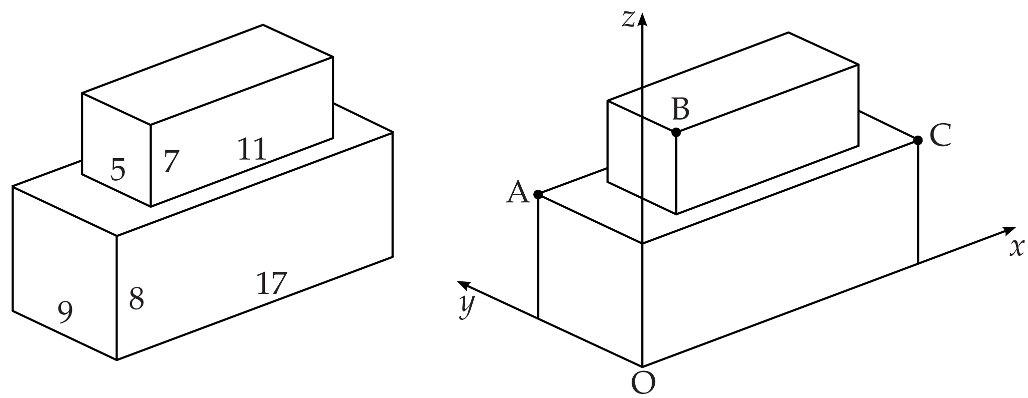
angle between vectors

[SQA] 1. A cuboid measuring 11 cm by 5 cm by 7 cm is placed centrally on top of another cuboid measuring 17 cm by 9 cm by 8 cm.

Coordinates axes are taken as shown.



(*a*) The point A has coordinates $(0, 9, 8)$ and C has coordinates $(17, 0, 8)$.

Write down the coordinates of B. **1**

(*b*) Calculate the size of angle ABC. **6**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	1	C	CN	G22	B(3, 2, 15)	2000 P2 Q9
(*b*)	6	C	CR	G28	92·5°	

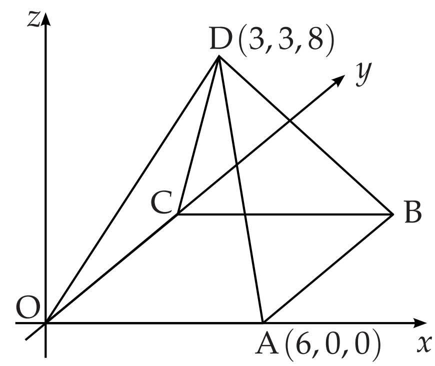
•¹ ic: interpret 3-d representation	•¹ $B = (3, 2, 15)$ *treat* $\begin{pmatrix} 3 \\ 2 \\ 15 \end{pmatrix}$ *as bad form*
•² ss: know to use scalar product	•² $\cos A\hat{B}C = \frac{\overrightarrow{BA}.\overrightarrow{BC}}{\lvert\overrightarrow{BA}\rvert\lvert\overrightarrow{BC}\rvert}$
•³ pd: process vectors	•³ $\overrightarrow{BA} = \begin{pmatrix} -3 \\ 7 \\ -7 \end{pmatrix}$
•⁴ pd: process vectors	•⁴ $\overrightarrow{BC} = \begin{pmatrix} 14 \\ -2 \\ -7 \end{pmatrix}$
•⁵ pd: process lengths	•⁵ $\lvert\overrightarrow{BA}\rvert = \sqrt{107}$, $\lvert\overrightarrow{BC}\rvert = \sqrt{249}$
•⁶ pd: process scalar product	•⁶ $\overrightarrow{BA}.\overrightarrow{BC} = -7$
•⁷ pd: evaluate scalar product	•⁷ $A\hat{B}C = 92{\cdot}5°$

[SQA] 2. The diagram shows a square-based pyramid of height 8 units.

Square OABC has a side length of 6 units.

The coordinates of A and D are $(6, 0, 0)$ and $(3, 3, 8)$.

C lies on the y-axis.



