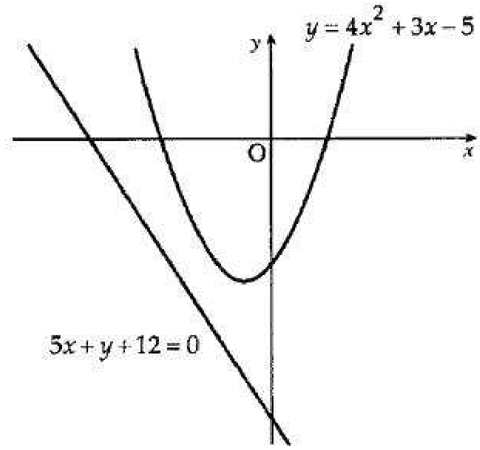
differentiate tangents

[SQA] 1. Find the coordinates of the point on the curve $y = 2x^2 - 7x + 10$ where the tangent to the curve makes an angle of $45°$ with the positive direction of the x-axis. **4**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	4	C	NC	G2, C4	$(2, 4)$	2002 P1 Q4

•1 sp: know to diff., and differentiate •2 pd: process gradient from angle •3 ss: equate equivalent expressions •4 pd: solve and complete	•1 $\frac{dy}{dx} = 4x - 7$ •2 $m_{\text{tang}} = \tan 45° = 1$ •3 $4x - 7 = 1$ •4 $(2, 4)$

[SQA] 2. The diagram below shows a parabola with equation $y = 4x^2 + 3x - 5$ and a straight line with equation $5x + y + 12 = 0$.
A tangent to the parabola is drawn parallel to the given straight line.
Find the x-coordinate of the point of contact of this tangent. **5**



Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	5	C	NC	G2, C4, C4		1997 P1 Q6

•1 equate gradients

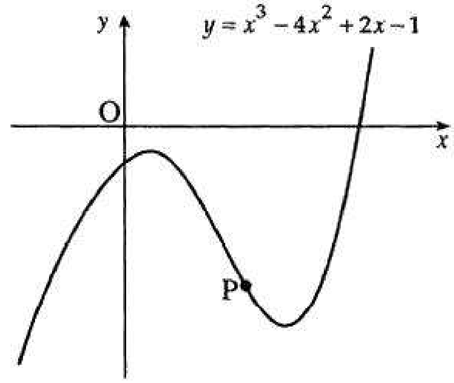
•2 $m = -5$

•3 $\frac{dy}{dx} = \ldots$

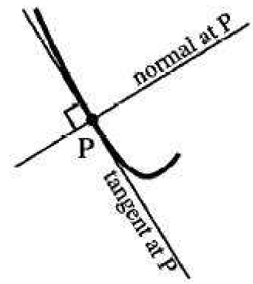
•4 $\frac{dy}{dx} = 8x + 3$

•5 $x = -1$

[SQA] 3. (*a*) The diagram shows an incomplete sketch of the curve with equation $y = x^3 - 4x^2 + 2x - 1$. Find the equation of the tangent to the curve at the point P where $x = 2$. **(5)**



(*b*) The normal to the curve at P is defined as the straight line through P which is perpendicular to the tangent to the curve at P.
Find the angle which the normal at P makes with the positive direction of the *x*-axis. **(2)**

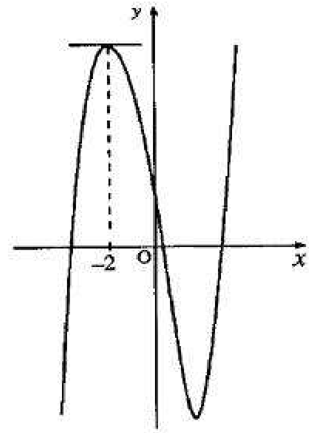


Part	Marks	Level	Calc.	Content	Answer	U1 OC3
(*a*)	5	C	CN	G3, C4, G3		1998 P2 Q3
(*b*)	2	C	CN	G2, G5		

(*a*)
- •[1] $\frac{dy}{dx} = \ldots$
- •[2] $3x^2 - 8x + 2$
- •[3] gradient $= -2$ (calculated from $\frac{dy}{dx}$)
- •[4] $y_A = -5$
- •[5] $y + 5 = -2(x - 2)$

(*b*)
- •[6] $m_{\text{normal}} = \frac{1}{2}$
- •[7] angle $= \tan^{-1}\frac{1}{2}$

[SQA] 4. The diagram shows a sketch of the curve $y = x^3 + kx^2 - 8x + 3$. The tangent to the curve at $x = -2$ is parallel to the x-axis.
Find the value of k. **4**



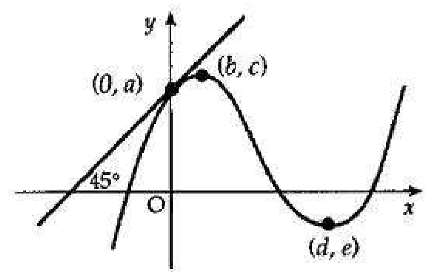
Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	4	C	NC	C1, C4		1998 P1 Q11

- •[1] $\frac{dy}{dx} = \ldots\ldots$
- •[2] $3x^2 + 2kx - 8$
- •[3] $3x^2 + 2kx - 8 = 0$ when $x = -2$
- •[4] $k = 1$

[SQA] 5. The diagram shows the graph of a cubic function with a maximum at (b, c) and a minimum at (d, e). The tangent at $(0, a)$ is inclined at 45° to the x-axis.

