roots of quadratics

[SQA] 1.

(i) Write down the condition for the equation $ax^2 + bx + c = 0$ to have no real roots. **1**

(ii) Hence or otherwise show that the equation $x(x+1) = 3x - 2$ has no real roots. **2**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
	3	C	CN	A17		1999 P1 Q8

•[1] $b^2 - 4ac = 0$
•[2] $x^2 + 6x + 9 = 0$
•[3] $b^2 - 4ac = 36 - 36 = 0$ *OR* •[3] $(x+3)(x+3) = 0$ so roots are $-3, -3$

[SQA] 2. Show that the roots of the equation $(k-2)x^2 - (3k-2)x + 2k = 0$ are real. **4**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
	1	C	CN	A17		1990 P1 Q18
	3	A/B	CN	A17		

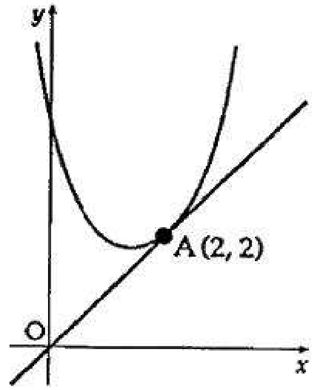
•[1] *use* discriminant Δ
•[2] $\Delta = (3k-2)^2 - 8k(k-2)$
•[3] $\Delta = k^2 + 4k + 4$
•[4] $(k+2)^2 \geq 0$ so roots real

[SQA] 3. Given that k is a real number, show that the roots of the equation $kx^2 + 3x + 3 = k$ are always real numbers. **5**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
	1	C	NC	A17		1991 P1 Q18
	4	A/B	NC	A17		

•[1] for realising "$b^2 - 4ac$"≥ 0
•[2] $kx^2 + 3x + (3-k) = 0$
•[3] $\Delta = 3^2 - 4k(3-k)$
•[4] $\Delta = (2k-3)^2$
•[5] for stating $(2k-3)^2$ is ≥ 0 for all real k

[SQA] 4. (*a*) The point A(2, 2) lies on the parabola $y = x^2 + px + q$.
Find a relationship between p and q. (1)



(*b*) The tangent to the parabola at A is the line $y = x$. Find the value of p.
Hence find the equation of the parabola. (6)

(*c*) Using your answers for p and q, find the value of the discriminant of $x^2 + px + q = 0$. What feature of the above sketch is confirmed by this value ? (2)

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(*a*)	1	C	CN	A6		1994 P2 Q9
(*b*)	2	C	CN	C4, CGD		
(*b*)	4	A/B	CN	C4, CGD		
(*c*)	2	A/B	CN	A17		

(*a*) •1 $2p + q = -2$

(*b*) •2 strategy

•3 $2x + p$

•4 gradient = 1, or equivalent

•5 $4 + p$

•6 $p = -3$

•7 $q = 4$

(*c*) •8 $\Delta = -7$

•9 $\sqrt{-7}$ means no roots

[SQA] 5. (*a*) $f(x) = 2x + 1$, $g(x) = x^2 + k$, where k is a constant.

(i) Find $g(f(x))$. **(2)**

(ii) Find $f(g(x))$. **(2)**

(*b*) (i) Show that the equation $g(f(x)) - f(g(x)) = 0$ simplifies to $2x^2 + 4x - k = 0$. **(2)**

(ii) Determine the nature of the roots of this equation when $k = 6$. **(2)**

(iii) Find the value of k for which $2x^2 + 4x - k = 0$ has equal roots. **(3)**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(*a*)	4	C	NC	A4		1996 P2 Q4
(*b*)	7	C	NC	A17, A18, A6		

(a)

•1 $g(2x+1)$

•2 $(2x+1)^2 + k$

•3 $f(x^2+k)$

•4 $2(x^2+k)+1$

(b)

•5 $4x^2 + 4x + k + 1$ *AND* $2x^2 + 2k + 1$

•6 $4x^2 + 4x + k + 1 - (2x^2 + 2k + 1) = 0$

so $2x^2 + 4x - k = 0$

•7 $b^2 - 4ac = 16 - 4 \times 2 \times (-6) = 64$

•8 so roots real & distinct

•9 $b^2 - 4ac = 16 - 4 \times 2 \times (-k)$

•10 $b^2 - 4ac = 0$ for equal roots

•11 $k = -2$

[SQA] 6. Diagram 1 shows a rectangular plate of transparent plastic moulded into a parabolic shape and pegged to the ground to form a cover for growing plants. Triangular metal frames are placed over the cover to support it and prevent it blowing away in the wind.

