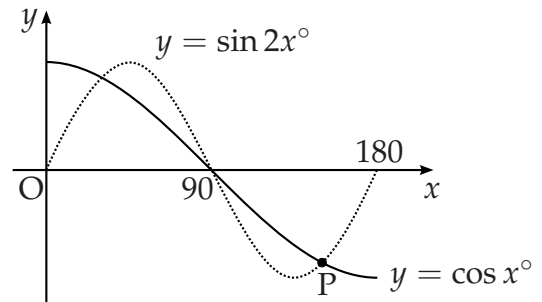


Exact Values (Non-Calculator)

[SQA] 1. (a) Solve the equation $\sin 2x^\circ - \cos x^\circ = 0$ in the interval $0 \leq x \leq 180$. 4

(b) The diagram shows parts of two trigonometric graphs, $y = \sin 2x^\circ$ and $y = \cos x^\circ$.

Use your solutions in (a) to write down the coordinates of the point P.



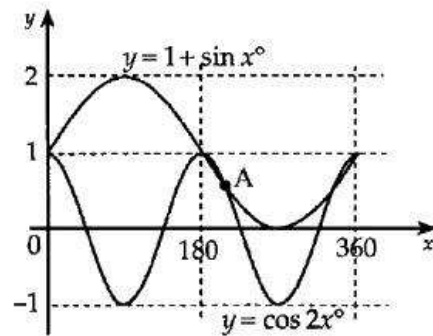
1

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
(a)	4	C	NC	T10	30, 90, 150	2001 P1 Q5
(b)	1	C	NC	T3	$(150, -\frac{\sqrt{3}}{2})$	

<ul style="list-style-type: none"> •¹ ss: use double angle formula •² pd: factorise •³ pd: process •⁴ pd: process •⁵ ic: interpret graph 	<ul style="list-style-type: none"> •¹ $2 \sin x^\circ \cos x^\circ$ •² $\cos x^\circ (2 \sin x^\circ - 1)$ •³ $\cos x^\circ = 0, \sin x^\circ = \frac{1}{2}$ •⁴ 90, 30, 150 <p style="text-align: center;">or</p> <ul style="list-style-type: none"> •³ $\sin x^\circ = \frac{1}{2}$ and $x = 30, 150$ •⁴ $\cos x^\circ = 0$ and $x = 90$ •⁵ $(150, -\frac{\sqrt{3}}{2})$
--	---

[SQA] 2. The diagram shows two curves with equations $y = \cos 2x^\circ$ and $y = 1 + \sin x^\circ$ where $0 \leq x \leq 360$. 4

Find the x-coordinate of the point of intersection at A.



Part	Marks	Level	Calc.	Content	Answer	U2 OC3
	1	C	NC	T10		1991 P1 Q20
	3	A/B	NC	T10		

<ul style="list-style-type: none"> •¹ $\cos 2x^\circ = 1 + \sin x^\circ$ •² $2 \sin^2 x^\circ + \sin x^\circ = 0$ •³ $\sin x^\circ = 0$ or $-\frac{1}{2}$ •⁴ $x = 210$

[SQA] 3. Solve the equation $\sin 2x^\circ + \sin x^\circ = 0$, $0 \leq x < 360$.

5

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
	5	C	NC	T10		1996 P1 Q10
<ul style="list-style-type: none"> •¹ $2 \sin x \cos x + \sin x = 0$ •² $\sin x(2 \cos x + 1) = 0$ •³ $\sin x = 0, \cos x = -\frac{1}{2}$ •⁴ 1st: $x = 0, 180$ •⁵ 2nd: $x = 120, 240$ 						

[SQA] 4.

(a) Solve $\cos 2x^\circ - 3 \cos x^\circ + 2 = 0$ for $0 \leq x < 360$.