(*a*) Write down the coordinates of B. **1**

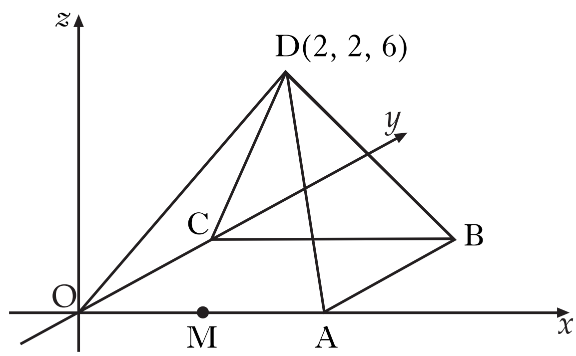
(*b*) Determine the components of $\overrightarrow{DA}$ and $\overrightarrow{DB}$. **2**

(*c*) Calculate the size of angle ADB. **4**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	1	C	CN	G22	$(6, 6, 0)$	2002 P2 Q2
(*b*)	2	C	CN	G17	$\overrightarrow{DA} = \begin{pmatrix} 3 \\ -3 \\ -8 \end{pmatrix}$, $\overrightarrow{DB} = \begin{pmatrix} 3 \\ 3 \\ -8 \end{pmatrix}$	
(*c*)	4	C	CR	G28	$38{\cdot}7^\circ$	

•[1] ic: interpret diagram	•[1] $B = (6, 6, 0)$
•[2] ic: write down components of a vector	•[2] $\overrightarrow{DA} = \begin{pmatrix} 3 \\ -3 \\ -8 \end{pmatrix}$
•[3] ic: write down components of a vector	•[3] $\overrightarrow{DB} = \begin{pmatrix} 3 \\ 3 \\ -8 \end{pmatrix}$
•[4] ss: use e.g. scalar product formula	•[4] $\cos A\hat{D}B = \frac{\overrightarrow{DA}.\overrightarrow{DB}}{\lvert\overrightarrow{DA}\rvert\lvert\overrightarrow{DB}\rvert}$
•[5] pd: process lengths	•[5] $\lvert\overrightarrow{DA}\rvert = \sqrt{82}$, $\lvert\overrightarrow{DB}\rvert = \sqrt{82}$
•[6] pd: process scalar product	•[6] $\overrightarrow{DA}.\overrightarrow{DB} = 64$
•[7] pd: process angle	•[7] $A\hat{D}B = 38{\cdot}7^\circ$

3. D,OABC is a square based pyramid as shown in the diagram below.



O is the origin, D is the point $(2, 2, 6)$ and OA $= 4$ units.

M is the mid-point of OA.

(*a*) State the coordinates of B. **1**

(*b*) Express $\overrightarrow{DB}$ and $\overrightarrow{DM}$ in component form. **3**

(*c*) Find the size of angle BDM. **5**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	1	C	CN	G22	$(4, 4, 0)$	2011 P2 Q1
(*b*)	3	C	CN	G20, G22	$\overrightarrow{DB} = \begin{pmatrix} 2 \\ 2 \\ -6 \end{pmatrix}, \overrightarrow{DM} = \begin{pmatrix} 0 \\ -2 \\ -6 \end{pmatrix}$	
(*c*)	5	C	CN	G28	$40{\cdot}3°$	

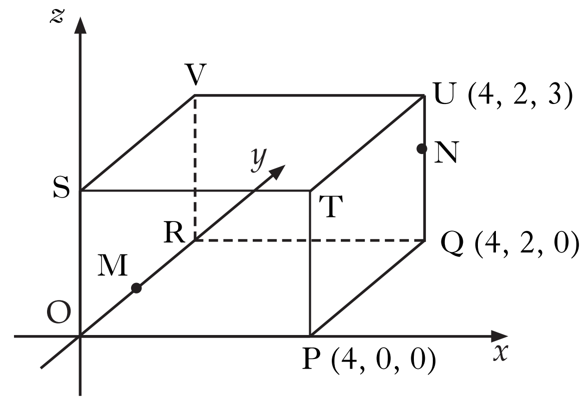
•[1] ic: state coordinates of B	•[1] $(4, 4, 0)$
•[2] pd: state components of $\overrightarrow{DB}$	•[2] $\begin{pmatrix} 2 \\ 2 \\ -6 \end{pmatrix}$
•[3] ic: state coordinates of M	•[3] $(2, 0, 0)$
•[4] pd: state components of $\overrightarrow{DM}$	•[4] $\begin{pmatrix} 0 \\ -2 \\ -6 \end{pmatrix}$
•[5] ss: know to use scalar product	•[5] $\cos \text{BDM} = \frac{\overrightarrow{DB} \cdot \overrightarrow{DM}}{\lvert\overrightarrow{DB}\rvert \lvert\overrightarrow{DM}\rvert}$
•[6] pd: find scalar product	•[6] $\overrightarrow{DB} \cdot \overrightarrow{DM} = 32$
•[7] pd: find magnitude of a vector	•[7] $\lvert\overrightarrow{DB}\rvert = \sqrt{44}$
•[8] pd: find magnitude of a vector	•[8] $\lvert\overrightarrow{DM}\rvert = \sqrt{40}$
•[9] pd: evaluate angle BDM	•[9] $40{\cdot}3°$ or $0{\cdot}703$ rads

4. The diagram shows a cuboid OPQR,STUV relative to the coordinate axes.

P is the point $(4, 0, 0)$, Q is $(4, 2, 0)$ and U is $(4, 2, 3)$.

