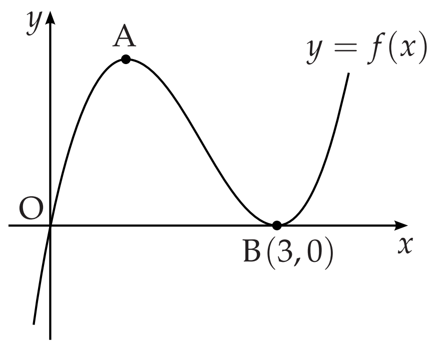
[SQA] 1. A sketch of the graph of $y = f(x)$ where $f(x) = x^3 - 6x^2 + 9x$ is shown below.

The graph has a maximum at A and a minimum at $B(3, 0)$.



(*a*) Find the coordinates of the turning point at A. **4**

(*b*) Hence sketch the graph of $y = g(x)$ where $g(x) = f(x + 2) + 4$.

Indicate the coordinates of the turning points. There is no need to calculate the coordinates of the points of intersection with the axes. **2**

(*c*) Write down the range of values of k for which $g(x) = k$ has 3 real roots. **1**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
(*a*)	4	C	NC	C8	$A(1, 4)$	2000 P1 Q2
(*b*)	2	C	NC	A3	sketch (translate 4 up, 2 left)	
(*c*)	1	A/B	NC	A2	$4 < k < 8$	

•[1] ss: know to differentiate •[2] pd: differentiate correctly •[3] ss: know gradient = 0 •[4] pd: process •[5] ic: interpret transformation •[6] ic: interpret transformation •[7] ic: interpret sketch	•[1] $\frac{dy}{dx} = \ldots$ •[2] $\frac{dy}{dx} = 3x^2 - 12x + 9$ •[3] $3x^2 - 12x + 9 = 0$ •[4] $A = (1, 4)$ translate $f(x)$ 4 units up, 2 units left •[5] sketch with coord. of $A'(-1, 8)$ •[6] sketch with coord. of $B'(1, 4)$ •[7] $4 < k < 8$ (accept $4 \leq k \leq 8$)

[SQA] 2. A function f is defined by the formula $f(x) = (x-1)^2(x+2)$ where $x \in \mathbb{R}$.

(*a*) Find the coordinates of the points where the curve with equation $y = f(x)$ crosses the x- and y-axes. **3**

(*b*) Find the stationary points of this curve $y = f(x)$ and determine their nature. **7**

(*c*) Sketch the curve $y = f(x)$. **2**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
(*a*)	3	C	NC	A6		1990 P2 Q1
(*b*)	7	C	NC	C8		
(*c*)	2	C	NC	C10		

(*a*)
- $\bullet^1$ $x = 1, -2$
- $\bullet^2$ $(1,0)$ and $(-2,0)$
- $\bullet^3$ $(0,2)$

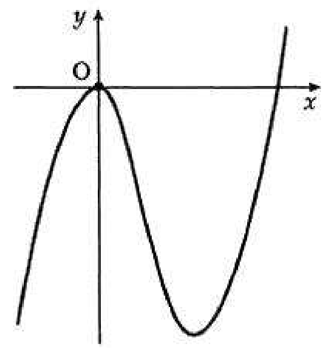
(*b*)
- $\bullet^4$ $f(x) = x^3 - 3x + 2$
- $\bullet^5$ $f'(x) = 3x^2 - 3$
- $\bullet^6$ $f'(x) = 0$ stated explicitly
- $\bullet^7$ $x = 1$ and -1
- $\bullet^8$

x	-1^-	-1	-1^+	1^-	1	1^+
$f'(x)$	$+$	0	$-$	$-$	0	$+$

- $\bullet^9$ max at $(-1, 4)$
- $\bullet^{10}$ min at $(1,0)$

(*c*)
- $\bullet^{11}$ correct shape of sketch
- $\bullet^{12}$ correct annotation of sketch(max, min, 2 axes intersections)

[SQA] 3. (*a*) The diagram shows a part of the curve with equation $y = 2x^2(x-3)$. Find the coordinates of the stationary points on the graph and determine their nature. **(5)**



(*b*) State the range of values of k for which $y = k$ intersects the graph in three distinct points. **(2)**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
(*a*)	5	C	CN	C8		1991 P2 Q1
(*b*)	2	C	CN	A1		

(*a*)

•1 $\frac{dy}{dx} = 6x^2 - 12x$

•2 $\frac{dy}{dx} = 0$

•3 $x = 0, \; x = 2$

•4

x	0^-	0	0^+	2^-	2	2^+
$\frac{dy}{dx}$	+	−	+	+	−	+

•5 max. at $(0,0)$ min at $(2,-8)$

(*b*)

•6 $k < 0$

•7 $k > -8$

[SQA] 4. A curve has equation $y = x^4 - 4x^3 + 3$.

