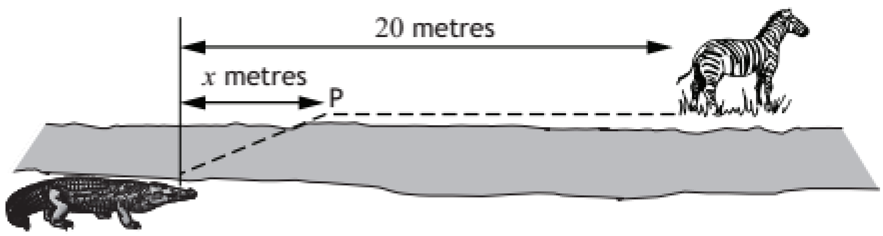
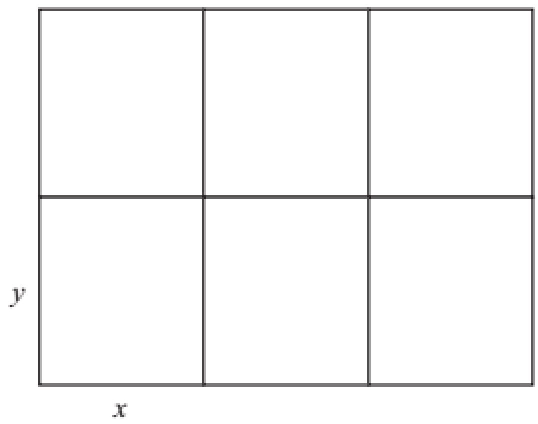
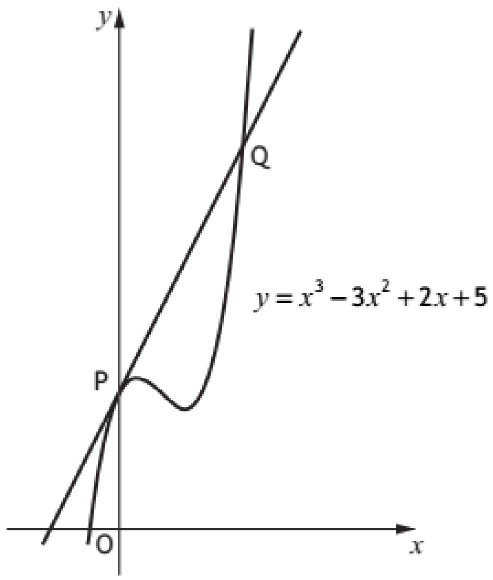
Y	Q	P	DIFFERENTIATION	
15	2	1	Find the equation of the tangent to the curve $y = 2x^3 + 3$ at the point where $x = -2$.	4
15	7	1	A function f is defined on a suitable domain by $f(x) = \sqrt{x}\left(3x - \frac{2}{x\sqrt{x}}\right)$. Find $f'(4)$.	4
15	8	2	A crocodile is stalking prey located 20 metres further upstream on the opposite bank of a river. Crocodiles travel at different speeds on land and in water. The time taken for the crocodile to reach its prey can be minimised if it swims to a particular point, P, x metres upstream on the other side of the river as shown in the diagram.  The time taken, T, measured in tenths of a second, is given by $T(x) = 5\sqrt{36 + x^2} + 4(20 - x)$ (a) (i) Calculate the time taken if the crocodile does not travel on land. **1** (ii) Calculate the time taken if the crocodile swims the shortest distance possible. **1** (b) Between these two extremes there is one value of x which minimises the time taken. Find this value of x and hence calculate the minimum possible time. **8**	
16	2	1	Given that $y = 12x^3 + 8\sqrt{x}$, where $x > 0$, find $\frac{dy}{dx}$.	3
16	9	1	(a) Find the x-coordinates of the stationary points on the graph with equation $y = f(x)$, where $f(x) = x^3 + 3x^2 - 24x$. **4** (b) Hence determine the range of values of x for which the function f is strictly increasing. **2**	

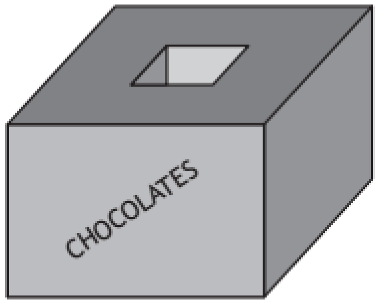
16	7	2	A council is setting aside an area of land to create six fenced plots where local residents can grow their own food. Each plot will be a rectangle measuring x metres by y metres as shown in the diagram.  (a) The area of land being set aside is $108\,\text{m}^2$. Show that the total length of fencing, L metres, is given by $$L(x) = 9x + \frac{144}{x}.$$	3
			(b) Find the value of x that minimises the length of fencing required.	6
16	10	2	(a) Given that $y = \left(x^2 + 7\right)^{\frac{1}{2}}$, find $\dfrac{dy}{dx}$.	2
16	11	2	(a) Show that $\sin 2x \tan x = 1 - \cos 2x$, where $\dfrac{\pi}{2} < x < \dfrac{3\pi}{2}$.	4
			(b) Given that $f(x) = \sin 2x \tan x$, find $f'(x)$.	2
17	3	1	Given $y = (4x-1)^{12}$, find $\dfrac{dy}{dx}$.	2
17	8	1	Calculate the rate of change of $d(t) = \dfrac{1}{2t}$, $t \neq 0$, when $t = 5$.	3
17	4	2	(a) Express $3x^2 + 24x + 50$ in the form $a(x+b)^2 + c$.	3
			(b) Given that $f(x) = x^3 + 12x^2 + 50x - 11$, find $f'(x)$.	2
			(c) Hence, or otherwise, explain why the curve with equation $y = f(x)$ is strictly increasing for all values of x.	2