M is the midpoint of OR.

N is the point on UQ such that $UN = \frac{1}{3}UQ$.



(*a*) State the coordinates of M and N. **2**

(*b*) Express the vectors $\overrightarrow{VM}$ and $\overrightarrow{VN}$ in component form. **2**

(*c*) Calculate the size of angle MVN. **5**

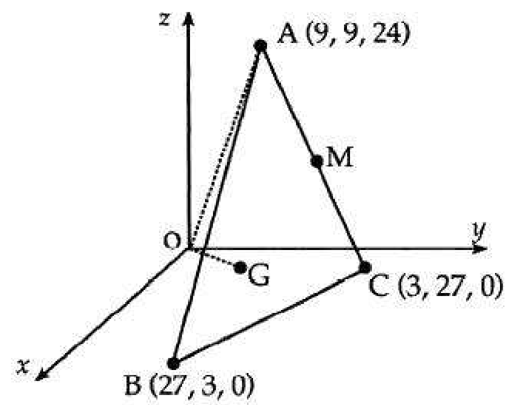
Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	2	C	CN	G22, G25	$M(0,1,0), N(4,2,2)$	2010 P2 Q1
(*b*)	2	C	CN	G17	$\overrightarrow{VM} = \begin{pmatrix} 0 \\ -1 \\ -3 \end{pmatrix}, \overrightarrow{VN} = \begin{pmatrix} 4 \\ 0 \\ -1 \end{pmatrix}$	
(*c*)	5	C	CN	G28	76·7° or 1·339 rad	

•1 ic: interpret midpoint for M	•1 $(0, 1, 0)$
•2 ic: interpret ratio for N	•2 $(4, 2, 2)$
•3 ic: interpret diagram	•3 $\overrightarrow{VM} = \begin{pmatrix} 0 \\ -1 \\ -3 \end{pmatrix}$
•4 pd: process vectors	•4 $\overrightarrow{VN} = \begin{pmatrix} 4 \\ 0 \\ -1 \end{pmatrix}$
•5 ss: know to use scalar product	•5 $\cos M\hat{V}N = \dfrac{\overrightarrow{VM}.\overrightarrow{VN}}{\lvert\overrightarrow{VM}\rvert\lvert\overrightarrow{VN}\rvert}$
•6 pd: find scalar product	•6 $\overrightarrow{VM}.\overrightarrow{VN} = 3$
•7 pd: find magnitude of a vector	•7 $\lvert\overrightarrow{VM}\rvert = \sqrt{10}$
•8 pd: find magnitude of a vector	•8 $\lvert\overrightarrow{VN}\rvert = \sqrt{17}$
•9 pd: evaluate angle	•9 76·7° or 1·339 rads or 85·2 grads

[SQA] 5. (*a*) Relative to mutually perpendicular axes Ox, Oy and Oz, the vertices of triangle ABC have coordinates A(9, 9, 24), B(27, 3, 0) and C(3, 27, 0). M is the mid-point of AC.

Find the coordinates of G which divides BM in the ratio 2:1. **(3)**

(*b*) Calculate the size of angle GOA. **(5)**



Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	3	C	CR	G25		1990 P2 Q4
(*b*)	5	C	CR	G28		

(*a*)

•[1] $M = (6,\ 18,\ 12)$

•[2] e.g. $\overrightarrow{BG} = \frac{2}{3}\begin{pmatrix}-21\\15\\12\end{pmatrix}$

•[3] $G = (13,\ 13,\ 8)$

(*b*)

•[4] $\cos A\hat{O}G = \dfrac{\overrightarrow{OA}.\overrightarrow{OG}}{|\overrightarrow{OA}|\,|\overrightarrow{OG}|}$

