circles points of intersection

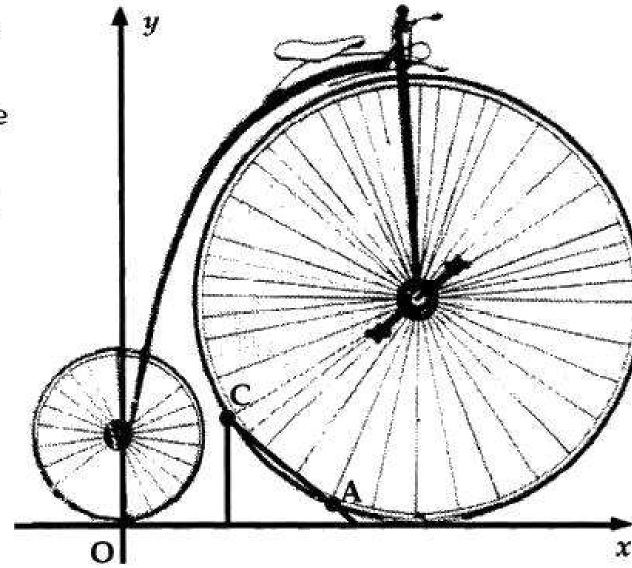
[SQA] 1. A penny-farthing bicycle on display in a museum is supported by a stand at points A and C. A and C lie on the front wheel.

With coordinate axes as shown and 1 unit = 5cm, the equation of the rear wheel (the small wheel) is

$x^2 + y^2 - 6y = 0$ and

the equation of the front wheel is

$x^2 + y^2 - 28x - 20y + 196 = 0$.



(*a*) (i) Find the distance between the centres of the two wheels.

(ii) Hence calculate the clearance, i.e. the smallest gap, between the front and rear wheels. Give your answer to the nearest millimetre. **(8)**

(*b*) B(7,3) is half-way between A and C, and P is the centre of the front wheel.

(i) Find the gradient of PB.

(ii) Hence find the equation of AC and the coordinates of A and C. **(8)**

Part	Marks	Level	Calc.	Content	Answer	U2 OC4
(*a*)	8	C	CR	G9, G1		1994 P2 Q4
(*b*)	8	C	CR	G2, G5, G12		

(*a*)

- •1 centre (0, 3)
- •2 centre (14, 10)
- •3 distance between centres = $\sqrt{245}$
- •4 radius = 3
- •5 radius = 10
- •6 strategy (clearance = distance between centres minus sum of radii)
- •7 $\sqrt{245} - 13$
- •8 133 mm or equivalent

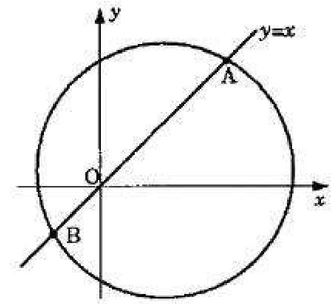
(*b*)

- •9 $m_{PB} = 1$
- •10 $m_{AC} = -1$
- •11 $y - 3 = -(x - 7)$ for AC
- •12 strategy: substitute
- •13 substituting correctly
- •14 *eg* $2x^2 - 28x + 96 = 0$
- •15 $x = 6,\ 8$ (*or* $y = 2, 4$)
- •16 $(6, 4)$ **and** $(8, 2)$

[SQA] 2. The straight line $y = x$ cuts the circle $x^2 + y^2 - 6x - 2y - 24 = 0$ at A and B.

(*a*) Find the coordinates of A and B. 3

(*b*) Find the equation of the circle which has AB as diameter. 3



Part	Marks	Level	Calc.	Content	Answer	U2 OC4
(*a*)	3	C	CN	G12		1994 P1 Q8
(*b*)	3	C	CN	G10		

•[1] $x^2 + x^2 - 6x - 2x - 24 = 0$

•[2] $(x+2)(x-6) = 0$

•[3] $(-2,-2)$ and $(6,6)$

OR

•[4] centre is $(2,2)$

•[5] radius is $\sqrt{32}$ or equivalent

•[6] $(x-2)^2 + (y-2)^2 = 32$

[SQA] 3. Circle P has equation $x^2 + y^2 - 8x - 10y + 9 = 0$. Circle Q has centre $(-2, -1)$ and radius $2\sqrt{2}$.

(*a*) (i) Show that the radius of circle P is $4\sqrt{2}$.