5

(b) Hence solve $\cos 4x^\circ - 3 \cos 2x^\circ + 2 = 0$ for $0 \leq x < 360$

Part	Marks	Level	Calc.	Content	Answer	U2 OC3		
(a)	5	C	NC	T7	$x = 0, 60, 300$	2011 P1 Q23		
(b)	2	A	NC	T11	$x = 0, 30, 150, 180, 210, 330$			
<table border="0" style="width: 100%;"> <tr> <td style="vertical-align: top; width: 50%;"> <ul style="list-style-type: none"> •¹ ss: know to use double angle formula •² ic: express as a quadratic in $\cos x^\circ$ •³ ss: start to solve •⁴ pd: reduce to equations in cos only •⁵ ic: process solutions in given domain •⁶ ic: interpret relationship with (a) •⁷ ic: interpret periodicity </td> <td style="vertical-align: top; width: 50%;"> <ul style="list-style-type: none"> •¹ $2 \cos^2 x^\circ - 1 \dots$ •² $2 \cos^2 x^\circ - 3 \cos x^\circ + 1 = 0$ •³ $(2 \cos x^\circ - 1)(\cos x^\circ - 1)$ •⁴ $\cos x^\circ = \frac{1}{2}, 1$ •⁵ $0, 60, 300$ •⁶ $2x = 0$ and 60 and 300 •⁷ $0, 30, 150, 180, 210$ and 330 </td> </tr> </table>							<ul style="list-style-type: none"> •¹ ss: know to use double angle formula •² ic: express as a quadratic in $\cos x^\circ$ •³ ss: start to solve •⁴ pd: reduce to equations in cos only •⁵ ic: process solutions in given domain •⁶ ic: interpret relationship with (a) •⁷ ic: interpret periodicity 	<ul style="list-style-type: none"> •¹ $2 \cos^2 x^\circ - 1 \dots$ •² $2 \cos^2 x^\circ - 3 \cos x^\circ + 1 = 0$ •³ $(2 \cos x^\circ - 1)(\cos x^\circ - 1)$ •⁴ $\cos x^\circ = \frac{1}{2}, 1$ •⁵ $0, 60, 300$ •⁶ $2x = 0$ and 60 and 300 •⁷ $0, 30, 150, 180, 210$ and 330
<ul style="list-style-type: none"> •¹ ss: know to use double angle formula •² ic: express as a quadratic in $\cos x^\circ$ •³ ss: start to solve •⁴ pd: reduce to equations in cos only •⁵ ic: process solutions in given domain •⁶ ic: interpret relationship with (a) •⁷ ic: interpret periodicity 	<ul style="list-style-type: none"> •¹ $2 \cos^2 x^\circ - 1 \dots$ •² $2 \cos^2 x^\circ - 3 \cos x^\circ + 1 = 0$ •³ $(2 \cos x^\circ - 1)(\cos x^\circ - 1)$ •⁴ $\cos x^\circ = \frac{1}{2}, 1$ •⁵ $0, 60, 300$ •⁶ $2x = 0$ and 60 and 300 •⁷ $0, 30, 150, 180, 210$ and 330 							

5. How many solutions does the equation

$$(4 \sin x - \sqrt{5})(\sin x + 1) = 0$$

have in the interval $0 \leq x < 2\pi$?

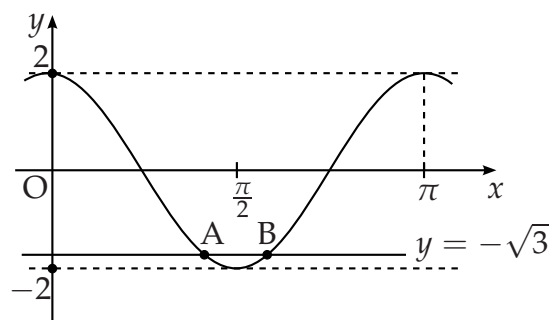
- A. 4
- B. 3
- C. 2
- D. 1

2

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
B	2.3	C	0	0	NC	T7	2009 P1 Q11

[SQA] 6. The diagram shows the graph of a cosine function from 0 to π .

