

Recurrence Relations		
1	<p>A sequence is defined by the recurrence relation <math>U_{n+1} = \frac{1}{4}U_n + 11</math></p> <p>(a) If <math>U_2 = 8</math>, find <math>U_3</math></p> <p>(b) Explain why this sequence approaches a limit as <math>n \rightarrow \infty</math></p> <p>(c) Calculate this limit</p>	<p>1</p> <p>1</p> <p>2</p>
2	<p>Two sequences are defined by the recurrence relations  <math>U_{n+1} = 3U_n - 0.4</math> with <math>U_0 = 1</math>, and <math>V_{n+1} = 0.3U_n + 4</math> with <math>V_0 = 1</math></p> <p>(a) Explain why only one of these sequences approaches a limit as <math>n \rightarrow \infty</math></p> <p>(b) Find algebraically the exact value of this limit</p> <p>(c) For the other sequence find the smallest value of <math>n</math> for which the <math>n</math>th term exceeds 1000</p>	<p>1</p> <p>2</p> <p>1</p>
3	<p>A sequence is defined by the recurrence relation  <math>U_{n+1} = 0.9U_n + 2</math> with <math>U_0 = 3</math></p> <p>(a) Calculate the value of <math>U_2</math></p> <p>(b) What is the smallest value of <math>n</math> for which <math>U_n &gt; 10</math></p> <p>(c) Find the limit of the sequence as <math>n \rightarrow \infty</math></p>	<p>4</p>
4	<p>A sequence is defined by the recurrence relation  <math>U_{n+1} = 0.5U_n + 3</math>     <math>U_{n+1} = aU_n + b</math></p> <p>(a) If <math>U_2 = 8</math>, <math>U_3 = 7</math> and <math>U_4 = 6.5</math>, find values for <math>a</math> and <math>b</math></p> <p>(b) Find the limit of this sequence as <math>n \rightarrow \infty</math></p>	<p>3</p> <p>2</p>
5	<p>A recurrence relation is defined by the formula <math>U_{n+1} = 0.6U_n + 24</math></p> <p>(a) Establish the limit of this sequence</p> <p>(b) Given that <math>U_1</math> is exactly half of this limit, find <math>U_0</math>, the initial value of this sequence</p> <p>(c) A second recurrence relation in the form <math>V_{n+1} = aU_n + b</math> has the same limit as the sequence above and is such that <math>b = 90a</math>.</p> <p>Find values for <math>a</math> and <math>b</math> for this second sequence</p>	<p>2</p> <p>2</p> <p>3</p>

Recurrence - Answers		
1	Calculate value State condition for limit Find Limit	$U_3 = 13$ A limit exists as $-1 < \frac{1}{4} < 1$ $L = \frac{11}{1-0.25} = \frac{44}{3}$
2(a)	State condition for limit	A limit exists for $V_n$ as $-1 < 0.3 < 1$
(b)	Find limit	$L = \frac{4}{1-0.3} = \frac{40}{7}$
(c)	Find $n$	$U_7 = 1749.8$
3(a)	Calculate value	$U_2 = 6.23$
(b)	Find $n$	$U_6 = 10.965503$
(c)	State condition for limit Find Limit	A limit exists as $-1 < 0.9 < 1$ $L = \frac{2}{1-0.9} = 20$
4(a)	Form simultaneous equations Solve for $a$ Find $b$	$U_3 = aU_2 + b \rightarrow 7 = 8a + b$ $U_4 = aU_3 + b \rightarrow 6.5 = 7a + b$ $0.5 = a$ $b = 3$
(b)	State condition for limit Find Limit	A limit exists as $-1 < 0.5 < 1$ $L = \frac{3}{1-0.5} = 6$
5(a)	Find Limit	$L = \frac{24}{1-0.6} = 60$
(b)	Calculate values for $U_1$ and $U_0$	$U_1 = 30, U_0 = 10$
(c)	Find limit in terms of $a$ and $b$ Substitute for $b$ and solve for $a$ Solve for $b$	$L = \frac{b}{1-a} = 60$ $60 = \frac{90a}{1-a}, a = 0.4$ $b = 36, V_{n+1} = 0.4U_n + 36$