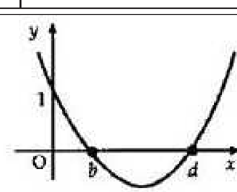
(a) State the values of $f'(b)$, $f'(d)$ and $f'(0)$. **2**

(b) Sketch the graph of the the derived function f'. **2**



Part	Marks	Level	Calc.	Content	Answer	U1 OC3
(a)	2	C	CN	C4		1989 P1 Q14
(b)	2	C	CN	A3		

- •[1] any one of $f'(b) = 0, f'(d) = 0, f'(0) = 1$
- •[2] remaining two answers
- •[3] shape of graph
- •[4] annotation



[SQA] 6. Find the x-coordinate of each of the points on the curve $y = 2x^3 - 3x^2 - 12x + 20$ at which the tangent is parallel to the x-axis. **4**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	4	C	NC	C4		1993 P1 Q4

•[1] $\frac{dy}{dx} = \ldots\ldots$

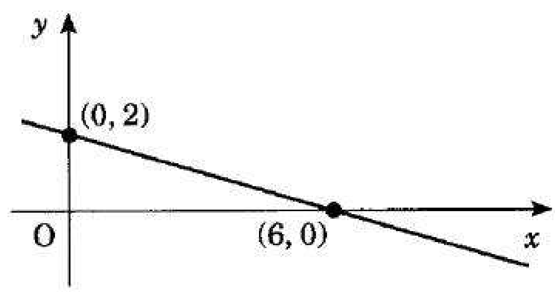
•[2] $6x^2 - 6x - 12$

•[3] $\ldots\ldots = 0$

•[4] $x = -1, 2$

[SQA] 7. The straight line shown in the diagram has equation $y = f(x)$.

Determine $f'(x)$. 2

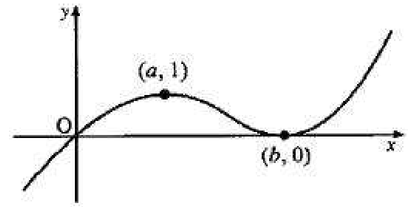


Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	2	C	NC	C4		1995 P1 Q14

•[1] gradient $= -\frac{1}{3}$

•[2] $f'(x) =$ gradient

[SQA] 8. A sketch of the graph of the cubic function f is shown. It passes through the origin, has a maximum turning point at $(a, 1)$ and a minimum turning point at $(b, 0)$.



(a) Make a copy of this diagram and on it sketch the graph of $y = 2 - f(x)$, indicating the coordinates of the turning points. 3

(b) On a separate diagram sketch the graph of $y = f'(x)$. 2

(c) The tangent to $y = f(x)$ at the origin has equation $y = \frac{1}{2}x$. Use this information to write down the coordinates of a point on the graph of $y = f'(x)$. 1

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
(a)	3	C	CN	A3		1998 P1 Q13
(b)	2	A/B	CN	A3		
(c)	1	A/B	CN	C4		

•1 clear evidence of reflection in $y = 0$

•2 clear evidence of translation $\begin{pmatrix}0\\2\end{pmatrix}$ subsequent to a reflection

•3 indication of passing through $(a,1)$ and $(b,2)$

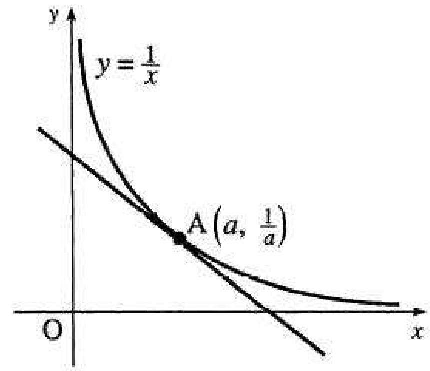
•4 roots at $x = a$ and $x = b$

•5 parabolic shape with min. turning point between the roots and no other turning points

•6 $\left(0, \frac{1}{2}\right)$

[SQA] 9. (a) A sketch of part of the graph of $y = \frac{1}{x}$ is shown in the diagram. The tangent at A $\left(a, \frac{1}{a}\right)$ has been drawn.

Find the gradient of this tangent. **(4)**



(b) Hence show that the equation of this tangent is $x + a^2y = 2a$. **(2)**

(c) This tangent cuts the y-axis at B and the x-axis at C.