(*a*) Find algebraically the coordinates of the stationary points. **6**

(*b*) Determine the nature of the stationary points. **2**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
(*a*)	6	C	NC	C8		1996 P2 Q1
(*b*)	2	C	NC	C8		

(a) $\bullet^1$ $\frac{dy}{dx} =$

$\bullet^2$ $4x^3 - 12x^2$

$\bullet^3$ $= 0$ stated explicitly

$\bullet^4$ *e.g.* $4x^2(x-3)$

$\bullet^5$ $x = 0,\ 3$

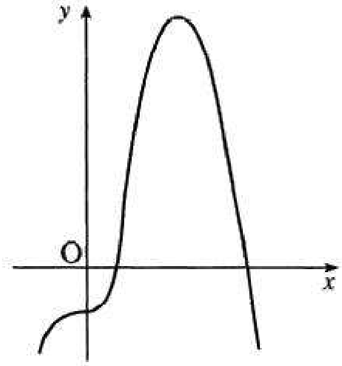
$\bullet^6$ $y = 3,\ -24$

(b) $\bullet^7$

x	0^-	0	0^+	3	3^+
$\frac{dy}{dx}$	$-$	0	$-$	0	$+$

$\bullet^8$ pt of inflection at $x = 0$
minimum at $x = 3$

[SQA] 5. A curve has equation $y = -x^4 + 4x^3 - 2$. An incomplete sketch of the graph is shown in the diagram.



(a) Find the coordinates of the stationary points. **(6)**

(b) Determine the nature of the stationary points. **(2)**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
(a)	6	C	CN	C8		1998 P2 Q2
(b)	2	C	CN	C8		

(a) •1 $\frac{dy}{dx} = \ldots\ldots$ *stated or implied* by •2

•2 $-4x^3 + 12x^2$

•3 $-4x^3 + 12x^2 = 0$ *or* $\frac{dy}{dx} = 0$ *explicitly stated*

•4 $-4x^2(x-3)$ (*accept* $x^2(-4x+12)$)

•5 $x = 0$ *and* 3

•6 $y = -2$ *and* 25

(b) •7

x	0^-	0	0^+	3	3^+
$\frac{dy}{dx}$					

•8 + 0 + 0 −

PI at $x = 0$, max *at* $x = 3$

[SQA] 6. A curve has equation $y = 2x^3 + 3x^2 + 4x - 5$.

Prove that this curve has no stationary points. **5**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	2	C	NC	C8, C7		1999 P1 Q16
	3	A/B	NC	C8, C7		

•1 $\frac{dy}{dx} = \ldots\ldots$

•2 $6x^2 + 6x + 4$

•3 *e.g.* "$b^2 - 4ac$" $= \ldots\ldots$

•4 -60 *or* -15 (from $3x^2 + 3x + 2$)

•5 Δ negative so no st. points

OR

•1 $\frac{dy}{dx} = \ldots\ldots$

•2 $6x^2 + 6x + 4$

•3 *e.g.* complete square……

•4 $S = 6\left(x + \frac{1}{2}\right)^2 + 2\frac{1}{2}$

•5 $S \geq 2\frac{1}{2}$ so no st. points

[SQA] 7. Find the coordinates of the turning points of the curve with equation $y = x^3 - 3x^2 - 9x + 12$ and determine their nature. **8**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	8	C	CN	C8, C9	max. at $(-1, 17)$ and min. at $(3, -15)$	2009 P2 Q1

•[1] ss: know to differentiate	•[1] $\frac{dy}{dx} = \cdots$ (1 term correct)
•[2] pd: differentiate	•[2] $3x^2 - 6x - 9$
•[3] ss: set derivative to zero	•[3] $\frac{dy}{dx} = 0$
•[4] pd: factorise	•[4] $3(x+1)(x-3)$
•[5] pd: solve for x	•[5] $x = -1$ or $x = 3$
•[6] pd: evaluate y-coordinates	•[6] $y = 17$ or $y = -15$
•[7] ss: know to, and justify turning points	•[7] $\begin{array}{c\|ccc\|ccc} x & \cdots & -1 & \cdots & \cdots & 3 & \cdots \\ \hline dy/dx & + & 0 & - & - & 0 & + \end{array}$
•[8] ic: interpret result	•[8] max. at $(-1, 17)$ and min. at $(3, -15)$

[SQA] 8. A function f is defined on the set of real numbers by $f(x) = (x-2)(x^2+1)$.

