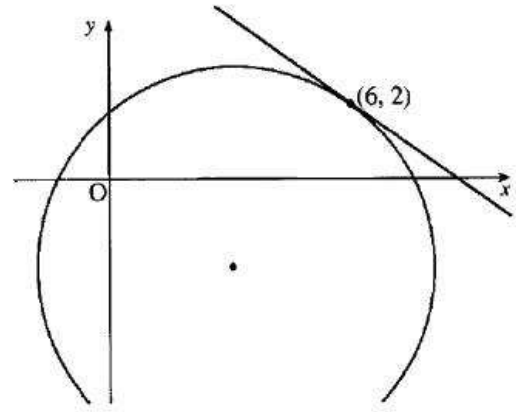
circle tangents

[SQA] 1. The circle shown has equation

$(x-3)^2+(y+2)^2=25$.

Find the equation of the tangent at the point (6, 2). 4



Part	Marks	Level	Calc.	Content	Answer	U2 OC4
	4	C	CN	G11		1998 P1 Q4

•1 Centre = (3,–2)

•2 $m_{rad}=\frac{4}{3}$

•3 $m_{tgt}=-\frac{3}{4}$

•4 $y-2=-\frac{3}{4}(x-6)$

[SQA] 2. Circle P has equation $x^2 + y^2 - 8x - 10y + 9 = 0$. Circle Q has centre $(-2, -1)$ and radius $2\sqrt{2}$.

(a) (i) Show that the radius of circle P is $4\sqrt{2}$.

(ii) Hence show that circles P and Q touch. **4**

(b) Find the equation of the tangent to the circle Q at the point $(-4, 1)$. **3**

(c) The tangent in (b) intersects circle P in two points. Find the x-coordinates of the points of intersection, expressing you answers in the form $a \pm b\sqrt{3}$. **3**

Part	Marks	Level	Calc.	Content	Answer	U2 OC4
(a)	2	C	CN	G9	proof	2001 P1 Q11
(a)	2	A/B	CN	G14		
(b)	3	C	CN	G11	$y = x + 5$	
(c)	3	C	CN	G12	$x = 2 \pm 2\sqrt{3}$	

$\bullet^1$ ic: interpret centre of circle (P)	$\bullet^1$ $C_P = (4, 5)$
$\bullet^2$ ss: find radius of circle (P)	$\bullet^2$ $r_P = \sqrt{16 + 25 - 9} = \sqrt{32} = 4\sqrt{2}$
$\bullet^3$ ss: find sum of radii	$\bullet^3$ $r_P + r_Q = 4\sqrt{2} + 2\sqrt{2} = 6\sqrt{2}$
$\bullet^4$ pd: compare with distance between centres	$\bullet^4$ $C_PC_Q = \sqrt{6^2 + 6^2} = 6\sqrt{2}$ **and** "so touch"
$\bullet^5$ ss: find gradient of radius	$\bullet^5$ $m_r = -1$
$\bullet^6$ ss: use $m_1 m_2 = -1$	$\bullet^6$ $m_{tgt} = +1$
$\bullet^7$ ic: state equation of tangent	$\bullet^7$ $y - 1 = 1(x + 4)$
$\bullet^8$ ss: substitute linear into circle	$\bullet^8$ $x^2 + (x+5)^2 - 8x - 10(x+5) + 9 = 0$
$\bullet^9$ pd: express in standard form	$\bullet^9$ $2x^2 - 8x - 16 = 0$
$\bullet^{10}$ pd: solve (quadratic) equation	$\bullet^{10}$ $x = 2 \pm 2\sqrt{3}$

[SQA] 3. The point P$(2, 3)$ lies on the circle $(x + 1)^2 + (y - 1)^2 = 13$. Find the equation of the tangent at P. **4**

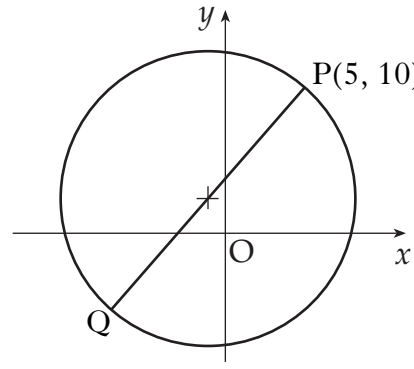
Part	Marks	Level	Calc.	Content	Answer	U2 OC4
	4	C	CN	G11	$2y + 3x = 12$	2002 P1 Q1

$\bullet^1$ ic: interpret centre from equ. of circle	$\bullet^1$ $C = (-1, 1)$
$\bullet^2$ ss: know to find gradient of radius	$\bullet^2$ $m_{rad} = \frac{2}{3}$
$\bullet^3$ ss: know to find perp. gradient	$\bullet^3$ $m_{tgt} = -\frac{3}{2}$
$\bullet^4$ ic: state equation of tangent	$\bullet^4$ $y - 3 = -\frac{3}{2}(x - 2)$

[SQA] 4.