(i) Calculate the area of triangle OBC **(3)**

(ii) Comment on your answer to c(i). **(1)**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
(a)	4	C	CN	C4		1997 P2 Q6
(b)	1	C	CN	G3		
(b)	1	A/B	CN	G3		
(c)	4	A/B	CN	CGD		

(a)
- •1 $\frac{1}{x} = x^{-1}$
- •2 $\frac{dy}{dx} = \ldots\ldots$
- •3 $\frac{dy}{dx} = -x^{-2}$
- •4 gradient $= -a^{-2}$

(b)
- •5 use $y - \frac{1}{a} = -\frac{1}{a^2}(x - a)$
- •6 $a^2y - a = -(x - a)$ and completes proof

(c)
- •7 $y_B = \frac{2a}{a^2}$
- •8 $x_A = 2a$
- •9 2
- •10 independent of a

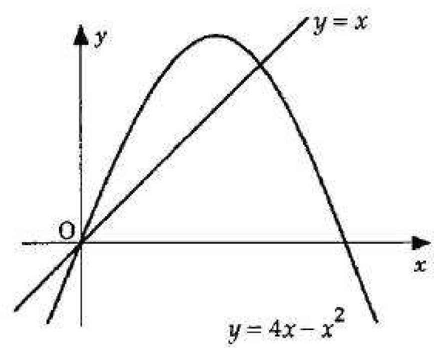
[SQA] 10. Calculate, to the nearest degree, the angle between the x-axis and the tangent to the curve with equation $y = x^3 - 4x - 5$ at the point where $x = 2$. **4**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	4	C	NC	C4, G2		1989 P1 Q13

- •[1] $\frac{dy}{dx} = 3x^2 - 4$
- •[2] $\frac{dy}{dx}_{x=2} = 8$
- •[3] $\tan\theta = 8$
- •[4] $83°$

[SQA] 11. Find the gradient of the tangent to the parabola $y = 4x - x^2$ at (0,0).

Hence calculate the size of the angle between the line $y = x$ and this tangent. 6



Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	6	C	NC	C4, G2		1994 P1 Q14

- •[1] know to differentiate
- •[2] $4 - 2x$
- •[3] $m = 4$
- •[4] $76°$
- •[5] $45°$
- •[6] $31°$

[SQA] 12. The point P$(-1, 7)$ lies on the curve with equation $y = 5x^2 + 2$. Find the equation of the tangent to the curve at P. **4**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	4	C	NC	C4, G3		1999 P1 Q9

- •[1] $\frac{dy}{dx} = \ldots\ldots$
- •[2] $10x$
- •[3] -10
- •[4] $y - 7 = -10(x - (-1))$

[SQA] 13. Find the equation of the tangent to the curve $y = 4x^3 - 2$ at the point where $x = -1$. **4**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	4	C	NC	C4, G3		1990 P1 Q2

•[1] strat: $\frac{dy}{dx} = \ldots\ldots$

•[2] $\frac{dy}{dx} = 12x^2$

•[3] $m = 12$

•[4] $y - (-6) = 12(x - (-1))$

[SQA] 14. Find the equation of the tangent to the curve with equation $y = 5x^3 - 6x^2$ at the point where $x = 1$. **4**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	4	C	NC	C4, G3		1992 P1 Q1

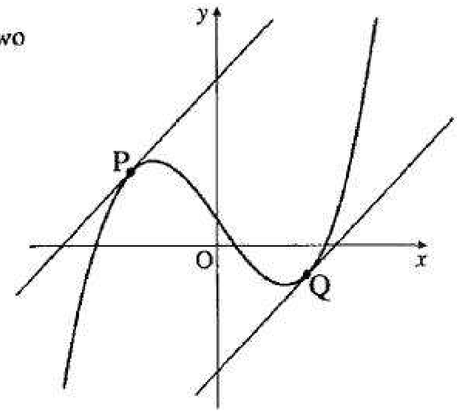
•[1] $y' = 15x^2 - 12x$

•[2] $y'(1) = 3$

•[3] $y(1) = -1$

•[4] $y - (-1) = 3(x - 1)$

[SQA] 15. The diagram shows a sketch of the graph of $y = x^3 - 9x + 4$ and two parallel tangents drawn at P and Q.