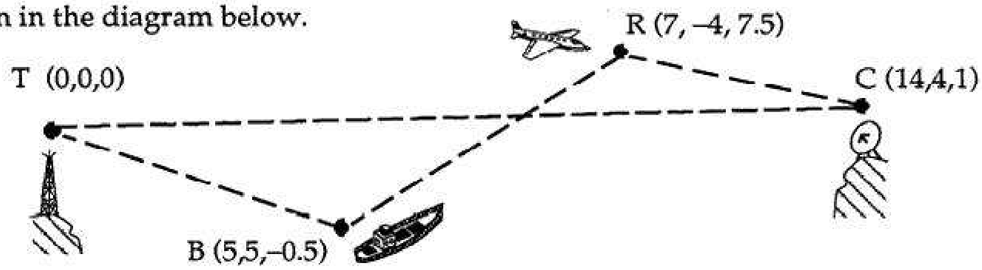
•[5] $\overrightarrow{OA} = \begin{pmatrix}9\\9\\24\end{pmatrix}$ and $\overrightarrow{OG} = \begin{pmatrix}13\\13\\8\end{pmatrix}$

•[6] $\overrightarrow{OA}.\overrightarrow{OG} = 426$

•[7] $|\overrightarrow{OA}| = \sqrt{738}$ and $|\overrightarrow{OG}| = \sqrt{402}$

•[8] $38.5°$

[SQA] 6. Relative to a suitable set of co-ordinate axes with a scale of 1 unit to 2 kilometres, the positions of a transmitter mast, ship, aircraft and satellite dish are shown in the diagram below.



The top T of the transmitter mast is the origin, the bridge B on the ship is the point (5, 5, –0·5), the centre C of the dish on the top of a mountain is the point (14, 4, 1) and the reflector R on the aircraft is the point (7, –4, 7·5).

(*a*) Find the distance from the bridge of the ship to the reflector on the aircraft. **(3)**

(*b*) Three minutes earlier the aircraft was at the point M(–2, 4, 8·5). Find the speed of the aircraft in kilometres per hour. **(2)**

(*c*) Prove that the direction of the beam TC is perpendicular to the direction of the beam BR. **(3)**

(*d*) Calculate the size of angle TCR. **(5)**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	3	C	CR	G16		1992 P2 Q2
(*b*)	2	C	CR	G16		
(*c*)	3	C	CR	G27		
(*d*)	5	C	CR	G28		

(*a*) •1 Strategy: use vectors or 3-D distance formula

•2 $\vec{BR} = \begin{pmatrix} 2 \\ 7 \\ 4 \end{pmatrix}$ *or* $BR^2 = 2^2 + 7^2 + 4^2$

•3 answer

(*b*) •4 $|\vec{MR}| = \sqrt{115.25}$ or equivalent

•5 answer

(*c*) •6 know to use a scalar product

•7 $\vec{TC}.\vec{BR} = 0$

•8 communication: 0 ⇔ perpendicularity

(*d*) •9 Strategy: know to use $\cos T\hat{C}R = \dfrac{\vec{TC}.\vec{RC}}{|TC||RC|}$ or equiv.

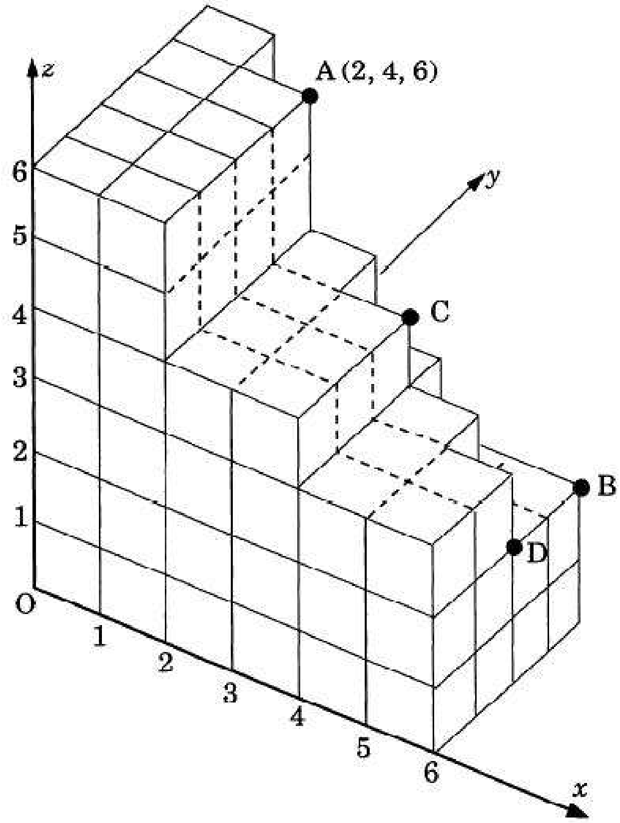
•10 $\vec{TC} = \begin{pmatrix} 12 \\ -4 \\ 1 \end{pmatrix}$ and $\vec{RC} = \begin{pmatrix} 5 \\ -6 \\ -2 \end{pmatrix}$

•11 $\sqrt{161}$ and $\sqrt{65}$

•12 $\vec{TC}.\vec{RC} = 82$

•13 36.7°

[SQA] 7. With coordinate axes as shown, the point A is (2,4,6).



