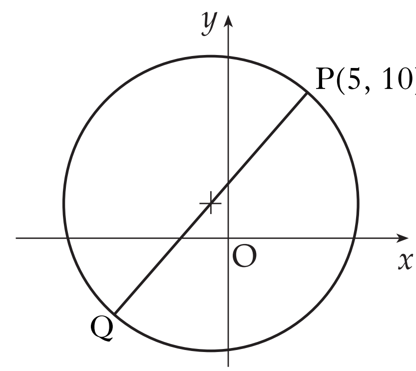
circles problems

[SQA] 1.

(a) Show that the point $\mathrm{P}(5, 10)$ lies on circle C_1 with equation $(x+1)^2 + (y-2)^2 = 100$. **1**

(b) PQ is a diameter of this circle as shown in the diagram. Find the equation of the tangent at Q. **5**



(c) Two circles, C_2 and C_3, touch circle C_1 at Q.

The radius of each of these circles is twice the radius of circle C_1.

Find the equations of circles C_2 and C_3. **4**

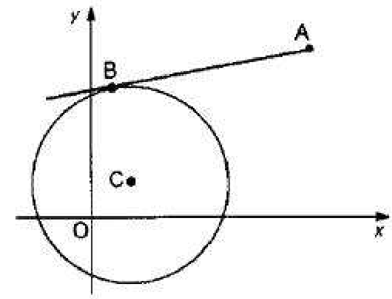
Part	Marks	Level	Calc.	Content	Answer	U2 OC4
(a)	1	C	CN	A6	proof	2009 P2 Q4
(b)	5	C	CN	G11	$3x + 4y + 45 = 0$	
(c)	4	A	NC	G15	$(x-5)^2 + (y-10)^2 = 400$, $(x+19)^2 + (y+22)^2 = 400$	

•[1] pd: substitute	•[1] $(5+1)^2 + (10-2)^2 = 100$
•[2] ic: find centre	•[2] centre $= (-1, 2)$
•[3] ss: use mid-point result for Q	•[3] $\mathrm{Q} = (-7, -6)$
•[4] ss: know to, and find gradient of radius	•[4] $m_{\text{rad}} = \frac{8}{6}$
•[5] ic: find gradient of tangent	•[5] $m_{\text{tgt}} = -\frac{3}{4}$
•[6] ic: state equation of tangent	•[6] $y - (-6) = -\frac{3}{4}(x - (-7))$
•[7] ic: state radius	•[7] radius $= 20$
•[8] ss: know how to find centre	•[8] centre $= (5, 10)$
•[9] ic: state equation of one circle	•[9] $(x-5)^2 + (y-10)^2 = 400$
•[10] ic: state equation of the other circle	•[10] $(x+19)^2 + (y+22)^2 = 400$

[SQA] 2. AB is a tangent at B to the circle with centre C and equation $(x-2)^2+(y-2)^2=25$.

The point A has co-ordinates (10, 8).

Find the area of triangle ABC. 5



Part	Marks	Level	Calc.	Content	Answer	U2 OC4
	5	C	CN	G9, G1, G15		1992 P1 Q16

•[1] *strat:* *i.e* find AC then AB

•[2] centre = (2,2) and radius = 5

•[3] $AC = 10$

•[4] $AB = \sqrt{75}$ units

•[5] area = $\frac{25}{2}\sqrt{3}$ square units

3. Circle C_1 has equation $(x+1)^2+(y-1)^2=121$.

A circle C_2 with equation $x^2+y^2-4x+6y+p=0$ is drawn inside C_1.

The circles have no points of contact.

What is the range of values of p? **9**

Part	Marks	Level	Calc.	Content	Answer	U2 OC4
	9	A	CN	G9, G15	$-23 < p < 13$	2011 P2 Q7

•[1] ic: state centre of C_1	•[1] $(-1,1)$
•[2] ic: state radius of C_1	•[2] 11 ($\sqrt{121}$ not accepted)
•[3] ic: state centre of C_2	•[3] $(2,-3)$
•[4] pd: find radius of C_2 in terms of p	•[4] $\sqrt{13-p}$
•[5] ic: interpret upper bound for p	•[5] $p < 13$
•[6] ic: find distance between centres, d	•[6] 5
•[7] ss: identify relevant relationship	•[7] $\sqrt{13-p} < 6$ or $r_2 + d < 11$ or $r_2 < 6$
•[8] ic: develop relationship by squaring	•[8] $13-p < 36$
•[9] pd: find lower bound for p	•[9] $p > -23$

[SQA] 4. Circle P has equation $x^2 + y^2 - 8x - 10y + 9 = 0$. Circle Q has centre $(-2, -1)$ and radius $2\sqrt{2}$.