(*a*) Find the equations of the tangents to the curve $y = x^3 - 9x + 4$ which have gradient 3. **6**

(*b*) Show that the shortest distance between the tangents is $\frac{16\sqrt{10}}{5}$. **6**

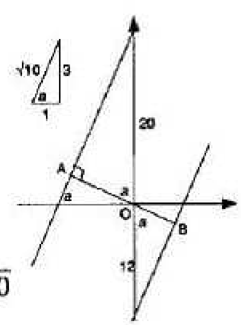
Part	Marks	Level	Calc.	Content	Answer	U1 OC3
(*a*)	6	C	CN	C4, G3		1999 P2 Q11
(*b*)	6	A/B	CN	G8, G1		

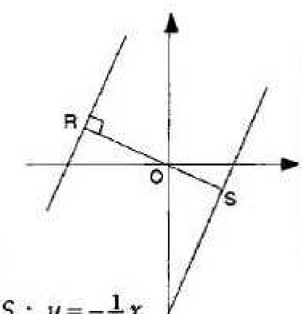
(*a*)
- •1 strategy: $\frac{dy}{dx} = \ldots\ldots = 3$
- •2 $3x^2 - 9$
- •3 $x = 2,\ -2$
- •4 $y = -6,\ 14$
- •5 $y + 6 = 3(x - 2)$
- •6 $y - 14 = 3(x + 2)$

OR

- •3 $x = 2,\ y = -6$
- •4 $x = -2,\ y = 14$

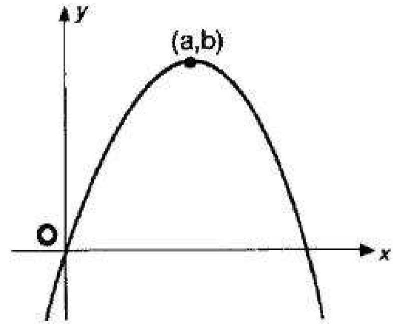
(*b*)
- •7 diagram with $y = -\frac{1}{3}x$
- •8 for 20 and 12
- •9 $AB = AO + OB$
- •10 $AB = 20\cos a + 12\cos a$
- •11 using $\tan a = \frac{3}{1}$
- •12 $AB = 32 \times \frac{1}{\sqrt{10}} = 32 \times \frac{\sqrt{10}}{10} = \frac{32}{5}\sqrt{10}$





- •7 $m_{RS} = -\frac{1}{3}$
- •8 *equ of RS* : $y = -\frac{1}{3}x$
- •9 $-\frac{1}{3}x = 3x - 12$ & $-\frac{1}{3}x = 3x + 20$
- •10 $R(-6, 2)$ and $S\left(\frac{18}{5}, -\frac{6}{5}\right)$
- •11 $d^2 = \left(-6 - \left(\frac{18}{5}\right)\right)^2 + \left(2 - \left(-\frac{6}{5}\right)\right)^2$
- •12 $d^2 = \frac{48^2}{25} + \frac{16^2}{25}$ and completes proof

[SQA] 16. The line with equation $y = x$ is a tangent at the origin to the parabola with equation $y = f(x)$. The parabola has a maximum turning point at (a, b).
Sketch the graph of $y = f'(x)$. **4**



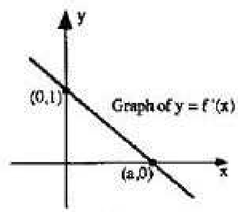
Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	1	C	CN	C4, A3		1992 P1 Q19
	3	A/B	CN	C4, A3		

•[1] $f'(a) = 0$

•[2] m_{tgt} at $(0,0) = 1$

•[3] $f'(0) = 1$

•[4] for the sketch



[SQA] 17. Diagram 1 shows 5 cars travelling up an incline on a roller-coaster. Part of the roller-coaster rail follows the curve with equation $y = 8 + 5\cos\frac{1}{2}x$.
Diagram 2 shows an enlargement of the last car and its position relative to a suitable set of axes. The floor of the car lies parallel to the tangent at P, the point of contact.
Calculate the acute angle a between the floor of the car and the horizontal when the car is at the point where $x_P = \frac{7\pi}{3}$.
Express your answer in degrees. **4**

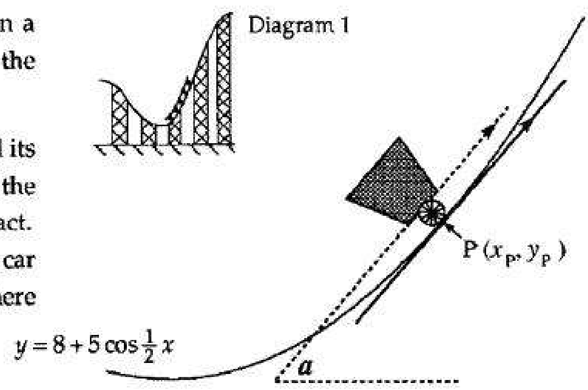


Diagram 1

Diagram 2

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	1	C	CR	C4, C4, G2		1997 P1 Q20
	3	A/B	CR	C4, C4, G2		

•[1] $\frac{dy}{dx} = \ldots$

•[2] $5 \times \left(-\frac{1}{2}\sin\frac{1}{2}x\right)$

•[3] $m = \frac{5}{4}$

•[4] $\theta = 51.3°$

[SQA] 18. A curve has equation $y = x - \frac{16}{\sqrt{x}}$, $x > 0$.

