composite functions

[SQA] 1. $f(x) = 3 - x$ and $g(x) = \frac{3}{x}, x \neq 0$.

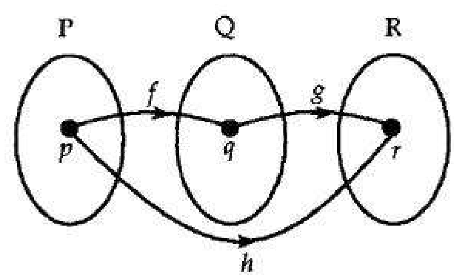
(a) Find $p(x)$ where $p(x) = f(g(x))$. **2**

(b) If $q(x) = \frac{3}{3-x}, x \neq 3$, find $p(q(x))$ in its simplest form. **3**

Part	Marks	Level	Calc.	Content	Answer	U1 OC2
(a)	2	C	CN	A4	$3 - \frac{3}{x}$	2000 P2 Q3
(b)	2	C	CN	A4	x	
(b)	1	A/B	CN	A4		

•[1] ic: interpret composite func.	•[1] $f\left(\frac{3}{x}\right)$ *stated or implied by* •[2]
•[2] pd: process	•[2] $3 - \frac{3}{x}$
•[3] ic: interpret composite func.	•[3] $p\left(\frac{3}{3-x}\right)$ *stated or implied by* •[4]
•[4] pd: process	•[4] $3 - \frac{3}{\frac{3}{3-x}}$
•[5] pd: process	•[5] x

[SQA] 2. The diagram illustrates three functions f, g and h. The functions are defined by $f(x) = 2x + 5$ and $g(x) = x^2 - 3$.



The function h is such that whenever $f(p) = q$ and $g(q) = r$ then $h(p) = r$.

(a) If $q = 7$, find the values of p and r. **2**

(b) Find a formula for $h(x)$, in terms of x. **2**

Part	Marks	Level	Calc.	Content	Answer	U1 OC2
(a)	2	C	NC	A4		1991 P1 Q19
(b)	2	A/B	NC	A4		

•[1] $p = 1$

•[2] $r = 46$

•[3] $h(x) = g(f(x))$

•[4] $h(x) = (2x+5)^2 - 3$

[SQA] 3. On a suitable set of real numbers, functions f and g are defined by $f(x) = \dfrac{1}{x+2}$ and $g(x) = \dfrac{1}{x} - 2$.

Find $f(g(x))$ in its simplest form. **3**

Part	Marks	Level	Calc.	Content	Answer	U1 OC2
	3	C	NC	A4		1992 P1 Q6

•1 $f\left(\frac{1}{x}-2\right)$

•2 $\dfrac{1}{\frac{1}{x}-2+2}$

•3 x

[SQA] 4. $f(x) = 2x - 1$, $g(x) = 3 - 2x$ and $h(x) = \frac{1}{4}(5 - x)$.

(*a*) Find a formula for $k(x)$ where $k(x) = f(g(x))$. **2**

(*b*) Find a formula for $h(k(x))$. **2**

(*c*) What is the connection between the functions h and k? **1**

Part	Marks	Level	Calc.	Content	Answer	U1 OC2
(*a*)	2	C	NC	A4		1993 P1 Q13
(*b*)	2	C	NC	A4		
(*c*)	1	A/B	NC	CGD		

•1 $f(3-2x)$

•2 $5-4x$

•3 $h(5-4x)$

•4 x

•5 inverse of each other

[SQA] 5. A function f is defined on the set of real numbers by $f(x) = \frac{x}{1-x}$, $x \neq 1$.

Find, in its simplest form, an expression for $f(f(x))$. **3**

Part	Marks	Level	Calc.	Content	Answer	U1 OC2
	1	C	NC	A4		1994 P1 Q19
	2	A/B	NC	A4		

•1 $f\left(\frac{x}{1-x}\right)$

•2 $\dfrac{\frac{x}{1-x}}{1-\frac{x}{1-x}}$

•3 $\frac{x}{1-2x}$

[SQA] 6. The functions f and g, defined on suitable domains, are given by $f(x) = \frac{1}{x^2-4}$ and $g(x) = 2x+1$.