- (a) State the equation of the graph.
- (b) The line with equation $y = -\sqrt{3}$ intersects this graph at point A and B.
Find the coordinates of B.



1

3

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
(a)	1	C	NC	T4	$y = 2 \cos 2x$	2002 P1 Q8
(b)	3	C	NC	T7	$B(\frac{7\pi}{12}, -\sqrt{3})$	

<ul style="list-style-type: none"> •¹ ic: interpret graph •² ss: equate equal parts •³ pd: solve linear trig equation in radians •⁴ ic: interpret result 	<ul style="list-style-type: none"> •¹ $2 \cos 2x$ •¹ $2 \cos 2x = -\sqrt{3}$ •² $2x = \frac{5\pi}{6}, \frac{7\pi}{6}$ •³ $x = \frac{7\pi}{12}$
--	--

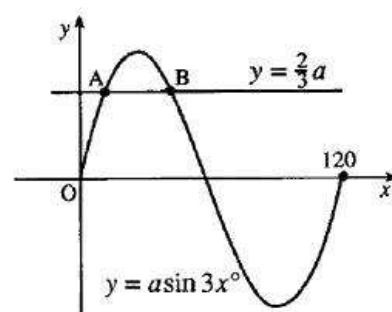
- [SQA] 7. Find the values of t , where $0 < t < 2\pi$, for which $4 \cos\left(2t - \frac{\pi}{4}\right)$ has its maximum value.

4

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
	4	C	NC	T7		1989 P1 Q15

<ul style="list-style-type: none"> •¹ $\cos\left(2t - \frac{\pi}{4}\right) = 1$ •² $2t - \frac{\pi}{4} = 0$ •³ $t = \frac{\pi}{8}$ •⁴ $\frac{\pi}{8}, \frac{9\pi}{8}$

- [SQA] 8. The diagram shows part of the graph of $y = a \sin 3x^\circ$ and the line with equation $y = \frac{2}{3}a$. Find the x -coordinates of A and B.

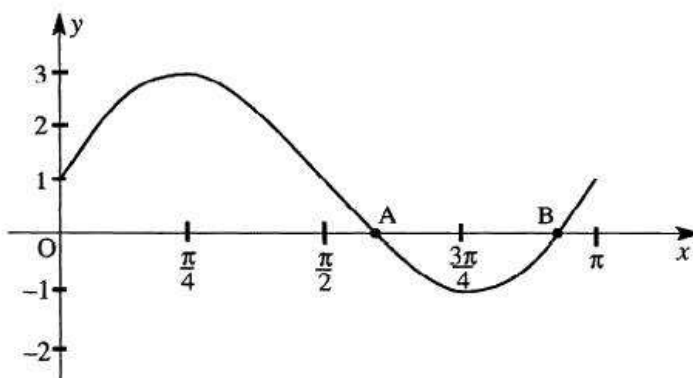


4

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
	4	C	NC	T7		1999 P1 Q14

<ul style="list-style-type: none"> •¹ $a \sin 3x = \frac{2}{3}a$ <i>stated or implied by</i> •² •² $\sin 3x = \frac{2}{3}$ •³ $3x = 41.8, 138.2$ (138.2 <i>stated or implied by</i> 46.1 in •⁴) •⁴ $13.9, 46.1$

- [SQA] 9. The diagram below shows the graph of $y = 2\sin 2x + 1$ for $0 \leq x \leq \pi$.



- (a) Find the coordinates of A and B (as shown in the diagram) by solving an appropriate equation algebraically. (5)
- (b) The points $(0, 2)$ and $(\pi, 0)$ are joined by a straight line l . In how many points does l intersect the given graph? (1)
- (c) C is the point on the given graph with an x -coordinate of $\frac{\pi}{2}$. Explain whether C is above, below or on the line l . (3)

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
(a)	3	C	NC	T7		1993 P2 Q6
(a)	2	A/B	NC	T7		
(b)	1	C	NC	CGD		
(c)	3	A/B	NC	CGD		