Find the equation of the tangent at the point where $x = 4$. **6**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	6	C	CN	C4, C5	$y = 2x - 12$	2001 P2 Q2

•1 ic: find corresponding y-coord.	•1 $(4, -4)$ *stated or implied by* •6
•2 ss: express in standard form	•2 $-16x^{-\frac{1}{2}}$
•3 ss: start to differentiate	•3 $\frac{dy}{dx} = 1\ldots$
•4 pd: diff. fractional negative power	•4 $\ldots + 8x^{-\frac{3}{2}}$
•5 ss: find gradient of tangent	•5 $m_{x=4} = 2$
•6 ic: write down equ. of tangent	•6 $y - (-4) = 2(x - 4)$

19. The derivative of a function f is given by $f'(x) = x^2 - 9$.

Here are two statements about f:

(1) f is increasing at $x = 1$;

(2) f is stationary at $x = -3$.

Which of the following is true?

A. Neither statement is correct.

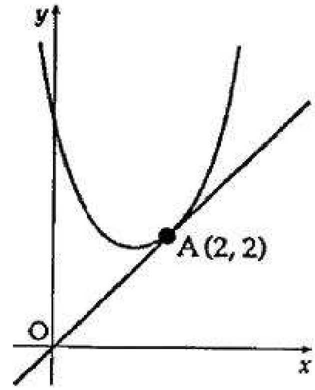
B. Only statement (1) is correct.

C. Only statement (2) is correct.

D. Both statements are correct. **2**

Key	Outcome	Grade	Facility	Disc.	Calculator	Content	Source
C	1.3	C	0	0	CN	C4, C7	2010 P1 Q15

[SQA] 20. (*a*) The point A(2, 2) lies on the parabola $y = x^2 + px + q$.
Find a relationship between p and q. **(1)**



(*b*) The tangent to the parabola at A is the line $y = x$. Find the value of p.
Hence find the equation of the parabola. **(6)**

(*c*) Using your answers for p and q, find the value of the discriminant of $x^2 + px + q = 0$. What feature of the above sketch is confirmed by this value ? **(2)**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(*a*)	1	C	CN	A6		1994 P2 Q9
(*b*)	2	C	CN	C4, CGD		
(*b*)	4	A/B	CN	C4, CGD		
(*c*)	2	A/B	CN	A17		

(*a*) •[1] $2p + q = -2$

(*b*) •[2] strategy

•[3] $2x + p$

•[4] gradient = 1, or equivalent

•[5] $4 + p$

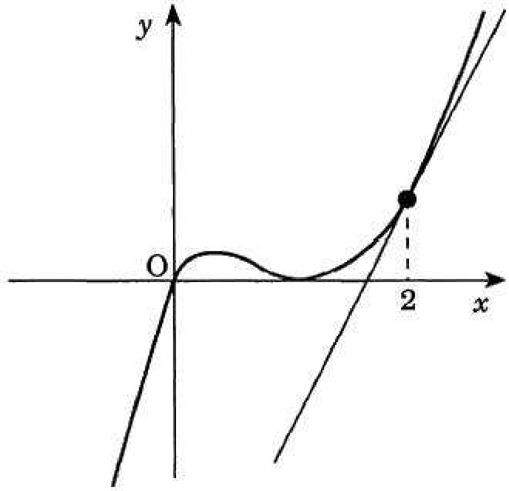
•[6] $p = -3$

•[7] $q = 4$

(*c*) •[8] $\Delta = -7$

•[9] $\sqrt{-7}$ means no roots

[SQA] 21. The diagram shows a sketch of part of the graph of $y = x^3 - 2x^2 + x$.



(a) Show that the equation of the tangent to the curve at $x = 2$ is $y = 5x - 8$. **(4)**

(b) Find algebraically the coordinates of the point where this tangent meets the curve again. **(5)**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(a)	4	C	NC	C4, G3		1995 P2 Q2
(b)	5	C	NC	A23		

(a) •[1] $\frac{dy}{dx} = \ldots\ldots\ldots$

•[2] $3x^2 - 4x + 1$

•[3] $m_{x=2} = 5$

•[4] $y - 2 = 5(x - 2)$

(b) •[5] equate 'y's

•[6] $x^3 - 2x^2 - 4x + 8 = 0$

•[7] e.g. synthetic division

•[8] the appearance of:

$x^2 - 4$

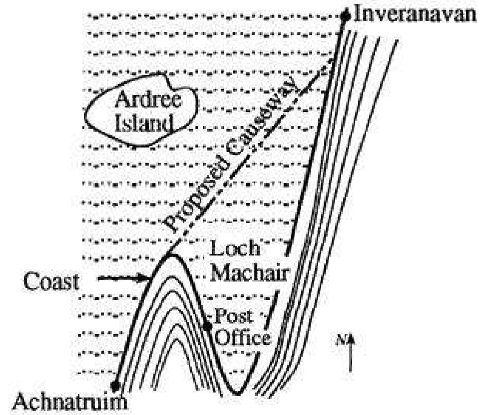
or $x^2 - 4x + 4$

or ± 2

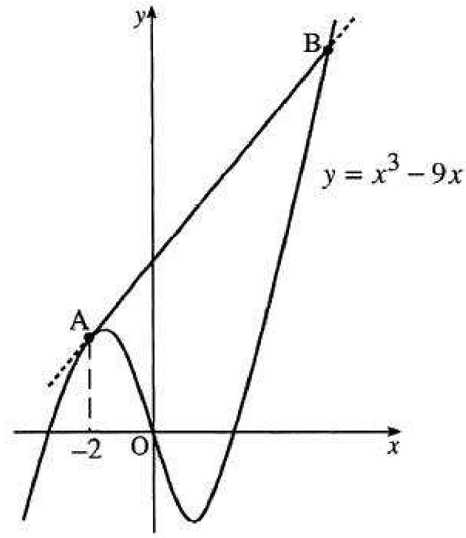
or $-2, 2, 2$

•[9] $x = -2,\ y = -18$

[SQA] 22. The map shows part of the coast road from Achnatruim to Inveranavan. In order to avoid the hairpin bends, it is proposed to build a straight causeway, as shown, with the southern end tangential to the existing road.



With the origin taken at the Post Office the part of the coast road shown lies along the curve with equation $y = x^3 - 9x$. The causeway is represented by the line AB. The southern end of the proposed causeway is at the point A where $x = -2$, and the line AB is a tangent to the curve at A.



(*a*) (i) Write down the coordinates of A. **(5)**

(ii) Find the equation of the line AB.

(*b*) Determine the coordinates of the point B which represents the northern end of the causeway. **(7)**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(*ai*)	1	C	NC	A6		1998 P2 Q5
(*aii*)	4	C	NC	C4, G3		
(*b*)	2	C	NC	A23		
(*b*)	5	A/B	NC	A23		

(a)

•1 $y_{x=-2} = 10$

•2 $\frac{dy}{dx} = \ldots\ldots$

•3 $3x^2 - 9$

•4 $m_{x=-2} = 3$

•5 $y - 10 - 3(x+2)$

(b)

•6 $y = 3x + 16$

•7 $3x + 16 = x^3 - 9x$

•8 $x^3 - 12x - 16 = 0$

•9 *e.g.*

−2	1	0	−12	−16
		−2	4	16
	1	−2	−8	0

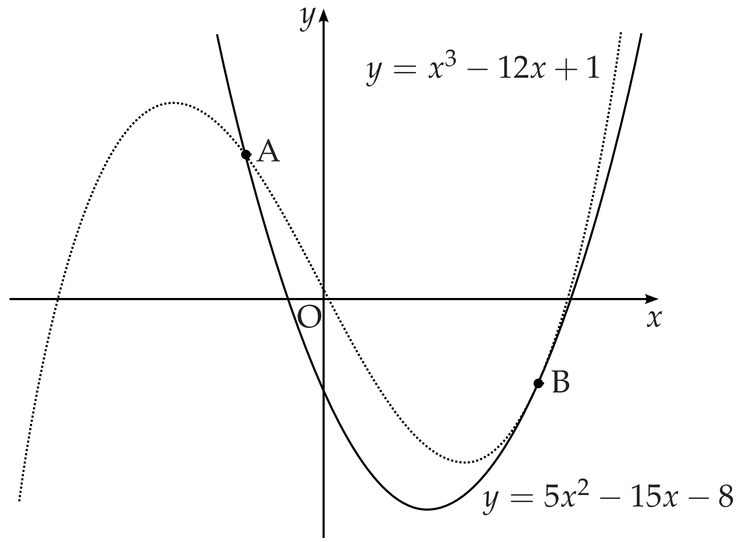
•10 *e.g.* $x^2 - 2x - 8$

•11 *e.g.* $(x+2)(x-4)$

•12 B is $(4, 28)$

[SQA] 23. The diagram shows a sketch of the graphs of $y = 5x^2 - 15x - 8$ and $y = x^3 - 12x + 1$.

The two curves intersect at A and touch at B, i.e. at B the curves have a common tangent.



(*a*) (i) Find the x-coordinates of the point of the curves where the gradients are equal. **4**

(ii) By considering the corresponding y-coordinates, or otherwise, distinguish geometrically between the two cases found in part (i). **1**

(*b*) The point A is $(-1, 12)$ and B is $(3, -8)$.