(a) Write down the coordinates of B,C and D. **(3)**

(b) Show that C is the midpoint of AD. **(1)**

(c) By using the components of the vectors $\vec{OA}$ and $\vec{OB}$, calculate the size of angle AOB, where O is the origin. **(4)**

(d) Hence calculate the size of angle OAB. **(2)**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(a)	3	C	CR	G16		1994 P2 Q3
(b)	1	C	CR	G25		
(c)	4	C	CR	G28		
(d)	2	C	CR	CGD		

(a) •[1] One of B, C or D

•[2] Remaining two of B, C and D

•[3] B (6,4,2), C (4,3,4), D (6,2,2)

(b) •[4] $\left(\frac{2+6}{2}, \frac{4+2}{2}, \frac{6+2}{2}\right)$

(c) •[5] $\cos A\hat{O}B = \frac{\vec{OA}.\vec{OB}}{|\vec{OA}||\vec{OB}|}$ or $\frac{OA^2 + OB^2 - AB^2}{2 \times OA \times OB}$ or equivalents

•[6] $\vec{OA}.\vec{OB} = 40$ or $AB^2 = 32$

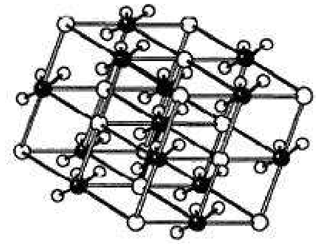
•[7] $OA = \sqrt{56} = OB$

•[8] 44°

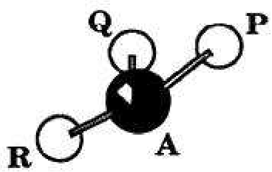
(d) •[9] strategy: e.g. use isosceles Δ

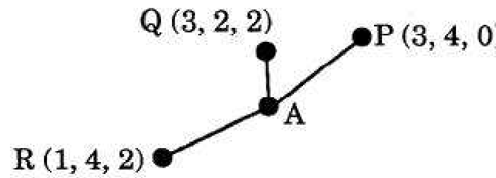
•[10] 68°

[SQA] 8. The diagram shows the rhombohedral crystal lattice of calcium carbonate.



The three oxygen atoms P, Q and R around the carbon atom A have coordinates as shown below.





(*a*) Calculate the size of angle PQR. **(4)**

(*b*) M is the midpoint of QR and T is the point which divides PM in the ratio 2:1.

(i) Find the coordinates of T.

(ii) Show that P, Q and R are equidistant from T. **(6)**

(*c*) The coordinates of A are (2, 3, 1).

(i) Show that P, Q and R are also equidistant from A

(ii) Explain why T, and not A, is the centre of the circle through P, Q and R. **(2)**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	4	C	CN	G28		1995 P2 Q5
(*b*)	6	C	CN	G25, G16		
(*c*)	1	C	CN	G16, G1		
(*c*)	1	A/B	CN	G16, CGD		

(a) •1 $PQ=\sqrt{8},\ RQ=\sqrt{8},$

•2 Use s.p.: $\overrightarrow{PQ}.\overrightarrow{RQ}=|\overrightarrow{PQ}|.|\overrightarrow{RQ}|\cos\theta$

•3 $\begin{pmatrix}0\\-2\\2\end{pmatrix}.\begin{pmatrix}2\\-2\\0\end{pmatrix}=4$

•4 60°

(b) •5 $M=(2,3,2)$

•6 $\overrightarrow{PT}=\frac{2}{3}\overrightarrow{PM}$ or equivalent

•7 $\overrightarrow{PT}=\frac{2}{3}\begin{pmatrix}-1\\-1\\2\end{pmatrix}$ or equiv.