(*a*) Find an expression for $h(x)$ where $h(x) = g(f(x))$. Give your answer as a single fraction. **3**

(*b*) State a suitable domain for h. **1**

Part	Marks	Level	Calc.	Content	Answer	U1 OC2
(*a*)	2	C	NC	A4		1995 P1 Q11
(*a*)	1	A/B	NC	A4		
(*b*)	1	A/B	NC	A1		

•1 $g\left(\frac{1}{x^2-4}\right)$

•2 $2\left(\frac{1}{x^2-4}\right)+1$

•3 $\frac{x^2-2}{x^2-4}$

•4 "any domain which excludes 2"

[SQA] 7. Functions f and g, defined on suitable domains, are given by $f(x) = 2x$ and $g(x) = \sin x + \cos x$.

Find $f(g(x))$ and $g(f(x))$. **4**

Part	Marks	Level	Calc.	Content	Answer	U1 OC2
	4	C	NC	A4		1997 P1 Q3

•1 $f(\sin x + \cos x)$

•2 $2(\sin x + \cos x)$

•3 $g(2x)$

•4 $\sin 2x + \cos 2x$

[SQA] 8. The functions f and g are defined on a suitable domain by $f(x) = x^2 - 1$ and $g(x) = x^2 + 2$.

(*a*) Find an expression for $f(g(x))$. **2**

(*b*) Factorise $f(g(x))$. **2**

Part	Marks	Level	Calc.	Content	Answer	U1 OC2
(*a*)	2	C	CN	A4		1998 P1 Q6
(*b*)	1	C	CN	A6		
(*b*)	1	A/B	CN	A6		

•1 $f(x^2+2)$

•2 $(x^2+2)^2-1$

•3 $((x^2+2)+1)((x^2+2)-1)$

•4 $(x^2+3)(x^2+1)$

OR

•3 x^4+4x^2+3

•4 $(x^2+3)(x^2+1)$

[SQA] 9. Functions f and g are defined by $f(x) = 2x + 3$ and $g(x) = \dfrac{x^2+25}{x^2-25}$ where $x \in \mathbb{R}$, $x \neq \pm 5$.

The function h is given by the formula $h(x) = g(f(x))$.

For which real values of x is the function h **undefined**? **4**

Part	Marks	Level	Calc.	Content	Answer	U1 OC2
	2	C	CN	A4, A1		1989 P1 Q19
	2	A/B	CN	A4, A1		

•1 $g(2x+3)$

•2 $\dfrac{(2x+3)^2+25}{(2x+3)^2-25}$

•3 $(2x+3)^2-25=0$

•4 $x=1,-4$

[SQA] 10. Functions f and g are given by $f(x) = 3x + 1$ and $g(x) = x^2 - 2$.

(*a*) (i) Find $p(x)$ where $p(x) = f(g(x))$.

(ii) Find $q(x)$ where $q(x) = g(f(x))$. **3**

(*b*) Solve $p'(x) = q'(x)$. **3**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
(*a*)	3	C	CN	A4	$3(x^2-2)+1$, $(3x+1)^2-2$	2009 P2 Q2
(*b*)	3	C	CN	C1	$x = -\frac{1}{2}$	

•1 ss: substitute for g(x) in f (x)	•1 $f(x^2-2)$
•2 ic: complete	•2 $3(x^2-2)+1$
•3 ic: sub. and complete for q(x)	•3 $(3x+1)^2-2$
•4 ss: simplify	•4 $p(x) = 3x^2-5$, $q(x) = 9x^2+6x-1$
•5 pd: differentiate	•5 $p'(x) = 6x$, $q'(x) = 18x+6$
•6 pd: solve	•6 $x = -\frac{1}{2}$

11. Functions f, g and h are defined on the set of real numbers by

- $f(x) = x^3 - 1$
- $g(x) = 3x + 1$
- $h(x) = 4x - 5$.

(*a*) Find $g(f(x))$. **2**

(*b*) Show that $g(f(x)) + xh(x) = 3x^3 + 4x^2 - 5x - 2$. **1**

(*c*) (i) Show that $(x - 1)$ is a factor of $3x^3 + 4x^2 - 5x - 2$.

