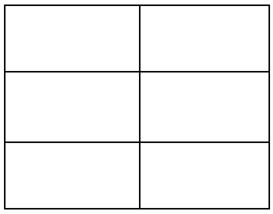
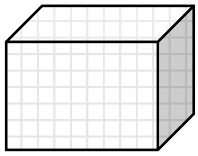
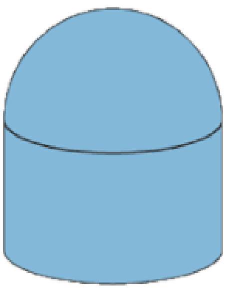
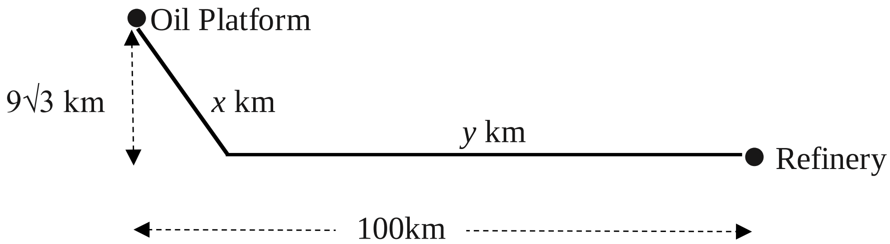
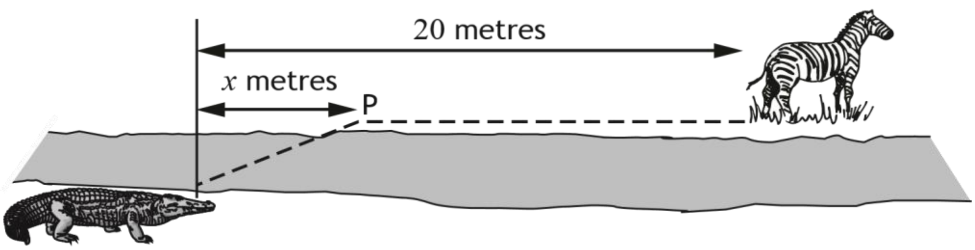
	Differentiation - Optimisation	
1.	A zoo keeper wants to fence off six individual pens. Each pen is a rectangle which measures *x* metres by *y* metres as shown.  *y* metres *x* metres (a) (i) Express the total length of fencing in terms of *x* and *y* (ii) Given that the total length of fencing used is 360m, show that the total area A m^2 of the six pens is given by $A(x) = 240x - \frac{16}{3}x^2$ (b) Find the values of *x* and *y* which give the **maximum** area and write down this maximum area	4 6
2.	The owners of a zoo intend to build a new aviary. This is in the shape of a cuboid with a square floor. The top of the aviary measures x metres by *x* metres. The height of the aviary is *h* metres. The volume of the aviary will be 500 m^3  (a) Show that the area in square metres of netting needed for the five sides of the aviary is given by $A(x) = \frac{2000}{x} + x^2$ (b) Find the dimensions of the aviary which ensure that the cost of the netting is **minimised** and find the total amount of netting needed for this aviary.	4 6
3	 A flask is in the shape of a cylinder with a hemi-sphere for a lid. The radius of the cylindrical base is *r* cm The height of the cylinder if *h* cm. The volume of the cylinder part of the flask is 400 cm^3. (a) Show that the surface area for the flask is given by $A(x) = 3\pi r^2 + \frac{800}{r}$ (b) Find the value of *r* which minimises the surface area. **Note** the surface area of a hemisphere is $2\pi r^2$	3 6

4	An oil platform which is $9\sqrt{3}$ km offshore is connected by a pipeline to a refinery on shore. The refinery is 100 km down the coast from the platform.  The length of the underwater pipe is *x* km and the length of the pipeline on land is *y* km. It costs £2 million to lay each km of the pipeline underwater and £1 million per km on land. (a) Show that the cost of the pipeline in millions is $C(x) = 2x + 100 - \sqrt{x^2 - 243}$ (b) Show that *x* = 18 gives a minimum cost for this pipeline. Hence find the total length of this pipeline and the minimum cost.	 4 7
5	A crocodile is stalking a zebra which is 20 metres further upstream on the opposite back of the river.  Crocodiles travel at different speeds on land and in the water. The time taken for the crocodile to reach its prey is minimised if swims part way across the river before continuing the chase on land. This time is given by: This time T, measured in tenths of a second is given by $T(x) = 5\sqrt{36 + x^2} + 4(20 - x)$ Where T is time in tenths of a second and *x* is the length of the underwater swim. Find the value for x which **minimises** the time take and hence calculate the minimum time.	 8