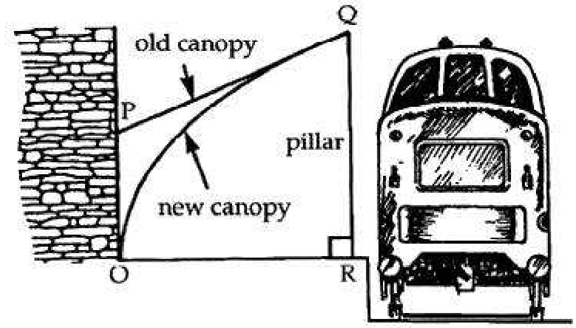
Find the area enclosed between the two curves. **5**

Part	Marks	Level	Calc.	Content	Answer	U2 OC2
(*ai*)	4	C	NC	C4	$x = \frac{1}{3}$ and $x = 3$	2000 P1 Q4
(*aii*)	1	C	NC	CGD	parallel and coincident	
(*b*)	5	C	NC	C17	$21\frac{1}{3}$	

•1 ss: know to diff. and equate	•1 find derivatives and equate
•2 pd: differentiate	•2 $3x^2 - 12$ **and** $10x - 15$
•3 pd: form equation	•3 $3x^2 - 10x + 3 = 0$
•4 ic: interpret solution	•4 $x = 3, x = \frac{1}{3}$
•5 ic: interpret diagram	•5 tangents at $x = \frac{1}{3}$ are parallel, at $x = 3$ coincident
•6 ss: know how to find area between curves	•6 $\int(\text{cubic} - \text{parabola})$ **or** $\int(\text{cubic}) - \int(\text{parabola})$
•7 ic: interpret limits	•7 $\int_{-1}^{3} \cdots dx$
•8 pd: form integral	•8 $\int(x^3 - 5x^2 + 3x + 9)dx$ or equiv.
•9 pd: process integration	•9 $\left[\frac{1}{4}x^4 - \frac{5}{3}x^3 + \frac{3}{2}x^2 + 9x\right]_{-1}^{3}$ or equiv.
•10 pd: process limits	•10 $21\frac{1}{3}$

[SQA] 24. The diagram shows a proposed replacement of the old platform canopy at the local railway station by a new parabolic canopy, while keeping the original pillars.

If OR and OP are taken as the x- and y- axes and Q has coordinates (1, 1), then OQ has equation $y = \sqrt{x}$ and PQ is the tangent at Q to the parabola.



The planners have received an objection that there is a reduction of more than 10% in the space under the canopy and wish to compare the two canopies.

(*a*) Find the equation of the tangent PQ and the coordinates of P. **(5)**

(*b*) Find the area of the trapezium OPQR. **(2)**

(*c*) Find the area under the parabola OQ. **(3)**

(*d*) Comment on the objection received. **(3)**

Part	Marks	Level	Calc.	Content	Answer	U2 OC2
(*a*)	5	C	NC	C4, G3		1991 P2 Q11
(*b*)	2	C	NC	CGD		
(*c*)	3	C	NC	C16		
(*d*)	1	C	NC	CGD		
(*d*)	2	A/B	NC	CGD		

(*a*)
- •1 $\frac{dy}{dx} = \ldots\ldots$
- •2 $\frac{1}{2}x^{-\frac{1}{2}}$
- •3 $m = \frac{dy}{dx}_{x=1} = \frac{1}{2}$
- •4 $y - 1 = \frac{1}{2}(x-1)$
- •5 $\mathrm{P} = (0, \frac{1}{2})$

(*b*)
- •6 method for area of trapezium
- •7 $\frac{3}{4}$

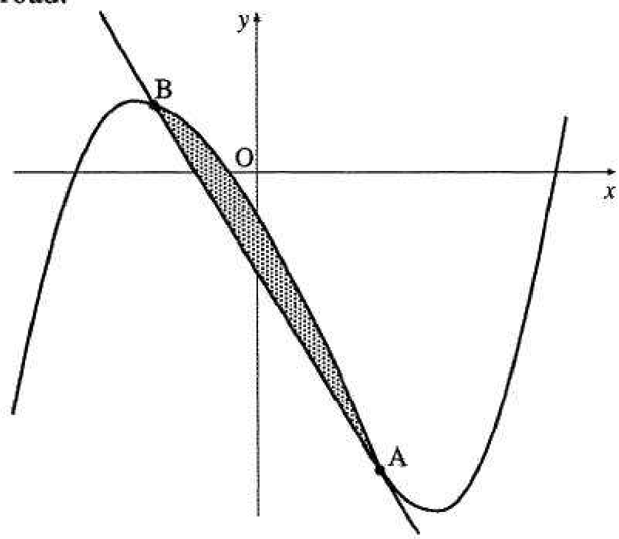
(*c*)
- •8 $\int_0^1 x^{\frac{1}{2}}\,dx$
- •9 $\frac{2}{3}x^{\frac{3}{2}}$
- •10 $\frac{2}{3}$

(*d*)
- •11 strategy: compare reduction with original
- •12 $\frac{3}{4} - \frac{2}{3} - \frac{1}{12}$ *and* $\frac{\frac{1}{12}}{\frac{3}{4}} - \frac{1}{9}$
- •13 $\frac{1}{9} = 11.1\% > 10\%$ so objection correct

[SQA] 25. In the diagram below a winding river has been modelled by the curve $y = x^3 - x^2 - 6x - 2$ and a road has been modelled by the straight line AB. The road is a tangent to the river at the point A(1, – 8).

