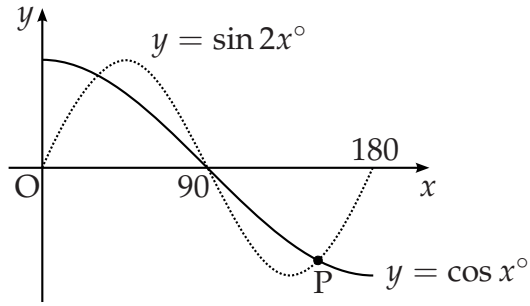
double angle

[SQA] 1. Solve the equation $3\cos 2x^\circ + \cos x^\circ = -1$ in the interval $0 \leq x \leq 360$. **5**

[SQA] 2. (*a*) Solve the equation $\sin 2x^\circ - \cos x^\circ = 0$ in the interval $0 \leq x \leq 180$. **4**

(*b*) The diagram shows parts of two trigonometric graphs, $y = \sin 2x^\circ$ and $y = \cos x^\circ$.

Use your solutions in (*a*) to write down the coordinates of the point P. **1**



[SQA] 3. Functions f and g are defined on suitable domains by $f(x) = \sin(x^\circ)$ and $g(x) = 2x$.

(*a*) Find expressions for:

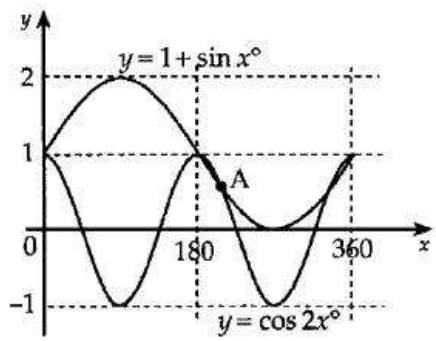
(i) $f(g(x))$;

(ii) $g(f(x))$. **2**

(*b*) Solve $2f(g(x)) = g(f(x))$ for $0 \leq x \leq 360$. **5**

[SQA] 4. The diagram shows two curves with equations $y = \cos 2x^\circ$ and $y = 1 + \sin x^\circ$ where $0 \leq x \leq 360$.

Find the x-coordinate of the point of intersection at A. **4**



[SQA] 5. Solve the equation $\cos 2x^\circ + 5\cos x^\circ - 2 = 0$, $0 \leq x < 360$. **5**

[SQA] 6. Solve the equation $\cos 2x^\circ + \cos x^\circ = 0$, $0 \leq x < 360$. **5**

[SQA] 7. Solve the equation $\sin 2x^\circ + \sin x^\circ = 0$, $0 \leq x < 360$. **5**

[SQA] 8.

(*a*) Show that $2\cos 2x^\circ - \cos^2 x^\circ = 1 - 3\sin^2 x^\circ$. **2**

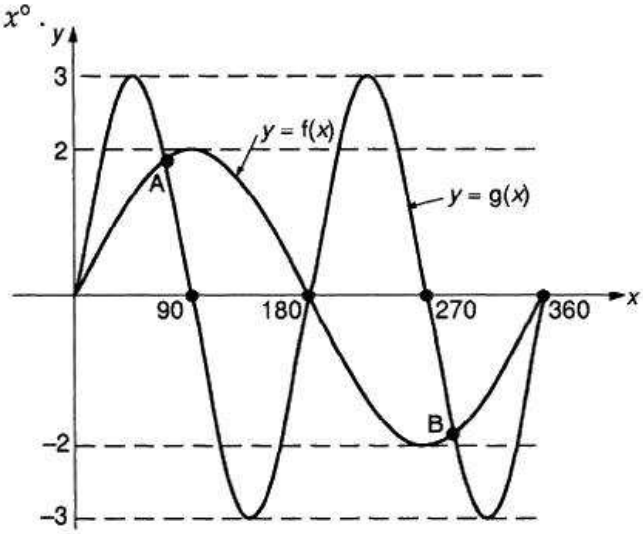
(*b*) Hence solve the equation $2\cos 2x^\circ - \cos^2 x^\circ = 2\sin x^\circ$ in the interval $0 \leq x < 360$. **4**

[SQA] 9. (*a*) Solve the equation $3\sin 2x^\circ = 2\sin x^\circ$ for $0 \leq x \leq 360$ **(4)**