(*a*) Show that the point $P(5, 10)$ lies on circle C_1 with equation $(x+1)^2 + (y-2)^2 = 100$. **1**

(*b*) PQ is a diameter of this circle as shown in the diagram. Find the equation of the tangent at Q. **5**



(*c*) Two circles, C_2 and C_3, touch circle C_1 at Q.

The radius of each of these circles is twice the radius of circle C_1.

Find the equations of circles C_2 and C_3. **4**

Part	Marks	Level	Calc.	Content	Answer	U2 OC4
(*a*)	1	C	CN	A6	proof	2009 P2 Q4
(*b*)	5	C	CN	G11	$3x + 4y + 45 = 0$	
(*c*)	4	A	NC	G15	$(x-5)^2 + (y-10)^2 = 400$, $(x+19)^2 + (y+22)^2 = 400$	

$\bullet^1$ pd: substitute	$\bullet^1$ $(5+1)^2 + (10-2)^2 = 100$
$\bullet^2$ ic: find centre	$\bullet^2$ centre $= (-1, 2)$
$\bullet^3$ ss: use mid-point result for Q	$\bullet^3$ $Q = (-7, -6)$
$\bullet^4$ ss: know to, and find gradient of radius	$\bullet^4$ $m_{\text{rad}} = \frac{8}{6}$
$\bullet^5$ ic: find gradient of tangent	$\bullet^5$ $m_{\text{tgt}} = -\frac{3}{4}$
$\bullet^6$ ic: state equation of tangent	$\bullet^6$ $y - (-6) = -\frac{3}{4}(x - (-7))$
$\bullet^7$ ic: state radius	$\bullet^7$ radius $= 20$
$\bullet^8$ ss: know how to find centre	$\bullet^8$ centre $= (5, 10)$
$\bullet^9$ ic: state equation of one circle	$\bullet^9$ $(x-5)^2 + (y-10)^2 = 400$
$\bullet^{10}$ ic: state equation of the other circle	$\bullet^{10}$ $(x+19)^2 + (y+22)^2 = 400$

[SQA] 5. Find the equation of the tangent at the point $(3,1)$ on the circle $x^2+y^2-4x+6y-4=0$. **5**

Part	Marks	Level	Calc.	Content	Answer	U2 OC4
	5	C	CN	G11		1991 P1 Q8

- $\bullet^1$ *strat*: use centre and tgt $\perp$ radius
- $\bullet^2$ *centre* $= (2,-3)$
- $\bullet^3$ $m_{radius} = 4$
- $\bullet^4$ $m_{tgt} = -\frac{1}{4}$
- $\bullet^5$ $y-1 = -\frac{1}{4}(x-3)$

[SQA] 6. (*a*) Find the equation of AB, the perpendicular bisector of the line joing the points $P(-3,1)$ and $Q(1,9)$. **4**

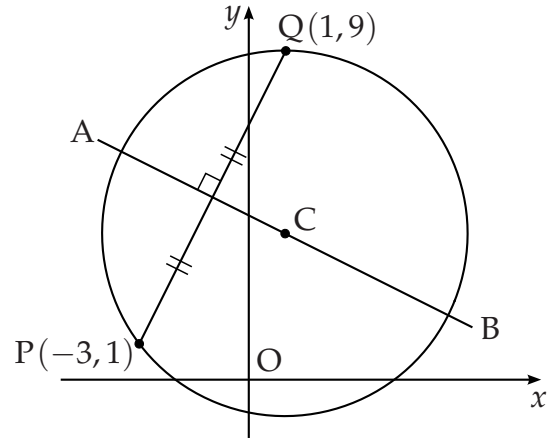
(*b*) C is the centre of a circle passing through P and Q. Given that QC is parallel to the y-axis, determine the equation of the circle. **3**

(*c*) The tangents at P and Q intersect at T.