(ii) Hence show that circles P and Q touch. **4**

(*b*) Find the equation of the tangent to the circle Q at the point $(-4, 1)$. **3**

(*c*) The tangent in (*b*) intersects circle P in two points. Find the x-coordinates of the points of intersection, expressing you answers in the form $a \pm b\sqrt{3}$. **3**

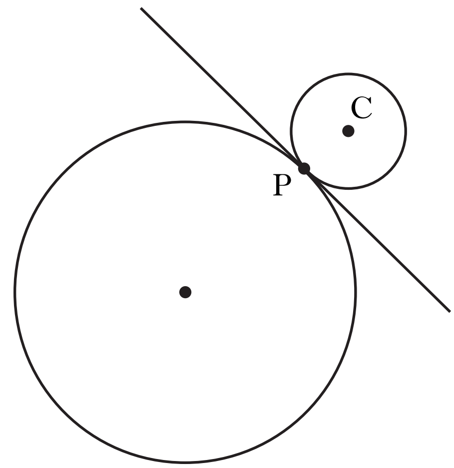
Part	Marks	Level	Calc.	Content	Answer	U2 OC4
(*a*)	2	C	CN	G9	proof	2001 P1 Q11
(*a*)	2	A/B	CN	G14		
(*b*)	3	C	CN	G11	$y = x + 5$	
(*c*)	3	C	CN	G12	$x = 2 \pm 2\sqrt{3}$	

•1 ic: interpret centre of circle (P)	•1 $C_P = (4, 5)$
•2 ss: find radius of circle (P)	•2 $r_P = \sqrt{16 + 25 - 9} = \sqrt{32} = 4\sqrt{2}$
•3 ss: find sum of radii	•3 $r_P + r_Q = 4\sqrt{2} + 2\sqrt{2} = 6\sqrt{2}$
•4 pd: compare with distance between centres	•4 $C_PC_Q = \sqrt{6^2 + 6^2} = 6\sqrt{2}$ **and** "so touch"
•5 ss: find gradient of radius	•5 $m_r = -1$
•6 ss: use $m_1 m_2 = -1$	•6 $m_{tgt} = +1$
•7 ic: state equation of tangent	•7 $y - 1 = 1(x + 4)$
•8 ss: substitute linear into circle	•8 $x^2 + (x+5)^2 - 8x - 10(x+5) + 9 = 0$
•9 pd: express in standard form	•9 $2x^2 - 8x - 16 = 0$
•10 pd: solve (quadratic) equation	•10 $x = 2 \pm 2\sqrt{3}$

4. (*a*) (i) Show that the line with equation $y = 3 - x$ is a tangent to the circle with equation $x^2 + y^2 + 14x + 4y - 19 = 0$.

(ii) Find the coordinates of the points of contact, P. **5**

(*b*) Relative to a suitable set of coordinate axes, the diagram below shows the circle from (*a*) and a second smaller circle with centre C.



The line $y = 3 - x$ is a common tangent at the point P.

The radius of the larger circle is three times the radius of the smaller circle.

Find the equation of the smaller circle. **6**

Part	Marks	Level	Calc.	Content	Answer	U2 OC4
(*ai*)	4	C	CN	G13	proof	2010 P2 Q3
(*aii*)	1	C	CN	G12	$P(-1, 4)$	
(*b*)	6	B	CN	G9, G15	$(x-1)^2 + (y-6)^2 = 8$	

$\bullet^1$	ss: substitute	$x^2 + (3-x)^2 + 14x + 4(3-x) - 19 = 0$
$\bullet^2$	pd: express in standard form	$2x^2 + 4x + 2 = 0$
$\bullet^3$	ic: start proof	$2(x+1)(x+1)$
$\bullet^4$	ic: complete proof	equal roots so line is a tangent
$\bullet^5$	pd: coordinates of P	$x = -1, y = 4$
$\bullet^6$	ic: state centre of larger circle	$(-7, -2)$
$\bullet^7$	ss: find radius of larger circle	$\sqrt{72}$
$\bullet^8$	pd: find radius of smaller circle	$\sqrt{8}$
$\bullet^9$	ss: strategy for finding centre	e.g. "Stepping out"
$\bullet^{10}$	ic: interpret centre of smaller circle	$(1, 6)$
$\bullet^{11}$	ic: state equation	$(x-1)^2 + (y-6)^2 = 8$

[SQA] 5. Diagram 1 shows a circle with equation $x^2 + y^2 + 10x - 2y - 14 = 0$ and a straight line, l_1, with equation $y = 2x + 1$.
The line intersects the circle at A and B.