- (a) •¹ $2\sin 2x + 1 = 0$
 •² $\sin 2x = -\frac{1}{2}$
 •³ for any valid sol of equ. in form $\sin ax = -\frac{b}{c}$
 •⁴ $(\frac{7\pi}{12}, 0)$
 •⁵ $(\frac{11\pi}{12}, 0)$
- (b) •⁶ 3
- (c) •⁷ $y_C = 1$
 •⁸ for a strategy to make a decision about C
 •⁹ for making a consistent decision about C

10. Solve $2 \cos x = \sqrt{3}$ for x , where $0 \leq x < 2\pi$.

- A. $\frac{\pi}{3}$ and $\frac{5\pi}{3}$
 B. $\frac{\pi}{3}$ and $\frac{2\pi}{3}$
 C. $\frac{\pi}{6}$ and $\frac{5\pi}{6}$
 D. $\frac{\pi}{6}$ and $\frac{11\pi}{6}$

2

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
D	2.3	C	0	0	NC	T7, T2, T3	2011 P1 Q10

[SQA] 11. Solve $2 \sin 3x^\circ - 1 = 0$ for $0 \leq x \leq 180$.

4

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
	4	C	NC	T7, T3		1989 P1 Q7

<ul style="list-style-type: none"> •¹ $\sin 3x^\circ = 0.5$ •² $3x = 30, 150$ •³ $x = 10, 50$ •⁴ solution is 10, 50, 130

[SQA] 12. Solve the equation $2 \cos^2 x = \frac{1}{2}$, for $0 \leq x \leq \pi$.

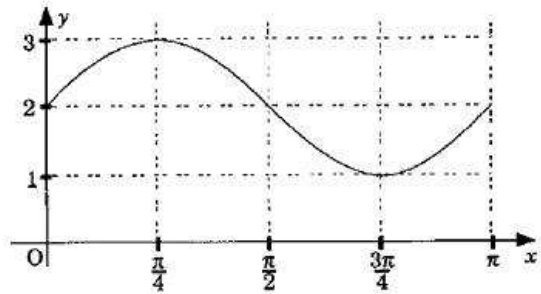
3

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
	3	C	NC	T7, T3		1990 P1 Q15

<ul style="list-style-type: none"> •¹ $\cos x = \pm \frac{1}{2}$ •² $x = \frac{\pi}{3}$ •³ $\frac{2\pi}{3}$
--

[SQA] 13. The diagram shows the graph of the function $y = a + b \sin cx$ for $0 \leq x \leq \pi$.

- (a) Write down the values of a , b and c .
 (b) Find algebraically the values of x for which $y = 2.5$.



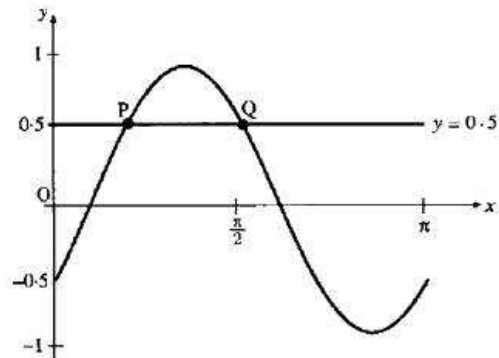
3
3

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
(a)	3	C	NC	T1		1994 P1 Q12
(b)	3	C	NC	T7, T3		

• ¹	$a = 2$	• ⁴	$2 + \sin 2x = 2\frac{1}{2}$	OR	• ⁴	$2 + \sin 2x = 2\frac{1}{2}$
• ²	$b = 1$	• ⁵	$2x = \frac{\pi}{6}, \frac{5\pi}{6}$		• ⁵	$2x = \frac{\pi}{6}, x = \frac{\pi}{12}$
• ³	$c = 2$	• ⁶	$x = \frac{\pi}{12}, \frac{5\pi}{12} (0.262, 1.309)$		• ⁶	$2x = \frac{5\pi}{6}, x = \frac{5\pi}{12}$

[SQA] 14. The diagram shows a sketch of the graph of $y = \sin(2x - \frac{\pi}{6})$, $0 \leq x \leq \pi$, and the straight line $y = 0.5$. These graphs intersect at P and Q.