(*b*) The diagram below shows parts of the graphs of sine functions f and g.
State expressions for $f(x)$ and $g(x)$. **(1)**

(*c*) Use your answers to part (*a*) to find the co-ordinates of A and B. **(2)**

(*d*) Hence state the values of x in the interval $0 \leq x \leq 360$ for which $3\sin 2x^\circ < 2\sin x^\circ$. **(3)**

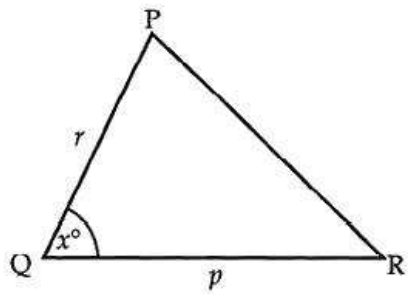


[SQA] 10. The diagram shows an isosceles triangle PQR in which PR = QR and angle PQR = x°.

(*a*) Show that $\frac{\sin x^\circ}{p} = \frac{\sin 2x^\circ}{r}$. **(3)**

(*b*) (i) State the value of x° when $p = r$.

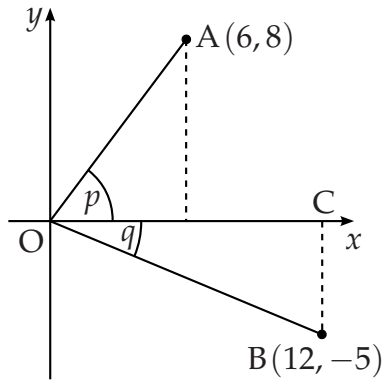
(ii) Using the fact that $p = r$, solve the equation in (*a*) above, to justify your stated value of x°. **(5)**



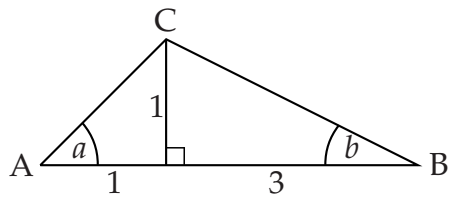
11. Solve $2\cos 2x - 5\cos x - 4 = 0$ for $0 \leq x < 2\pi$. **5**

[SQA] 12. On the coordinate diagram shown, A is the point $(6, 8)$ and B is the point $(12, -5)$. Angle AOC $= p$ and angle COB $= q$.

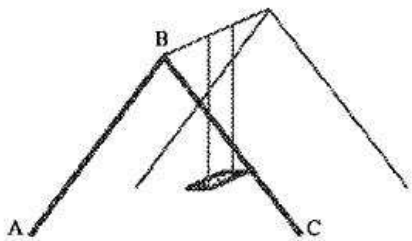
Find the exact value of $\sin(p + q)$. **4**

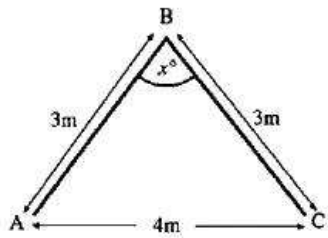


[SQA] 13. In triangle ABC, show that the exact value of $\sin(a + b)$ is $\frac{2}{\sqrt{5}}$. **4**

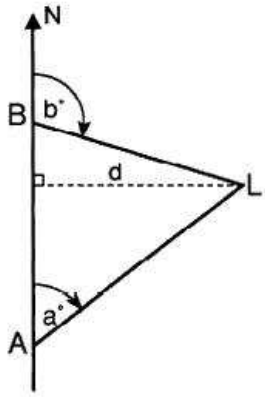


[SQA] 14. **The framework of a child's swing has dimensions as shown in the diagram on the right. Find the exact value of $\sin x°$.** **5**





[SQA] 15. A ship is sailing due north at a constant speed. When at position A, lighthouse L is observed on a bearing of $a°$. One hour later, when the ship is at position B, the lighthouse is on a bearing of $b°$. The shortest distance between the ship and the lighthouse during this hour was d miles.



(*a*) Prove that $AB = \dfrac{d}{\tan a°} - \dfrac{d}{\tan b°}$. **(2)**

(*b*) Hence prove that $AB = \dfrac{d \sin(b-a)°}{\sin a° \sin b°}$. **(3)**

(*c*) Calculate the shortest distance from the ship to the lighthouse when the bearings $a°$ and $b°$ are 060° and 135° respectively and the constant speed of the ship is 14 miles per hour. **(3)**

[SQA] 16.