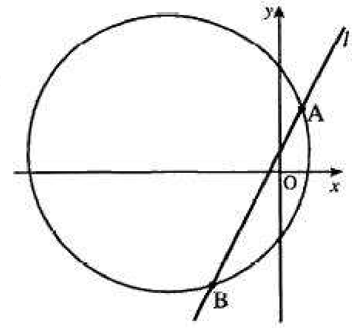


Diagram 1

(*a*) Find the coordinates of the points A and B. **(5)**

(*b*) Diagram 2 shows a second line, l_2, which passes through the centre of the circle, C, and is at right angles to line l_1.

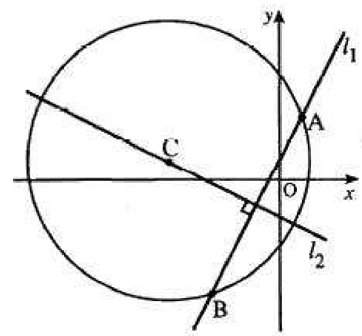


Diagram 2

(i) Write down the coordinates of C. **(1)**

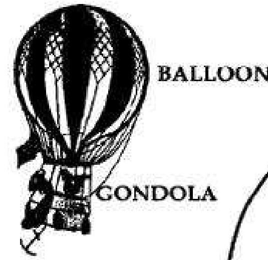
(ii) Find the equation of the line l_2. **(3)**

Part	Marks	Level	Calc.	Content	Answer	U2 OC4
(*a*)	5	C	CN	G12		1997 P2 Q1
(*bi*)	1	C	CN	G9		
(*bii*)	3	C	CN	G8, G3		

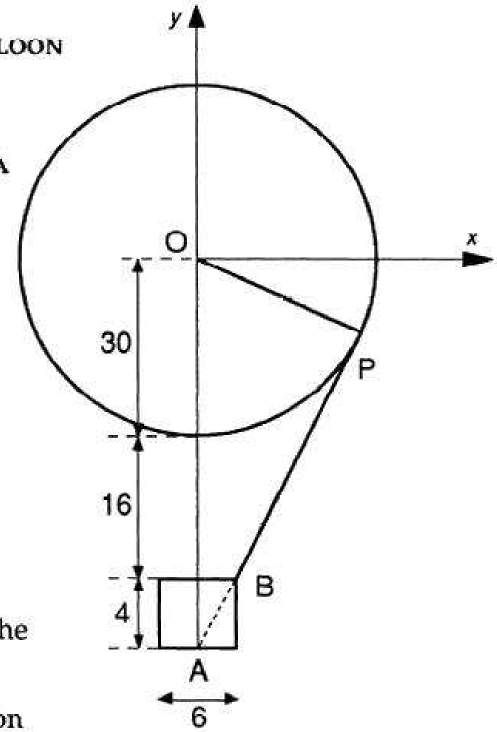
(*a*)
- •[1] know to substitute
- •[2] correct substitution
- •[3] a "quadratic" = 0
- •[4] $x = -3,\ 1$
- •[5] $y - -5,\ 3$

(*b*)
- •[6] $m_{\text{diameter}} = 2$
- •[7] $m_{\text{perpendicular}} = -\frac{1}{2}$
- •[8] centre = $(-1, -1)$
- •[9] equation: $y + 1 = -\frac{1}{2}(x + 1)$

[SQA] 6. A spherical hot-air balloon has radius 30 feet. Cables join the balloon to the gondola which is cylindrical with diameter 6 feet and height 4 feet. The top of the gondola is 16 feet below the bottom of the balloon.



Co-ordinate axes are chosen as shown in the diagram. One of the cables is represented by PB and PBA is a straight line.



(*a*) Find the equation of the cable PB. **(3)**

(*b*) State the equation of the circle representing the balloon. **(1)**

(*c*) Prove that this cable is a tangent to the balloon and find the co-ordinates of the point P. **(5)**

Part	Marks	Level	Calc.	Content	Answer	U2 OC4
(*a*)	3	C	CN	G2, G3		1992 P2 Q9
(*b*)	1	C	CN	G10		
(*c*)	2	C	CN	G12, G13		
(*c*)	3	A/B	CN	G12, G13		

(*a*) •1 Strategy: know to find m

•2 $m = \frac{4}{3}$

•3 $y + 46 = \frac{4}{3}(x - 3)$

(*b*) •4 $x^2 + y^2 = 900$ or equivalent

(*c*) •5 Strategy: know to substitute

•6 $x^2 + \left(\frac{4}{3}x - 50\right)^2 = 900$

•7 $(x - 24)^2$ or evaluate the discriminant

•8 communication relating to tangency

•9 $(24, -18)$

[END OF QUESTIONS]