(*a*) (i) Show that the radius of circle P is $4\sqrt{2}$.

(ii) Hence show that circles P and Q touch. **4**

(*b*) Find the equation of the tangent to the circle Q at the point $(-4, 1)$. **3**

(*c*) The tangent in (*b*) intersects circle P in two points. Find the x-coordinates of the points of intersection, expressing you answers in the form $a \pm b\sqrt{3}$. **3**

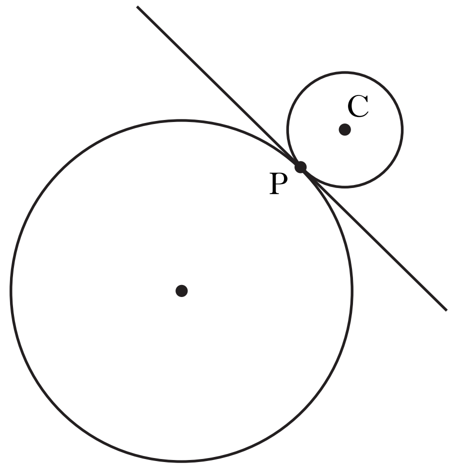
Part	Marks	Level	Calc.	Content	Answer	U2 OC4
(*a*)	2	C	CN	G9	proof	2001 P1 Q11
(*a*)	2	A/B	CN	G14		
(*b*)	3	C	CN	G11	$y = x + 5$	
(*c*)	3	C	CN	G12	$x = 2 \pm 2\sqrt{3}$	

•1 ic: interpret centre of circle (P)	•1 $C_P = (4, 5)$
•2 ss: find radius of circle (P)	•2 $r_P = \sqrt{16 + 25 - 9} = \sqrt{32} = 4\sqrt{2}$
•3 ss: find sum of radii	•3 $r_P + r_Q = 4\sqrt{2} + 2\sqrt{2} = 6\sqrt{2}$
•4 pd: compare with distance between centres	•4 $C_PC_Q = \sqrt{6^2 + 6^2} = 6\sqrt{2}$ **and** "so touch"
•5 ss: find gradient of radius	•5 $m_r = -1$
•6 ss: use $m_1 m_2 = -1$	•6 $m_{tgt} = +1$
•7 ic: state equation of tangent	•7 $y - 1 = 1(x + 4)$
•8 ss: substitute linear into circle	•8 $x^2 + (x+5)^2 - 8x - 10(x+5) + 9 = 0$
•9 pd: express in standard form	•9 $2x^2 - 8x - 16 = 0$
•10 pd: solve (quadratic) equation	•10 $x = 2 \pm 2\sqrt{3}$

5. (*a*) (i) Show that the line with equation $y = 3 - x$ is a tangent to the circle with equation $x^2 + y^2 + 14x + 4y - 19 = 0$.

(ii) Find the coordinates of the points of contact, P. **5**

(*b*) Relative to a suitable set of coordinate axes, the diagram below shows the circle from (*a*) and a second smaller circle with centre C.



The line $y = 3 - x$ is a common tangent at the point P.

The radius of the larger circle is three times the radius of the smaller circle.

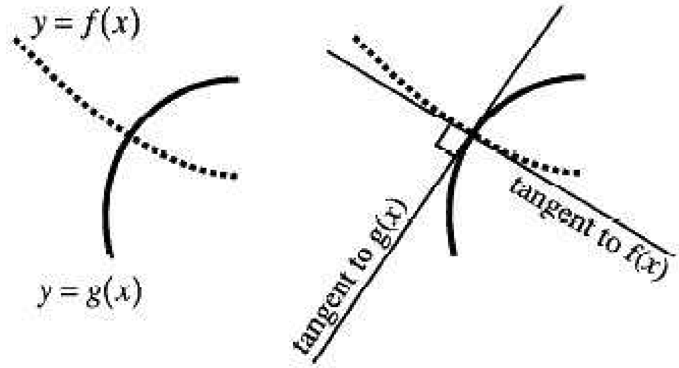
Find the equation of the smaller circle. **6**