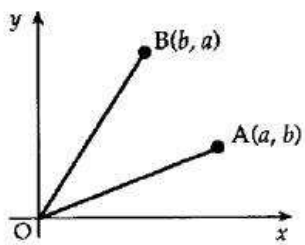
(*a*) Using the fact that $\frac{7\pi}{12} = \frac{\pi}{3} + \frac{\pi}{4}$, find the exact value of $\sin\left(\frac{7\pi}{12}\right)$. **3**

(*b*) Show that $\sin(A+B) + \sin(A-B) = 2\sin A \cos B$. **2**

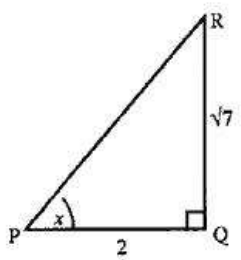
(*c*) (i) Express $\frac{\pi}{12}$ in terms of $\frac{\pi}{3}$ and $\frac{\pi}{4}$.

(ii) Hence or otherwise find the exact value of $\sin\left(\frac{7\pi}{12}\right) + \sin\left(\frac{\pi}{12}\right)$. **4**

[SQA] 17. In the diagram, A and B have coordinates as shown.
Express $\sin A\hat{O}B$ in terms of a and b. **4**



[SQA] 18. Using triangle PQR, as shown, find the exact value of $\cos 2x$. **3**



[SQA] 19. Given that $\cos D = \frac{2}{\sqrt{5}}$ and $0 < D < \frac{\pi}{2}$, find the exact values of $\sin D$ and $\cos 2D$. **3**

[SQA] 20. Given that $\sin A = \frac{3}{4}$, where $0 < A < \frac{\pi}{2}$, find the exact value of $\sin 2A$. **3**

[SQA] 21. For acute angles P and Q, $\sin P = \frac{12}{13}$ and $\sin Q = \frac{3}{5}$.

Show that the exact value of $\sin(P + Q)$ is $\frac{63}{65}$. **3**

[SQA] 22. Find the exact value of $\sin\theta^\circ + \sin(\theta^\circ + 120^\circ) + \cos(\theta^\circ + 150^\circ)$. **3**

[SQA] 23. If $\cos\theta = \frac{4}{5}$, $0 \leq \theta < \frac{\pi}{2}$, find the exact value of

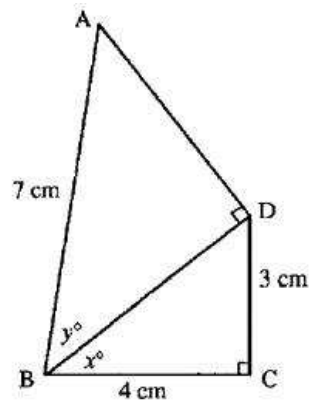
(*a*) $\sin 2\theta$ **2**

(*b*) $\sin 4\theta$. **3**

[SQA] 24. Given that $\tan\alpha = \frac{\sqrt{11}}{3}$, $0 < \alpha < \frac{\pi}{2}$, find the exact value of $\sin 2\alpha$. **3**

[SQA] 25. The diagram shows two right-angled triangles ABD and BCD with AB = 7cm, BC = 4cm and CD = 3cm. Angle DBC = $x°$ and angle ABD = $y°$.

Show that the exact value of $\cos(x+y)^\circ$ is $\frac{20-6\sqrt{6}}{35}$. **3**



[SQA] 26. If x° is an acute angle such that $\tan x^\circ = \frac{4}{3}$, show that the exact value of $\sin(x^\circ + 30^\circ)$ is $\frac{4\sqrt{3}+3}{10}$. **3**

[SQA] 27. A and B are acute angles such that $\tan A = \frac{3}{4}$ and $\tan B = \frac{5}{12}$.