•8 $T=\left(\frac{7}{3},\frac{10}{3},\frac{4}{3}\right)$

•9 $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$ stated or implied

•10 $PT=2\sqrt{\frac{2}{3}}, QT=2\sqrt{\frac{2}{3}}, RT=2\sqrt{\frac{2}{3}}$ or equivalent

(c) •11 $PA=QA=RA=\sqrt{3}$

•12 A is in a different plane

[SQA] 9. The first four levels of a stepped pyramid with a square base are shown in Diagram 1.

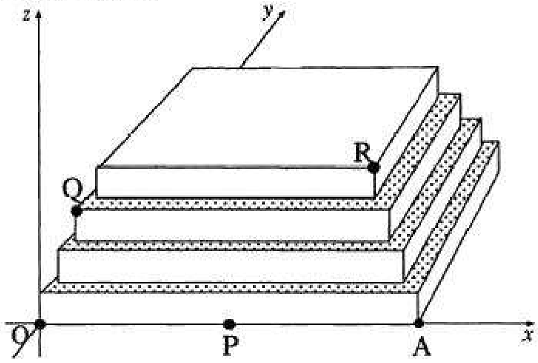


Diagram 1

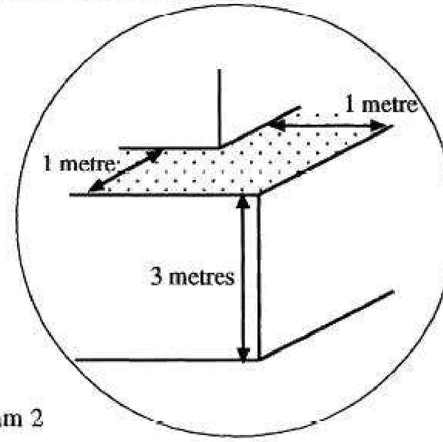


Diagram 2

Each level is a square-based cuboid with a height of 3 m. The shaded parts indicate the steps which have a "width" of 1 m.
The height and "width" of a step at a corner are shown in the enlargement in Diagram 2.
With coordinate axes as shown and 1 unit representing 1 metre, the coordinates of P and A are (12, 0, 0) and (24, 0, 0).

(*a*) Find the coordinates of Q and R. **(2)**

(*b*) Find the size of angle QPR. **(7)**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	2	C	CR	G16		1996 P2 Q3
(*b*)	7	C	CR	G28		

(a) •1 $Q=(2,2,9)$

•2 $R=(21,3,12)$

(b) •3 $\cos\theta=\frac{a.b}{|a||b|}$ *with some subsequent use*

eg $\cos Q\hat{P}R=\frac{\overrightarrow{PQ}.\overrightarrow{PR}}{|\overrightarrow{PQ}||\overrightarrow{PR}|}$

•4 $\overrightarrow{PQ}=\begin{pmatrix}-10\\2\\9\end{pmatrix}$ •5 $\overrightarrow{PR}=\begin{pmatrix}9\\3\\12\end{pmatrix}$

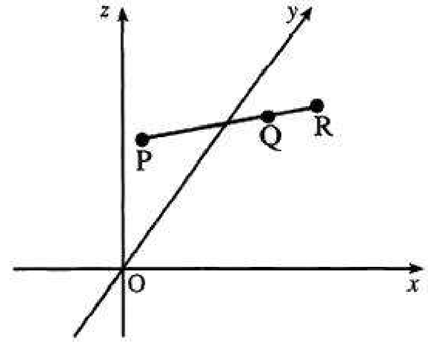
•6 $|\overrightarrow{PQ}|=\sqrt{185}$

•7 $|\overrightarrow{PR}|=\sqrt{234}$

•8 $\overrightarrow{PQ}.\overrightarrow{PR}=24$

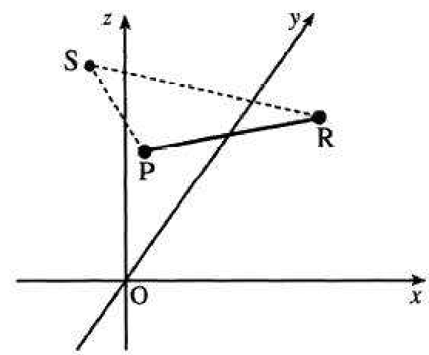
•9 $Q\hat{P}R=83\cdot4^{\circ}$

[SQA] 10. Relative to the axes shown and with an appropriate scale, P(−1, 3, 2) and Q(5, 0, 5) represent points on a road. The road is then extended to the point R such that $\overrightarrow{PR} = \frac{4}{3}\overrightarrow{PQ}$.