Differentiation - Answers

1

Form an expression for the fence	$9y + 8x$
Form an expression for the area	A = $2x \times 3y$
Rearrange in terms fence in terms of *x*	$360 = 9y + 8x$, $y = \frac{1}{9}(360 - 8x)$
Substitute into area expression	$2x \times 3\left(\frac{1}{9}(360 - 8x)\right)$
	$= 2x \times \left(120 - \frac{8}{3}x\right) = 240x - \frac{8}{3}x^2$
Know to differentiate	$A'(x) =$
Differentiate and set = 0	$240 - \frac{32}{3}x = 0$
Solve for *x*	$x = 22.5$
Use a nature table	

x	22.5^-	22.5	22.5^+
$\frac{dy}{dx}$	+	0	-

State values: **$x = 22.5$, $y = 20$, A = 2700m^2**

2

Form an expression for the surface area	SA = $x^2 + 4xh$
Form an expression for the volume	V = x^2h
Rearrange volume in terms of *h*	$500 = x^2h$, $h = \frac{500}{x^2}$
Substitute into area expression	SA = $x^2 + 4x\left(\frac{500}{x^2}\right)$
	$= x^2 + \frac{2000}{x}$ as required
Know to differentiate	$A'(x) =$
Differentiate and set = 0	$2x - \frac{2000}{x^2} = 0$
Solve for *x*	$x = 10$
Use a nature table	

x	10^-	10	10^+
$\frac{dy}{dx}$	-	0	+

State values: **x = 10 m, h = 5m, A = 300m^2**

3

Form an expression for the surface area	SA = $\pi r^2 + 2\pi rh + 2\pi r^2$
Form an expression for the volume	V = $\pi r^2 h$, $400 = \pi r^2 h$, $h = \frac{400}{\pi r^2}$
=	
Substitute into area expression	SA = $3\pi r^2 + 2\pi r\left(\frac{400}{\pi r^2}\right)$
	$= 3\pi r^2 + \frac{800}{r}$ as required
Know to differentiate	$A'(x) =$
Complete differentiation = 0	$6\pi r - \frac{800}{r^2} = 0$
Solve for *x*	$x = 3.488$. or $x = 3.5$

	Use a nature table State values	$\begin{array}{c\|ccc} x & 3.5^- & 3.5 & 3.5^+ \\ \hline \frac{dy}{dx} & - & 0 & + \end{array}$ **x = 3.5cm Area is = 344m^2**
4	Form an expression for cost Use Pythagoras Rearrange volume in terms of h Substitute into cost Express in differentiable form Start differentiation Complete differentiation Substitute x = 18 into the derivative Use a nature table State values	C(x) = 2x + y $b^2 = x^2 - (9\sqrt{3})^2$ where b + y = 100 $b = \sqrt{x^2 - (9\sqrt{3})^2}$ where y = 100 - b C(x) = $2x + 100 - \sqrt{x^2 - 243}$ $C(x) = 2x + 100 - (x^2 - 243)^{1/2}$ 2 - $\frac{1}{2}(x^2 - 243)^{-0.5} \times 2x$ ... $2 - x(x^2 - 243)^{-0.5}$ C'(18) = 0, hence x = 18 is a min/max $\begin{array}{c\|ccc} x & 18^- & 18 & 18^+ \\ \hline \frac{dy}{dx} & - & 0 & + \end{array}$ **x = 18 km, y = 91km, length is 109km= 98, Cost is £127 million**
5	Write the function in differentiable form Differentiate the composite function Complete differentiation and = 0 Solve for x Use a nature table State values	$T(x) = 5(36 + x^2)^{0.5} + 80 - 4x$ $\frac{5}{2}(36 + x^2)^{-0.5} \times 2x$ ... $5x(36 + x^2)^{-0.5} - 4 = 0$ $\frac{5x}{(36 + x^2)^{-0.5}} = 4,$ $\frac{5x}{4} = (36 + x^2)^{0.5}$ $\frac{25x^2}{16} = 36 + x^2, \quad x^2 = 64,$ x = 8 $\begin{array}{c\|ccc} x & 8^- & 8 & 8^+ \\ \hline \frac{dy}{dx} & - & 0 & + \end{array}$ **x = 8m, T = 98 or 9.8 seconds**