17	7	2	(a) Find the x-coordinate of the stationary point on the curve with equation $y = 6x - 2\sqrt{x^3}$. **4** (b) Hence, determine the greatest and least values of y in the interval $1 \leq x \leq 9$. **3**
17	11	2	(a) Show that $\frac{\sin 2x}{2\cos x} - \sin x \cos^2 x = \sin^3 x$, where $0 < x < \frac{\pi}{2}$. **3** (b) Hence, differentiate $\frac{\sin 2x}{2\cos x} - \sin x \cos^2 x$, where $0 < x < \frac{\pi}{2}$. **3**
18	3	1	A function, f, is defined on the set of real numbers by $f(x) = x^3 - 7x - 6$. Determine whether f is increasing or decreasing when $x = 2$. **3**
18	7	1	The curve with equation $y = x^3 - 3x^2 + 2x + 5$ is shown on the diagram.  (a) Write down the coordinates of P, the point where the curve crosses the y-axis. **1** (b) Determine the equation of the tangent to the curve at P. **3** (c) Find the coordinates of Q, the point where this tangent meets the curve again. **4**
18	3	2	A function, f, is defined on the set of real numbers by $f(x) = x^3 - 7x - 6$. Determine whether f is increasing or decreasing when $x = 2$. **3**

18	9	2	A sector with a particular fixed area has radius x cm. The perimeter, P cm, of the sector is given by $$P = 2x + \frac{128}{x}.$$ Find the minimum value of P.	6
19	1	1	Find the x-coordinates of the stationary points on the curve with equation $y = \frac{1}{2}x^4 - 2x^3 + 6$.	4
19	6	1	Given that $y = \frac{1}{(1-3x)^5}$, $x \neq \frac{1}{3}$, find $\frac{dy}{dx}$.	3

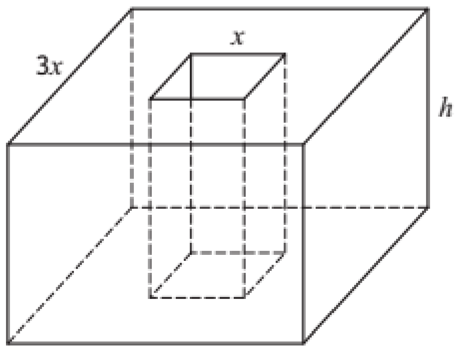
19 11 2

A manufacturer of chocolates is launching a new product in novelty shaped cardboard boxes.



The box is a cuboid with a cuboid shaped tunnel through it.

- The height of the box is h centimetres
- The top of the box is a square of side $3x$ centimetres
- The end of the tunnel is a square of side x centimetres
- The volume of the box is $2000\,\text{cm}^3$



(a) Show that the total surface area, A cm^2, of the box is given by

$$A = 16x^2 + \frac{4000}{x}.$$ **3**

(b) To minimise the cost of production, the surface area, A, of the box should be as small as possible.

Find the minimum value of A. **6**

22 4 1

Differentiate $y = \sqrt{x^3} - 2x^{-1}$, where $x > 0$. **3**

22 12 1

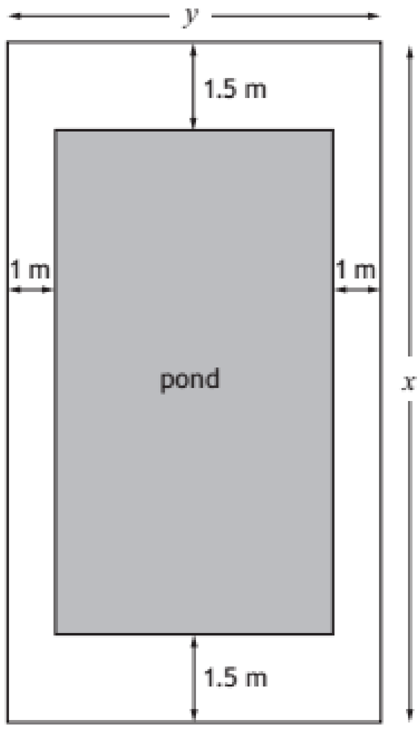
Given that $f(x) = 4\sin\left(3x - \frac{\pi}{3}\right)$, evaluate $f'\left(\frac{\pi}{6}\right)$. **3**

22	8	2

A rectangular plot consists of a rectangular pond surrounded by a path.

The length and breadth of the plot are x metres and y metres respectively.

The path is 1.5 metres wide at the ends of the pond and 1 metre wide along the other sides as shown.



The total area of the **pond and path together** is 150 square metres.

(a) Show that the area of the pond, A square metres, is given by

$$A(x) = 156 - 2x - \frac{450}{x}.$$ **3**

(b) Determine the maximum area of the pond. **6**