Write down

(i) the equation of the tangent at Q

(ii) the coordinates of T. **2**



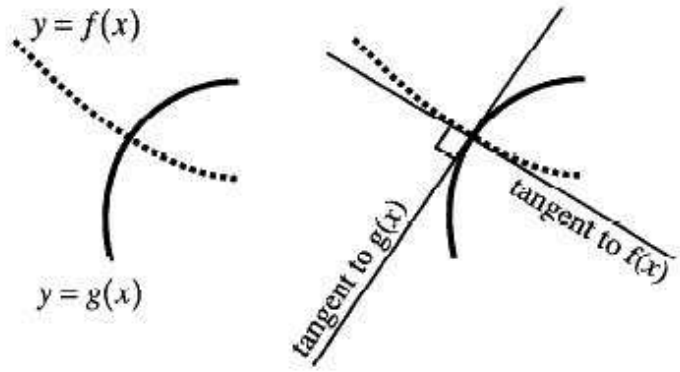
Part	Marks	Level	Calc.	Content	Answer	U2 OC4
(*a*)	4	C	CN	G7	$x+2y=9$	2000 P2 Q2
(*b*)	3	C	CN	G10	$(x-1)^2+(y-4)^2=25$	
(*c*)	2	C	CN	G11, G8	(i) $y=9$, (ii) $T(-9,9)$	

- $\bullet^1$ ss: know to use midpoint
- $\bullet^2$ pd: process gradient of PQ
- $\bullet^3$ ss: know how to find perp. gradient
- $\bullet^4$ ic: state equ. of line
- $\bullet^5$ ic: interpret "parallel to y-axis"
- $\bullet^6$ pd: process radius
- $\bullet^7$ ic: state equ. of circle
- $\bullet^8$ ic: interpret diagram
- $\bullet^9$ ss: know to use equ. of AB

- $\bullet^1$ midpoint $= (-1,5)$
- $\bullet^2$ $m_{PQ} = \frac{9-1}{1-(-1)}$
- $\bullet^3$ $m_\perp = -\frac{1}{2}$
- $\bullet^4$ $y-5 = -\frac{1}{2}(x-(-1))$
- $\bullet^5$ $y_C = 4$ *stated or implied by* $\bullet^7$
- $\bullet^6$ radius $= 5$ or equiv. *stated or implied by* $\bullet^7$
- $\bullet^7$ $(x-1)^2+(y-4)^2=25$
- $\bullet^8$ $y=9$
- $\bullet^9$ T$=(-9,9)$

[SQA] 7.

Two curves, $y = f(x)$ and $y = g(x)$, are called orthogonal if, at each point of intersection, their tangents are at right angles to each other.



(*a*) Diagram 1 shows the parabola with equation $y = 6 + \frac{1}{9}x^2$ and the circle M with equation $x^2 + (y-5)^2 = 13$.
These two curves intersect at (3, 7) and (–3, 7).
Prove that these curves are orthogonal. **(6)**

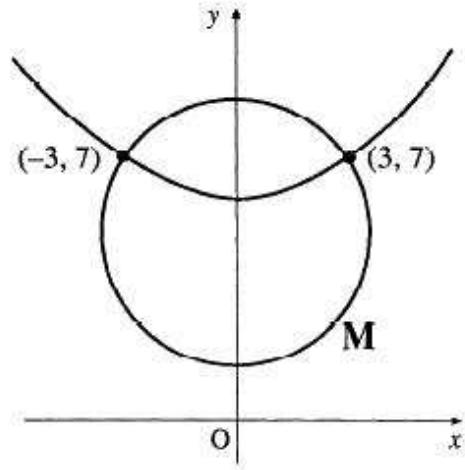


Diagram 1

(*b*) Diagram 2 shows the circle M, from (*a*) above, which is orthogonal to the circle N. The circles intersect at (3, 7) and (–3, 7).

(i) Write down the equation of the tangent to circle M at the point (–3, 7). **(1)**

(ii) Hence find the equation of circle N. **(3)**

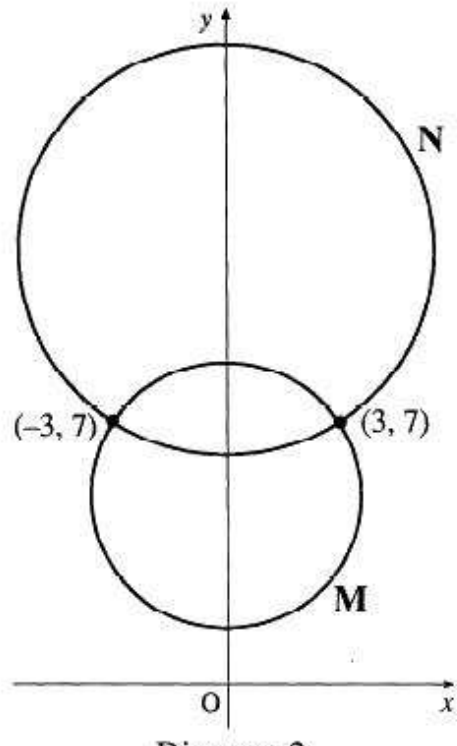


Diagram 2

Part	Marks	Level	Calc	Content	Answer	U2 OC4