	Properties of the discriminant $b^2 - 4ac$	
1	Find the range of values of p such that $x^2 - 2x + 3 - p = 0$ has no real roots	3
2	Prove that the roots of the equation $2x^2 + px - 3 = 0$ are real for all values of p	3
3	Find the range of values of p such that $x^2 + px + 4 = 0$ has no real roots	4
4	Show algebraically that there is only one real root for $2x^3 - 3x^2 + 2x - 8 = 0$	5
5	Functions f and g are defined on suitable domains by $f(x) = x(x - 1) + q$ and $g(x) = x + 3$ (a) Find an expression for $f(g(x))$ (b) Hence, find the value of q such that the equation $f(g(x)) = 0$ has real roots	 2 4
6	Given that the equation $x^2 + y^2 - 2px - 4py + 2p + 3 = 0$ Represents a circle, determine the range of values of p	5
7	(a) Write the equation $\cos 2\theta + 8\cos\theta + 9 = 0$ in terms of $\cos\theta$ and show that for $\cos\theta$ it has equal roots (b) Show that there are no real roots for θ	3 2

	Properties of the discriminant – Answers	
1	Statement	for no real roots $b^2 - 4ac < 0$,
	Form an inequality	$(-2)^2 - 4(1)(3 - p) < 0$, $4p - 8 < 0$
	Solve for p	$p < 2$
2	Statement	for real roots $b^2 - 4ac \geq 0$,
	Identify the discriminant	$(p)^2 - 4(2)(-3)$, $p^2 + 24$
	Proof	p^2 is positive for all values of p so roots are always real
3	Statement	for no real roots $b^2 - 4ac < 0$,
	Form an inequality	$(p)^2 - 4(1)(4) < 0$, $p^2 - 16 < 0$
	Factorise and solve for = 0	$(p + 4)(p - 4) = 0$, $p = \pm 4$
	Correct range	$p < -4$ and $p > 4$
4	Establish a real root using substitution or synthetic division	
	2 | 2 -3 2 -8 0 4 2 8 2 1 4 |0	No remainder, $x = 2$ is a root, $(x - 2)$ is a factor $2x^3 - 3x^2 + 2x - 8 = (x - 2)(2x^2 + x + 4)$
		For $2x^2 + x + 4 = 0$, $b^2 - 4ac = 1 - 16 = -15$
	Proof	$b^2 - 4ac < 0$ means no real roots Hence there is only one real root at $x = 2$
5		
(a)	Substitute $g(x)$ into $f(x)$	$f(x + 3) = (x + 3)(x + 3 - 1) + q$
	Simplify	$x^2 + 5x + 6 + q$
(b)	Statement	for real roots $b^2 - 4ac \geq 0$,
	Form an inequality	$(5)^2 - 4(1)(6 + q) \geq 0$
	Simplify the inequality	$1 - 4q \geq 0$
	Solve for p	$\mathbf{1 \geq 4q, \quad q \leq \frac{1}{4}}$
6	$x^2 + y^2 - 2px - 4py + 2p + 3 = 0$	
	Find values for g, f and c	$g = p$, $f = 2$ and $c = 2p + 3$
	Use radius $r = \sqrt{g^2 + f^2 - c}$	$g^2 + f^2 - c > 0$
	Form an inequality	$p^2 + 4p^2 - (2p + 3) > 0$
(b)	Simplify, factorise	$5p^2 - 2p - 3 > 0$, $(5p + 3)(p - 1) > 0$
	Correct range	For $(5p + 3)(p - 1) = 0$, $p = -3/5$ and $p = 1$ For $(5p + 3)(p - 1) > 0$, **$p < -3/5$ and $p > 1$**
7(a)	Show substitution for $\cos 2\theta$	$2\cos^2\theta - 1 + 8\cos\theta + 9 = 0$
	Collect terms	$2\cos^2\theta + 8\cos\theta + 8 = 0$
	Identify the discriminant	$b^2 - 4ac = (8)^2 - 4(2)(8) = 0$ hence equal roots
(b)	Factorise	$2\cos^2\theta + 8\cos\theta + 8 = 0$# $2(\cos\theta + 2)^2 = 0$
	Proof	$\cos\theta + 2 = 0$, $\cos\theta = -2$ has no solutions