(ii) Factorise $3x^3 + 4x^2 - 5x - 2$ fully. **5**

(*d*) Hence solve $g(f(x)) + xh(x) = 0$. **1**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(*a*)	2	C	CN	A4	$3(x^3 - 1) + 1$	2011 P2 Q2
(*b*)	1	C	CN	A6	proof	
(*c*)	5	C	CN	A21	$(x-1)(3x+1)(x+2)$	
(*d*)	1	C	CN	A22	$-2, -\frac{1}{3}, 1$	

•[1] ic: interpret notation	•[1] $g(x^3 - 1)$
•[2] ic: complete process	•[2] $3(x^3 - 1) + 1$
•[3] ic: substitute and complete	•[3] $3(x^3 - 1) + 1 + x(4x - 5)$ $= 3x^3 + 4x^2 - 5x - 2$
•[4] ss: know to use $x = 1$	•[4] evaluating at $x = 1$...
•[5] pd: complete evaluation	•[5] $3 + 4 - 5 - 2 = 0$
•[6] ic: state conclusion	•[6] $(x - 1)$ is a factor
•[7] ic: find quadratic factor	•[7] $(x-1)(2x^2 + 7x + 2)$
•[8] pd: factorise completely	•[8] $(x-1)(3x+1)(x+2)$
•[9] ic: interpret and solve equation in (d)	•[9] $-2, -\frac{1}{3}, 1$

[SQA] 12.

(*a*) The function f is defined by $f(x) = x^3 - 2x^2 - 5x + 6$.

The function g is defined by $g(x) = x - 1$.

Show that $f(g(x)) = x^3 - 5x^2 + 2x + 8$. **4**

(*b*) Factorise fully $f(g(x))$. **3**

(*c*) The function k is such that $k(x) = \dfrac{1}{f(g(x))}$.

For what values of x is the function k not defined? **3**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(*a*)	4	C	NC	A4		1990 P2 Q6
(*b*)	3	C	NC	A21		
(*c*)	2	C	NC	A1		

(*a*)

•[1] $f(g(x)) = f(x-1)$

•[2] $(x-1)^3 - 2(x-1)^2 - 5(x-1) + 6$

•[3] $(x-1)^3 = x^3 - 3x^2 + 3x - 1$

•[4] $-2x^2 + 4x - 2 - 5x + 5 + 6$ and completing argument

(*b*)

•[5] *first "0" e.g.*

$$\begin{array}{r|rrrr} 2 & 1 & -5 & 2 & 8 \\ & & 2 & -6 & -8 \\ \hline & 1 & -3 & -4 & 0 \end{array}$$

•[6] $x^2 - 3x - 4 = (x+1)(x-4)$

•[7] $(x-2)(x+1)(x-4)$

(*c*)

•[8] denominator $(= (x-2)(x+1)(x-4)) \neq 0$

•[9] $-1,\ 2,\ 4$

[SQA] 13. (*a*) $f(x) = 2x + 1, \quad g(x) = x^2 + k$, where k is a constant.

(i) Find $g(f(x))$. **(2)**

(ii) Find $f(g(x))$. **(2)**

(*b*) (i) Show that the equation $g(f(x)) - f(g(x)) = 0$ simplifies to $2x^2 + 4x - k = 0$. **(2)**

(ii) Determine the nature of the roots of this equation when $k = 6$. **(2)**

(iii) Find the value of k for which $2x^2 + 4x - k = 0$ has equal roots. **(3)**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(*a*)	4	C	NC	A4		1996 P2 Q4
(*b*)	7	C	NC	A17, A18, A6		

(*a*)

•1 $g(2x+1)$

•2 $(2x+1)^2 + k$

•3 $f(x^2 + k)$

•4 $2(x^2 + k) + 1$

(*b*)

•5 $4x^2 + 4x + k + 1$ *AND* $2x^2 + 2k + 1$

•6 $4x^2 + 4x + k + 1 - (2x^2 + 2k + 1) = 0$

so $2x^2 + 4x - k = 0$

•7 $b^2 - 4ac = 16 - 4 \times 2 \times (-6) = 64$

•8 so roots real & distinct

•9 $b^2 - 4ac = 16 - 4 \times 2 \times (-k)$

•10 $b^2 - 4ac = 0$ for equal roots

•11 $k = -2$

[SQA] 14. Functions f and g are defined on the set of real numbers by $f(x) = x - 1$ and $g(x) = x^2$.

(a) Find formulae for

(i) $f(g(x))$

(ii) $g(f(x))$. **4**

(b) The function h is defined by $h(x) = f(g(x)) + g(f(x))$.