Find algebraically the coordinates of P and Q.



4

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
	4	C	NC	T7, T3		1996 P1 Q12

• ¹	$\sin(2x - \frac{\pi}{6}) = 0.5$	<i>stated or implied by 2nd mark</i>				
• ²	$2x - \frac{\pi}{6} = \frac{\pi}{6}$					
• ³	$2x - \frac{\pi}{6} = \frac{5\pi}{6}$					
• ⁴	$(\frac{\pi}{6}, 0.5), (\frac{\pi}{2}, 0.5)$					

[SQA] 15. Solve the equation $2 \sin\left(2x - \frac{\pi}{6}\right) = 1, 0 \leq x < 2\pi$.

4

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
	4	C	NC	T7, T3		1998 P1 Q9

<p>•¹ $\sin\left(2x - \frac{\pi}{6}\right) = \frac{1}{2}$</p> <p>•² $2x - \frac{\pi}{6} = \frac{\pi}{6}, \frac{5\pi}{6}$ (accept 30, 150)</p> <p>•³ $x = \frac{\pi}{6}, \frac{\pi}{2}$</p> <p>•⁴ $x = \frac{7\pi}{6}, \frac{3\pi}{2}$</p>	<p style="text-align: center;">Alternative for 2nd and 3rd marks</p> <p>•² $2x - \frac{\pi}{6} = \frac{\pi}{6}, x = \frac{\pi}{6}$</p> <p>•³ $2x - \frac{\pi}{6} = \frac{5\pi}{6}, x = \frac{\pi}{2}$</p>
---	---

[SQA] 16.

(a) Using the fact that $\frac{7\pi}{12} = \frac{\pi}{3} + \frac{\pi}{4}$, find the exact value of $\sin\left(\frac{7\pi}{12}\right)$.

3

(b) Show that $\sin(A + B) + \sin(A - B) = 2 \sin A \cos B$.

2

(c) (i) Express $\frac{\pi}{12}$ in terms of $\frac{\pi}{3}$ and $\frac{\pi}{4}$.

(ii) Hence or otherwise find the exact value of $\sin\left(\frac{7\pi}{12}\right) + \sin\left(\frac{\pi}{12}\right)$.

4

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
(a)	3	C	NC	T8, T3	$\frac{\sqrt{3} + 1}{2\sqrt{2}}$	2009 P1 Q24
(b)	2	C	CN	T8	proof	
(c)	3	B	NC	T11	$\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$	
(c)	1	C	NC	T11	$\frac{\sqrt{6}}{2}$ or $\sqrt{\frac{3}{2}}$	

<p>•¹ ss: expand compound angle</p> <p>•² ic: substitute exact values</p> <p>•³ pd: process to a single fraction</p> <p>•⁴ ic: start proof</p> <p>•⁵ ic: complete proof</p> <p>•⁶ ss: identify steps</p> <p>•⁷ ic: start process (identify 'A' & 'B')</p> <p>•⁸ ic: substitute</p> <p>•⁹ pd: process</p>	<p>•¹ $\sin \frac{\pi}{3} \cos \frac{\pi}{4} + \cos \frac{\pi}{3} \sin \frac{\pi}{4}$</p> <p>•² $\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \frac{1}{\sqrt{2}}$</p> <p>•³ $\frac{\sqrt{3} + 1}{2\sqrt{2}}$ or equivalent</p> <p>•⁴ $\sin A \cos B + \cos A \sin B + \dots$</p> <p>•⁵ $\dots + \sin A \cos B - \cos A \sin B$ and complete</p> <p>•⁶ $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$</p> <p>•⁷ $2 \times \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}}$</p> <p>•⁸ $\frac{\sqrt{6}}{2}$ or $\sqrt{\frac{3}{2}}$</p>
---	---

[END OF QUESTIONS]