(*a*) Find the coordinates of R. **(3)**

(*b*) Roads from P and R are built to meet at the point S (−2, 2, 5). Calculate the size of angle PSR. **(7)**



Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	3	C	CR	G25		1997 P2 Q2
(*b*)	7	C	CR	G28		

(a)

•1 $\overrightarrow{PQ} = \begin{pmatrix} 6 \\ -3 \\ 3 \end{pmatrix}$ •2 $\begin{pmatrix} 8 \\ -4 \\ 4 \end{pmatrix}$

•3 $R = (7, -1, 6)$

(b)

•4 $\overrightarrow{SP}.\overrightarrow{SR} = |SP||SR|\cos P\hat{S}R$

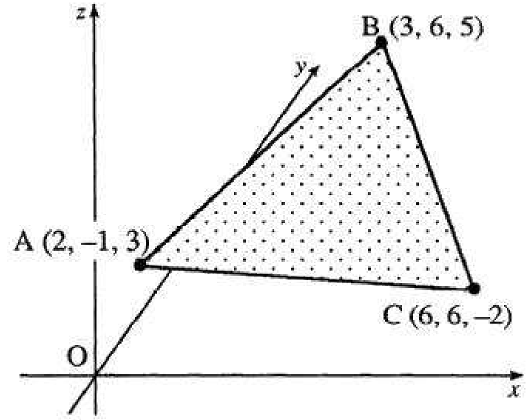
•5 $\overrightarrow{SP} = \begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix}$ •6 $\overrightarrow{SR} = \begin{pmatrix} 9 \\ -3 \\ 1 \end{pmatrix}$

•7 $|SP| = \sqrt{11}$ •8 $|SR| = \sqrt{91}$

•9 $\overrightarrow{SP}.\overrightarrow{SR} = 3$

•10 $P\hat{S}R = 84 \cdot 6°$

[SQA] 11. A triangle ABC has vertices A (2, –1, 3), B(3, 6, 5) and C (6, 6, –2).



(*a*) Find $\overrightarrow{AB}$ and $\overrightarrow{AC}$. **(2)**

(*b*) Calculate the size of angle BAC. **(5)**

(*c*) Hence find the area of the triangle. **(2)**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	2	C	CR	G16		1998 P2 Q1
(*b*)	5	C	CR	G28		
(*c*)	2	C	CR	CGD		

(*a*) •1 $\overrightarrow{AB} = \begin{pmatrix} 1 \\ 7 \\ 2 \end{pmatrix}$

•2 $\overrightarrow{AC} = \begin{pmatrix} 4 \\ 7 \\ -5 \end{pmatrix}$

(*b*) •3 $\cos B\hat{A}C = \dfrac{\overrightarrow{AB}.\overrightarrow{AC}}{|\overrightarrow{AB}|\,|\overrightarrow{AC}|}$ *stated or implied by responses to •4 to •7*

•4 $\overrightarrow{AB}.\overrightarrow{AC} = 4 + 49 - 10$

•5 $\overrightarrow{AB} = \sqrt{54}$

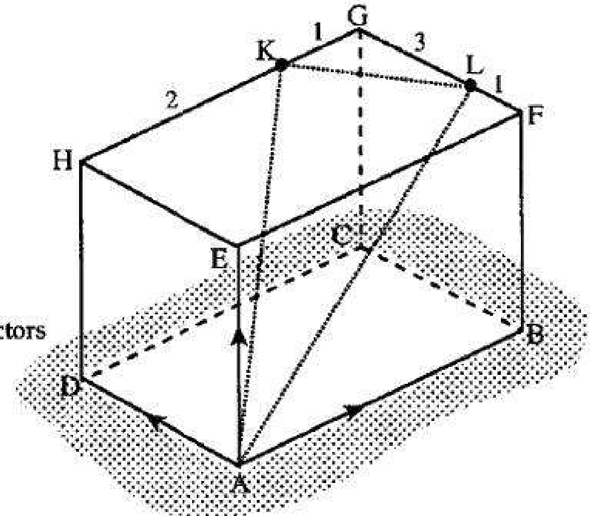
•6 $\overrightarrow{AC} = \sqrt{90}$

•7 $B\hat{A}C = 51 \cdot 9°$

(*c*) •8 **identify** 2 sides and included angle *e.g.* $\sqrt{54}$, $\sqrt{90}$, $B\hat{A}C$

•9 $27 \cdot 4$

[SQA] 12. ABCDEFGH is a cuboid.



