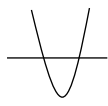
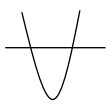


	Properties of the discriminant $b^2 - 4ac$	
1	Find the range of values of p such that $x^2 - 2x + 3 - p = 0$ has no real roots	3
2	Prove that the roots of the equation $2x^2 + px - 3 = 0$ are real for all values of p	3
3	Find the range of values of p such that $x^2 + px + 4 = 0$ has no real roots	4
4	Show algebraically that there is only one real root for $2x^3 - 3x^2 + 2x - 8 = 0$	5
5	Functions f and g are defined on suitable domains by $f(x) = x(x - 1) + q$ and $g(x) = x + 3$ (a) Find an expression for $f(g(x))$ (b) Hence, find the value of q such that the equation $f(g(x)) = 0$ has real roots	2 4
6	Given that the equation $x^2 + y^2 - 2px - 4py + 2p + 3 = 0$ Represents a circle, determine the range of values of p	5
7	(a) Write the equation $\cos 2\theta + 8\cos\theta + 9 = 0$ in terms of $\cos\theta$ and show that for $\cos\theta$ it has equal roots (b) Show that there are no real roots for θ	3 2

Properties of the discriminant – Answers	
1	<p>Statement for no real roots $b^2 - 4ac < 0$, Form an inequality $(-2)^2 - 4(1)(3 - p) < 0$, $4p - 8 < 0$ Solve for p $p < 2$</p>
2	<p>Statement for real roots $b^2 - 4ac \geq 0$, Identify the discriminant $(p)^2 - 4(2)(-3)$, $p^2 + 24$ Proof p^2 is positive for all values of p so roots are always real</p>
3	<p>Statement for no real roots $b^2 - 4ac < 0$, Form an inequality $(p)^2 - 4(1)(4) < 0$, $p^2 - 16 < 0$ Factorise and solve for $= 0$ $(p + 4)(p - 4) = 0$, $p = \pm 4$ Correct range $p < -4$ and $p > 4$</p> 
4	<p>Establish a real root using substitution or synthetic division</p> $\begin{array}{r rrrr} 2 & 2 & -3 & 2 & -8 \\ & & 0 & 4 & 2 & 8 \\ \hline & 2 & 1 & 4 & 0 \end{array}$ <p>No remainder, $x = 2$ is a root, $(x - 2)$ is a factor $2x^3 - 3x^2 + 2x - 8 = (x - 2)(2x^2 + x + 4)$</p> <p>For $2x^2 + x + 4 = 0$, $b^2 - 4ac = 1 - 16 = -15$ $b^2 - 4ac < 0$ means no real roots Hence there is only one real root at $x = 2$</p> <p>Proof</p>
5	<p>(a) Substitute $g(x)$ into $f(x)$ Simplify $f(x + 3) = (x + 3)(x + 3 - 1) + q$ $x^2 + 5x + 6 + q$</p> <p>(b) Statement for real roots $b^2 - 4ac \geq 0$, Form an inequality $(5)^2 - 4(1)(6 + q) \geq 0$ Simplify the inequality $1 - 4q \geq 0$ Solve for p $1 \geq 4q$, $q \leq \frac{1}{4}$</p>
6	<p>$x^2 + y^2 - 2px - 4py + 2p + 3 = 0$</p> <p>Find values for g, f and c $g = p$, $f = 2$ and $c = 2p + 3$</p> <p>Use radius $r = \sqrt{g^2 + f^2 - c}$ $g^2 + f^2 - c > 0$</p> <p>Form an inequality $p^2 + 4p^2 - (2p + 3) > 0$</p> <p>(b) Simplify, factorise $5p^2 - 2p - 3 > 0$, $(5p + 3)(p - 1) > 0$ Correct range For $(5p + 3)(p - 1) = 0$, $p = -3/5$ and $p = 1$</p>  <p>For $(5p + 3)(p - 1) > 0$, $p < -3/5$ and $p > 1$</p>
7(a)	<p>Show substitution for $\cos 2\theta$ $2\cos^2\theta - 1 + 8\cos\theta + 9 = 0$ Collect terms $2\cos^2\theta + 8\cos\theta + 8 = 0$ Identify the discriminant $b^2 - 4ac = (8)^2 - 4(2)(8) = 0$ hence equal roots</p>
(b)	<p>Factorise $2\cos^2\theta + 8\cos\theta + 8 = 0$# $2(\cos\theta + 2)^2 = 0$ Proof $\cos\theta + 2 = 0$, $\cos\theta = -2$ has no solutions</p>