(*a*) Find the equation of the tangent at A and hence find the coordinates of B. **(8)**

(*b*) Find the area of the shaded part which represents the land bounded by the river and the road. **(3)**



Part	Marks	Level	Calc.	Content	Answer	U2 OC2
(*a*)	5	C	CN	C4, G3, A23		1996 P2 Q8
(*a*)	3	A/B	CN	C4, G3, A23		
(*b*)	3	A/B	CN	C17		

(*a*)

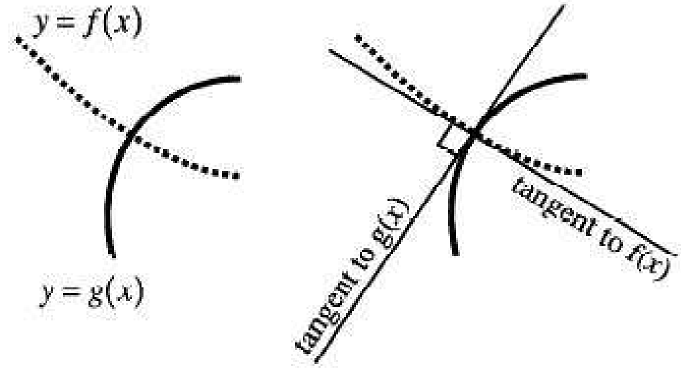
- •[1] *strat:* $\frac{dy}{dx} = \ldots$
- •[2] $\frac{dy}{dx} = 3x^2 - 2x - 6$
- •[3] $m_{tgt} = -5$
- •[4] $y + 8 = -5(x - 1)$
- •[5] *strat:* attempt to simplify and equate y's
- •[6] $x^3 - x^2 - x + 1 = 0$
- •[7] *strat:* *e.g.* try to factorise
- •[8] $B = (-1, 2)$

(*b*)

- •[9] $\int \left(x^3 - x^2 - 6x - 2\right) - (-5x - 3)\ dx$
- •[10] $\left[\frac{1}{4}x^4 - \frac{1}{3}x^3 - \frac{1}{2}x^2 + x\right]$
- •[11] $1\frac{1}{3}$

[SQA] 26.

Two curves, $y = f(x)$ and $y = g(x)$, are called orthogonal if, at each point of intersection, their tangents are at right angles to each other.



(*a*) Diagram 1 shows the parabola with equation $y = 6 + \frac{1}{9}x^2$ and the circle M with equation $x^2 + (y-5)^2 = 13$.
These two curves intersect at (3, 7) and (–3, 7).
Prove that these curves are orthogonal. **(6)**

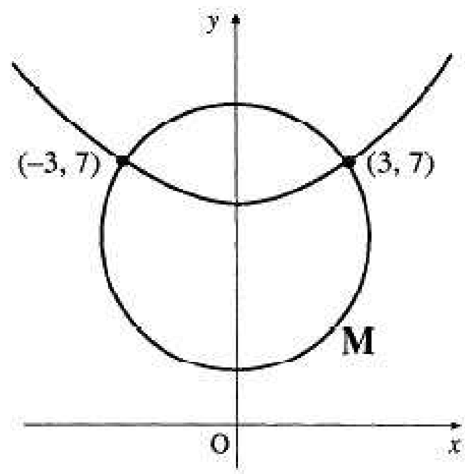


Diagram 1

(*b*) Diagram 2 shows the circle M, from (*a*) above, which is orthogonal to the circle N. The circles intersect at (3, 7) and (–3, 7).

(i) Write down the equation of the tangent to circle M at the point (–3, 7). **(1)**

(ii) Hence find the equation of circle N. **(3)**

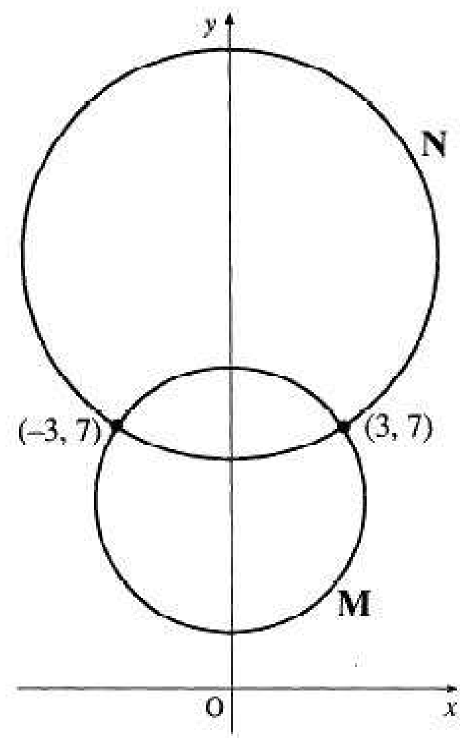


Diagram 2

Part	Marks	Level	Calc.	Content	Answer	U2 OC4