Part	Marks	Level	Calc.	Content	Answer	U2 OC4
(*ai*)	4	C	CN	G13	proof	2010 P2 Q3
(*aii*)	1	C	CN	G12	$P(-1, 4)$	
(*b*)	6	B	CN	G9, G15	$(x-1)^2 + (y-6)^2 = 8$	

•[1] ss: substitute	•[1] $x^2 + (3-x)^2 + 14x + 4(3-x) - 19 = 0$
•[2] pd: express in standard form	•[2] $2x^2 + 4x + 2 = 0$
•[3] ic: start proof	•[3] $2(x+1)(x+1)$
•[4] ic: complete proof	•[4] equal roots so line is a tangent
•[5] pd: coordinates of P	•[5] $x = -1, y = 4$
•[6] ic: state centre of larger circle	•[6] $(-7, -2)$
•[7] ss: find radius of larger circle	•[7] $\sqrt{72}$
•[8] pd: find radius of smaller circle	•[8] $\sqrt{8}$
•[9] ss: strategy for finding centre	•[9] e.g. "Stepping out"
•[10] ic: interpret centre of smaller circle	•[10] $(1, 6)$
•[11] ic: state equation	•[11] $(x-1)^2 + (y-6)^2 = 8$

[SQA] 6.

Two curves, $y = f(x)$ and $y = g(x)$, are called orthogonal if, at each point of intersection, their tangents are at right angles to each other.



(*a*) Diagram 1 shows the parabola with equation $y = 6 + \frac{1}{9}x^2$ and the circle M with equation $x^2 + (y-5)^2 = 13$.
These two curves intersect at $(3, 7)$ and $(-3, 7)$.
Prove that these curves are orthogonal. **(6)**

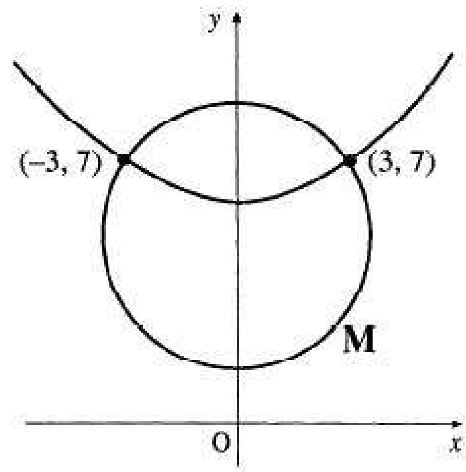


Diagram 1

(*b*) Diagram 2 shows the circle M, from (*a*) above, which is orthogonal to the circle N. The circles intersect at $(3, 7)$ and $(-3, 7)$.

(i) Write down the equation of the tangent to circle M at the point $(-3, 7)$. **(1)**

(ii) Hence find the equation of circle N. **(3)**

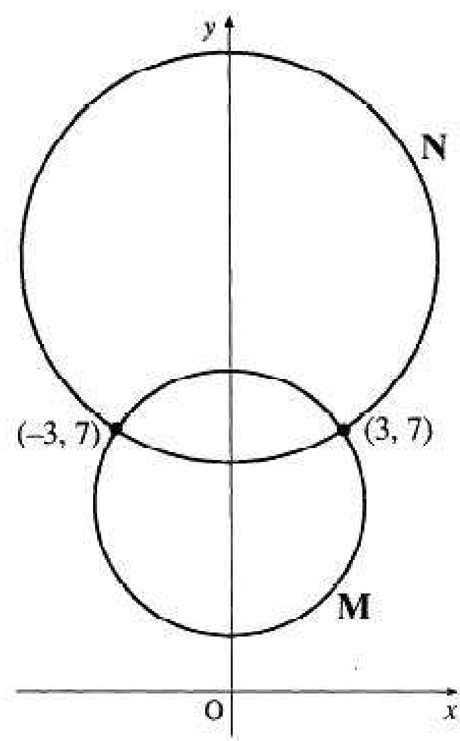


Diagram 2

Part	Marks	Level	Calc.	Content	Answer	U2 OC4