of individual areas — use of diagrams, graphing packages. Reducing area to be determined to smaller components in order to estimate segment of area between curve and $x$axis. Use of area formulae (triangle/rectangle) in solving such problems. A practical application of the integral of $1/x^2$ is to calculate the energy required to lift an object from the earth's surface into space. The work energy required is $E = Fdr$ where F is the force due to the earth's gravity and r is the distance from the centre of the earth. For a 1 kg object $E = - (GM / r^2)dr$ where M is the mass of the earth and G is the universal gravitational constant. $GM = 4.0 \text{ x } 10^{14} \text{ m}^3\text{s}^{-2}$. The integration extends from

Reasoning skills for all units		
Interpreting a situation where mathematics can be used and identifying a valid strategy	Can be attached to a skill of Outcome 1 to require analysis of a situation.	This should be a mathematical or real-life context problem in which some analysis is required. The learner should be required to choose an appropriate strategy and employ mathematics to the situation.
Explaining a solution and, where appropriate, relating it to context	Can be attached to a skill of Outcome 1 to require explanation of the solution given.	The learner should be required to give meaning to the determined solution in everyday language.
Additional Information		

Symbols, terms and sets:

the symbols: ∈, ∉, { } the terms: set, subset, empty set,

member, element the conventions for representing

sets, namely: $\mathbb{N}$, the set of natural numbers, {1, 2, 3, ...}

W, the set of whole numbers, {0, 1, 2, 3, ...}

$\mathbb{Z}$, the set of integers

$\mathbb{Q}$, the set of rational numbers

$\mathbb{R}$, the set of real numbers

The content listed above is not examinable but learners are expected to be able to understand its use.