Diagram 2 shows an end view of the cover and the triangular frame related to the origin O and axes Ox and Oy. (All dimensions are given in centimetres.)

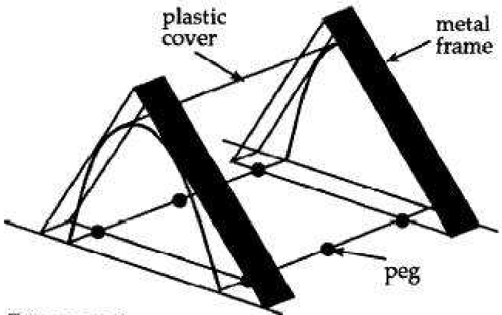


Diagram 1

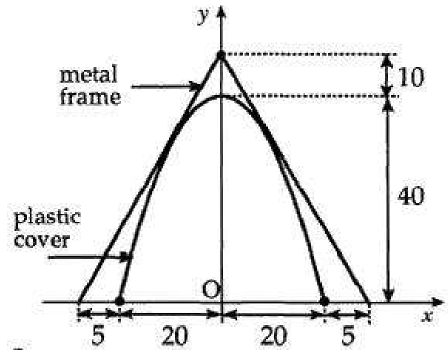


Diagram 2

(*a*) Show that the equation of the parabolic end is $y = 40 - \frac{x^2}{100}$, $-20 \le x \le 20$. **(4)**

(*b*) Show that the triangular frame touches the cover without disturbing the parabolic shape. **(7)**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(*a*)	1	C	CN	A7		1991 P2 Q10
(*a*)	3	A/B	CN	A7		
(*b*)	3	C	CN	G2, G3		
(*b*)	4	A/B	CN	A23, A17, A20		

(*a*)
- •1 $y = ax^2 + bx + c$
- •2 $(0,40) \Rightarrow c = 40$
- •3 symmetry $\Rightarrow b = 0$
- •4 $(20,0) \Rightarrow a = -\frac{1}{10}$

(*b*)
- •5 strategy: find equ of line and solve with parabola
- •6 *e.g.* gradient of left line = 2
- •7 $y = 2x + 50$
- •8 $2x + 50 = 40 - \frac{1}{10}x^2$
- •9 $x^2 + 20x + 100 = 0$
- •10 $b^2 - 4ac = 0$ *or* $(x-10)^2 = 0$
- •11 equal roots so line is tangent to parabola

[SQA] 7.

(a) Write the equation $\cos 2\theta + 8\cos\theta + 9 = 0$ in terms of $\cos\theta$ and show that, for $\cos\theta$, it has equal roots. **3**

(b) Show that there are no real roots for θ. **1**

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
(a)	1	C	CN	T8, A17		1998 P1 Q18
(a)	2	A/B	CN	T8, A17		
(b)	1	A/B	CN	A1		

•[1] $2\cos^2\theta - 1 + 8\cos\theta + 9$

•[2] $2(\cos\theta + 2)^2 = 0$

or "$b^2 - 4ac$" $= 16 - 4\times1\times4$

•[3] $\cos\theta = -2$ twice or "$b^2 - 4ac$" $= 0$

•[4] $\cos\theta = -2$ has no solution

[SQA] 8. For what range of values of k does the equation $x^2 + y^2 + 4kx - 2ky - k - 2 = 0$ represent a circle? **5**

Part	Marks	Level	Calc.	Content	Answer	U2 OC4
	5	A	NC	G9, A17	for all k	2000 P1 Q6