K lies two thirds of the way along HG.
(i.e. HK:KG = 2:1).
L lies one quarter of the way along FG.
(i.e. FL:LG = 1:3).

$\overrightarrow{AB}$, $\overrightarrow{AD}$ and $\overrightarrow{AE}$ can be represented by the vectors

$\begin{pmatrix}3\\6\\3\end{pmatrix}$, $\begin{pmatrix}-8\\4\\4\end{pmatrix}$ and $\begin{pmatrix}1\\-3\\5\end{pmatrix}$ respectively.

(a) Calculate the components of $\overrightarrow{AK}$. **2**

(b) Calculate the components of $\overrightarrow{AL}$. **2**

(c) Calculate the size of angle KAL. **5**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(a)	2	C	CN	G20		1999 P2 Q3
(b)	2	C	CN	G20		
(c)	5	C	CN	G28		

(a) •1 obtaining for example $\begin{pmatrix}2\\4\\2\end{pmatrix}$

•2 $\overrightarrow{AK} = \begin{pmatrix}-5\\5\\11\end{pmatrix}$

(b) •3 obtaining for example $\begin{pmatrix}-2\\1\\1\end{pmatrix}$

•4 $\overrightarrow{AL} = \begin{pmatrix}2\\4\\9\end{pmatrix}$

(c) •5 *strategy* *e.g.* $\cos K\hat{A}L = \frac{\overrightarrow{AK}.\overrightarrow{AL}}{|AK|\times|AL|}$

•6 109

•7 $\sqrt{171}$

•8 $\sqrt{101}$

•9 $\hat{A} = 34.0$

OR

•5 *strategy* *e.g.* $\cos K\hat{A}L = \frac{AK^2 + AL^2 - KL^2}{2AK \times AL}$

•6 $\sqrt{54}$

•7 $\sqrt{171}$

•8 $\sqrt{101}$

•9 $\hat{A} = 34.0$

[SQA] 13. Diagram 1 shows a christmas tree decoration which is made of coloured glass rods in the shape of a square-based prism topped by a square pyramid. Diagram 2 shows the decoration relative to the origin and rectangular coordinate axes OX, OY and OZ.

The vertex F has position vector $\begin{pmatrix} 2 \\ 2 \\ -7 \end{pmatrix}$

and the vertex V has position vector $\begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$

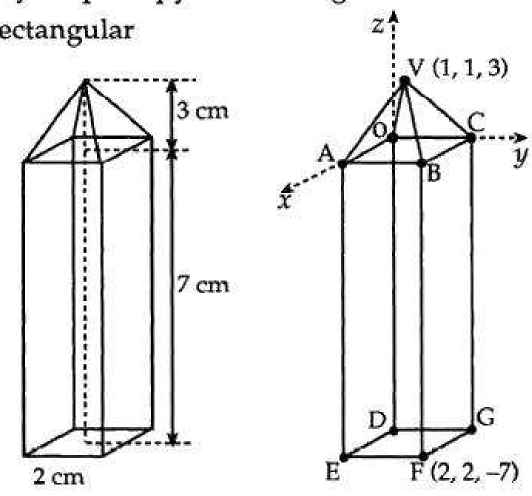


Diagram 1 Diagram 2

(*a*) Find

(i) the components of the vectors represented by $\vec{VF}$ and $\vec{VE}$;

(ii) the size of angle EVF. **(7)**

(*b*) To make the decoration more attractive, triangular sheets of coloured glass VEF and VDG are added to it.

Calculate the area of the glass triangle VEF. **(3)**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	7	C	CR	G28, G16		1991 P2 Q5
(*b*)	3	C	CR	CGD		

(*a*)

•1 $\vec{VF} = \begin{pmatrix} 1 \\ 1 \\ -10 \end{pmatrix}$

•2 $E = (2, 0, -7)$

•3 $\vec{VE} = \begin{pmatrix} 1 \\ -1 \\ -10 \end{pmatrix}$

•4 $\cos E\hat{V}F = \frac{\vec{VE}.\vec{VF}}{|\vec{VE}||\vec{VF}|}$ This may appear as $\frac{100}{102}$ after the completion of •5 and •6.

•5 $\vec{VE}.\vec{VF} = 100$

•6 $|\vec{VE}||\vec{VF}| = 102$

•7 $11.4°$

(*b*)

•8 $\frac{1}{2} VE \times VF \sin E\hat{V}F$

•9 $\frac{1}{2} \times 102 \times \sin 11.4°$

•10 10.02

[END OF QUESTIONS]