Show that $h(x) = 2x^2 - 2x$ and sketch the graph of h. **3**

(c) Find the area enclosed between this graph and the x-axis. **4**

Part	Marks	Level	Calc.	Content	Answer	U2 OC2
(a)	4	C	NC	A4		1999 P2 Q6
(b)	3	C	NC	A4		
(c)	4	C	NC	C16		

(a) •1 $f(x^2)$ *stated or implied by •2*

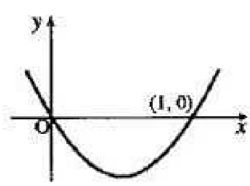
•2 $x^2 - 1$

•3 $g(x-1)$ *stated or implied by •4*

•4 $(x-1)^2$

(b) •5 $(x-1)^2 + x^2 - 1$ and complete proof

•6 sketch as shown



•7 minimum at $\left(\frac{1}{2}, -\frac{1}{2}\right)$ calculated or on sketch

(c) •8 $\int_0^1 (2x^2 - 2x)\,dx$

•9 $\left[\frac{2}{3}x^3 - x^2\right]$

•10 $-\frac{1}{3}$

•11 dealing with – ve

[SQA] 15. Functions $f(x) = \sin x$, $g(x) = \cos x$ and $h(x) = x + \frac{\pi}{4}$ are defined on a suitable set of real numbers.

(*a*) Find expressions for:

(i) $f(h(x))$;

(ii) $g(h(x))$. **2**

(*b*) (i) Show that $f(h(x)) = \frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x$.

(ii) Find a similar expression for $g(h(x))$ and hence solve the equation $f(h(x)) - g(h(x)) = 1$ for $0 \leq x \leq 2\pi$. **5**

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
(*a*)	2	C	NC	A4	(i) $sin(x + \frac{\pi}{4})$, (ii) $cos(x + \frac{\pi}{4})$	2001 P1 Q7
(*b*)	5	C	NC	T8, T7	(i) proof, (ii) $x = \frac{\pi}{4}, \frac{3\pi}{4}$	

•1 ic: interpret composite functions	•1 $\sin(x + \frac{\pi}{4})$
•2 ic: interpret composite functions	•2 $\cos(x + \frac{\pi}{4})$
•3 ss: expand $\sin(x + \frac{\pi}{4})$	•3 $\sin x \cos \frac{\pi}{4} + \cos x \sin \frac{\pi}{4}$ **and complete**
•4 ic: interpret	•4 $g(h(x)) = \frac{1}{\sqrt{2}} \cos x - \frac{1}{\sqrt{2}} \sin x$
•5 ic: substitute	•5 $(\frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x) - (\frac{1}{\sqrt{2}} \cos x - \frac{1}{\sqrt{2}} \sin x)$
•6 pd: start solving process	•6 $\frac{2}{\sqrt{2}} \sin x$
•7 pd: process	•7 $x = \frac{\pi}{4}, \frac{3\pi}{4}$ *accept only radians*

[SQA] 16. Functions f and g are defined on suitable domains by $f(x) = \sin(x°)$ and $g(x) = 2x$.

(*a*) Find expressions for:

(i) $f(g(x))$;

(ii) $g(f(x))$. **2**

(*b*) Solve $2f(g(x)) = g(f(x))$ for $0 \leq x \leq 360$. **5**

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
(*a*)	2	C	CN	A4	(i) $\sin(2x°)$, (ii) $2\sin(x°)$	2002 P1 Q3
(*b*)	5	C	CN	T10	$0°, 60°, 180°, 300°, 360°$	

•1 ic: interpret $f(g(x))$ •2 ic: interpret $g(f(x))$ •3 ss: equate for intersection •4 ss: substitute for $\sin 2x$ •5 pd: extract a common factor •6 pd: solve a 'common factor' equation •7 pd: solve a 'linear' equation	•1 $\sin(2x°)$ •2 $2\sin(x°)$ •3 $2\sin(2x°) = 2\sin(x°)$ •4 appearance of $2\sin(x°)\cos(x°)$ •5 $2\sin(x°)(2\cos(x°) - 1)$ •6 $\sin(x°) = 0$ **and** $0, 180, 360$ •7 $\cos(x°) = \frac{1}{2}$ **and** $60, 300$ **or** •6 $\sin(x°) = 0$ **and** $\cos(x°) = \frac{1}{2}$ •7 $0, 60, 180, 300, 360$

[END OF QUESTIONS]