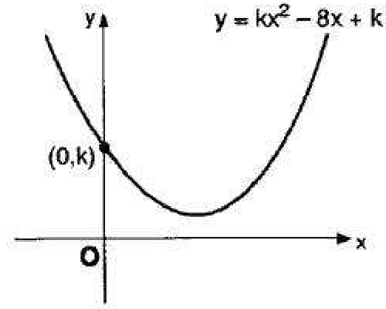
•[1] ss: know to examine radius •[2] pd: process •[3] pd: process •[4] ic: interpret quadratic inequation •[5] ic: interpret quadratic inequation	•[1] $g = 2k$, $f = -k$, $c = -k - 2$ *stated or implied by* •[2] •[2] $r^2 = 5k^2 + k + 2$ •[3] (real $r \Rightarrow$) $5k^2 + k + 2 > 0$ (*accept* $\geq$) •[4] use discr. **or** complete sq. **or** diff. •[5] true for all k

[SQA] 9. For what value of k does the equation $x^2 - 5x + (k+6) = 0$ have equal roots? **3**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
	3	C	CN	A18	$k = \frac{1}{4}$	2001 P1 Q2

•[1] ss: know to set disc. to zero •[2] ic: substitute a, b and c into discriminant •[3] pd: process equation in k	•[1] $b^2 - 4ac = 0$ *stated or implied by* •[2] •[2] $(-5)^2 - 4\times(k+6)$ •[3] $k = \frac{1}{4}$

[SQA] 10. Calculate the least positive integer value of k so that the graph of $y = kx^2 - 8x + k$ does not cut or touch the x-axis. **4**



Part	Marks	Level	Calc.	Content	Answer	U2 OC1
	1	C	NC	A18		1992 P1 Q17
	3	A/B	NC	A18		

- •[1] *strat*: use discriminant
- •[2] $b^2 - 4ac < 0$
- •[3] $64 - 4k^2$
- •[4] $k = 5$

[SQA] 11. Find the values of k for which the equation $2x^2 + 4x + k = 0$ has real roots. **2**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
	2	C	NC	A18		1993 P1 Q3

- •[1] discriminant $= 16 - 4 \times 2 \times k$
- •[2] $16 - 8k \geq 0$ for real roots $\Rightarrow k \leq 2$

[SQA] 12. The roots of the equation $(x - 1)(x + k) = -4$ are equal.

Find the values of k. **5**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
	1	C	CN	A18		1995 P1 Q20
	4	A/B	CN	A18	$k = -5, 3$	

- •[1] $x^2 + kx - x + 4 - k = 0$
- •[2] $b^2 - 4ac = 0$
- •[3] $(k-1)^2 - 4(4-k)$
- •[4] $k^2 + 2k - 15 = 0$
- •[5] $k = -5,\ k = 3$

[SQA] 13. For what value of a does the equation $ax^2 + 20x + 40 = 0$ have equal roots? **2**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
	2	C	NC	A18		1996 P1 Q2

•[1] $b^2 - 4ac = 0$

•[2] $a = 2\frac{1}{2}$

[SQA] 14. Show that the equation $(1-2k)x^2 - 5kx - 2k = 0$ has real roots for all integer values of k. **5**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
	5	A/B	CN	A18, A16, CGD	proof	2002 P2 Q9

•[1] ss: know to use discriminant •[2] ic: pick out discriminant •[3] pd: simplify to quadratic •[4] ss: choose to draw table or graph •[5] pd: complete proof using disc.≥ 0	•[1] discriminant $= \ldots$ •[2] disc $= (-5k)^2 - 4(1-2k)(-2k)$ •[3] $9k^2 + 8k$ •[4] e.g. draw a table, graph, complete the square •[5] complete proof and conclusion relating to disc.≥ 0

[SQA] 15. Find the possible values of k for which the line $x - y = k$ is a tangent to the circle $x^2 + y^2 = 18$. **5**

Part	Marks	Level	Calc.	Content	Answer	U2 OC4
	2	C	CN	A18, A20		1989 P1 Q18
	3	A/B	CN	G13		

•[1] $x^2 + (x-k)^2 = 18$

•[2] $2x^2 - 2kx + k^2 - 18 = 0$

•[3] strat: "$b^2 - 4ac$" $= 0$

•[4] $(-2k)^2 - 4.2\left(k^2 - 18\right)$

•[5] $k = \pm 6$

[END OF QUESTIONS]