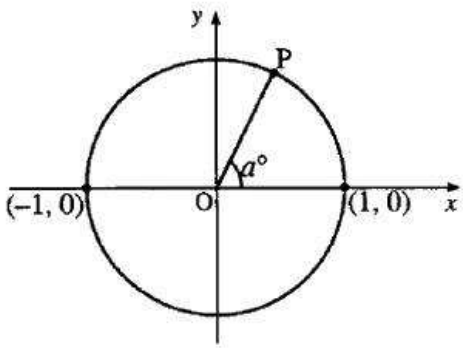
Find the exact value of

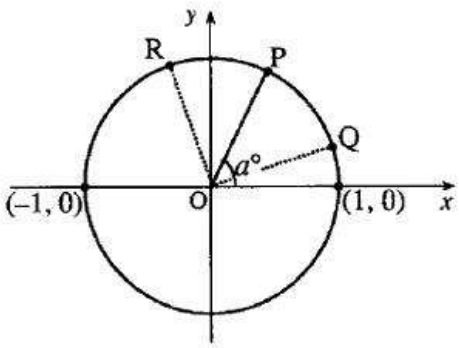
(a) $\sin 2A$ **2**

(b) $\cos 2A$ **1**

(c) $\sin(2A + B)$. **2**

[SQA] 28. **The diagram shows a circle of radius 1 unit and centre the origin. The radius OP makes an angle $a°$ with the positive direction of the x-axis.**





(a) Show that P is the point $(\cos a°, \sin a°)$. **1**

(b) If $\hat{POQ} = 45°$, deduce the coordinates of Q in terms of a. **1**

(c) If $\hat{POR} = 45°$, deduce the coordinates of R in terms of a. **1**

(d) Hence find an expression for the gradient of QR in its simplest form. **4**

(e) Show that the tangent to the circle at P is parallel to QR. **2**

29. (*a*) Diagram 1 shows a right angled triangle, where the line OA has equation $3x - 2y = 0$.

(i) Show that $\tan a = \frac{3}{2}$.

(ii) Find the value of $\sin a$. **4**

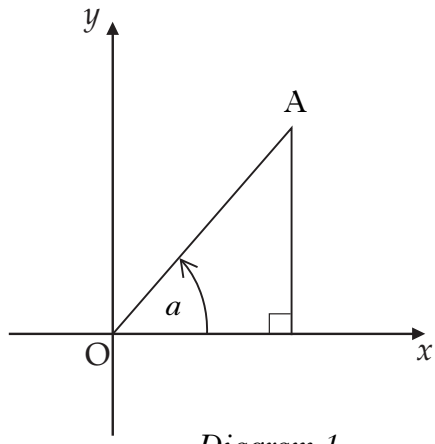


Diagram 1

(*b*) A second right angled triangle is added as shown in Diagram 2.

The line OB has equation $3x - 4y = 0$.

Find the values of $\sin b$ and $\cos b$. **4**

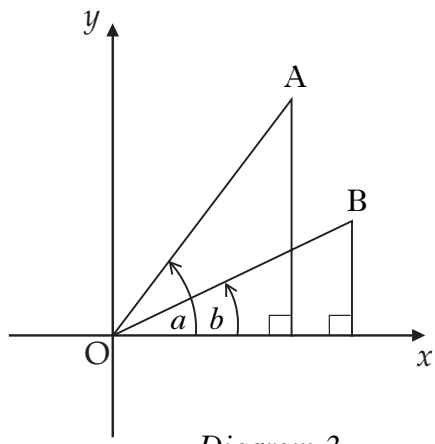


Diagram 2

(*c*) (i) Find the value of $\sin(a - b)$.

(ii) State the value of $\sin(b - a)$. **4**

[SQA] 30.

(*a*) Write the equation $\cos 2\theta + 8\cos\theta + 9 = 0$ in terms of $\cos\theta$ and show that, for $\cos\theta$, it has equal roots. **3**

(*b*) Show that there are no real roots for θ. **1**

[SQA] 31. Functions $f(x) = \sin x$, $g(x) = \cos x$ and $h(x) = x + \frac{\pi}{4}$ are defined on a suitable set of real numbers.

(*a*) Find expressions for:

(i) $f(h(x))$;

(ii) $g(h(x))$. **2**

(*b*) (i) Show that $f(h(x)) = \frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x$.

(ii) Find a similar expression for $g(h(x))$ and hence solve the equation $f(h(x)) - g(h(x)) = 1$ for $0 \leq x \leq 2\pi$. **5**

[END OF QUESTIONS]