(a) Find where the graph of $y = f(x)$ cuts:

(i) the x-axis;

(ii) the y-axis. **2**

(b) Find the coordinates of the stationary points on the curve with equation $y = f(x)$ and determine their nature. **8**

(c) On separate diagrams sketch the graphs of:

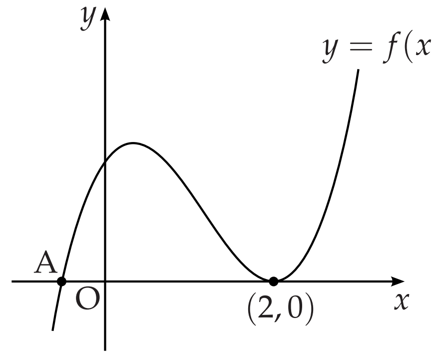
(i) $y = f(x)$;

(ii) $y = -f(x)$. **3**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
(a)	2		CN	A6	$(2,0)$, $(0,-2)$	2011 P1 Q22
(b)	8		CN	C8, C9	max: $(\frac{1}{3}, -\frac{50}{27})$, min: $(1,-2)$	
(ci)	2		CN	A8, A7	sketch	
(cii)	1		CN	A3	reflect in x-axis	

•1 ic: interpret x intercept	•1 $(2,0)$
•2 ic: interpret y intercept	•2 $(0,-2)$
•3 ic: write in differentiable form	•3 x^3-2x^2+x-2
•4 ss: know to and start to differentiate	•4 $3x^2\ldots$
•5 pd: complete derivative and equate to 0	•5 $3x^2-4x+1=0$
•6 pd: factorise derivative	•6 $(3x-1)(x-1)$
•7 pd: process for x	•7 $\frac{1}{3}$ and 1
•8 pd: evaluate y-coordinates	•8 $-\frac{50}{27}$ and -2
•9 ic: justify nature of stationary points	•9 $\begin{array}{c\|ccccc} x & \rightarrow & \frac{1}{3} & \rightarrow & 1 & \rightarrow \\ \hline f'(x) & + & 0 & - & 0 & + \end{array}$
•10 ic: interpret and state conclusions	•10 max. at $(\frac{1}{3} - \frac{50}{27})$, min. at $(1,-2)$
•11 ic: curve showing points from (a) and (b) without annotation	•11 sketch
•12 ic: **cubic** curve showing **all** intercepts and stationary points annotated	•12 sketch
•13 ic: curve from (i) reflected in x-axis	•13 reflected sketch

[SQA] 9. The diagram shows part of the graph of the curve with equation $y = 2x^3 - 7x^2 + 4x + 4$.



(*a*) Find the x-coordinate of the maximum turning point. **5**

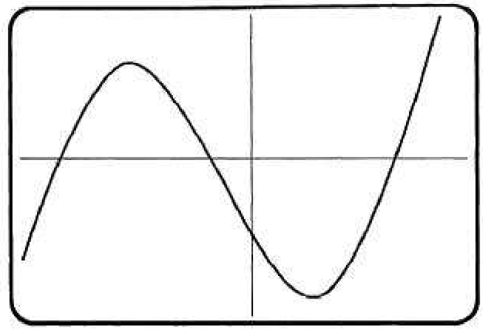
(*b*) Factorise $2x^3 - 7x^2 + 4x + 4$. **3**

(*c*) State the coordinates of the point A and hence find the values of x for which $2x^3 - 7x^2 + 4x + 4 < 0$. **2**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(*a*)	5	C	NC	C8	$x = \frac{1}{3}$	2002 P2 Q3
(*b*)	3	C	NC	A21	$(x-2)(2x+1)(x-2)$	
(*c*)	2	C	NC	A6	$\mathrm{A}(-\frac{1}{2}, 0)$, $x < -\frac{1}{2}$	

•1 ss: know to differentiate	•1 $f'(x) = \ldots$
•2 pd: differentiate	•2 $6x^2 - 14x + 4$
•3 ss: know to set derivative to zero	•3 $6x^2 - 14x + 4 = 0$
•4 pd: start solving process of equation	•4 $(3x-1)(x-2)$
•5 pd: complete solving process	•5 $x = \frac{1}{3}$
•6 ss: strategy for cubic, e.g. synth. division	•6 $\begin{array}{c\|cccc} \cdots & 2 & -7 & 4 & 4 \\ & & \cdots & \cdots & \cdots \\ \hline & \cdots & \cdots & \cdots & \boxed{0} \end{array}$
•7 ic: extract quadratic factor	•7 $2x^2 - 3x - 2$
•8 pd: complete the cubic factorisation	•8 $(x-2)(2x+1)(x-2)$
•9 ic: interpret the factors	•9 $\mathrm{A}(-\frac{1}{2}, 0)$
•10 ic: interpret the diagram	•10 $x < -\frac{1}{2}$

[SQA] 10. The diagram shows part of the graph of the curve with equation $f(x) = x^3 + x^2 - 16x - 16$.



(*a*) Factorise $f(x)$. **(3)**

(*b*) Write down the co-ordinates of the four points where the curve crosses the x and y axes. **(2)**

(*c*) Find the turning points and justify their nature. **(6)**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(*a*)	3	C	NC	A21		1992 P2 Q1
(*b*)	2	C	NC	A6		
(*c*)	6	C	NC	C8		

(a) •1 any linear factor

•2 corresponding quadratic factor

•3 $f(x) = (x+1)(x-4)(x+4)$

(b) •4 For all 3 points on x-axis

•5 $(0, -16)$

(c) •6 $f'(x) = 3x^2 + 2x - 16$

•7 use $f'(x) = 0$

•8 $x = 2$, and $x = -\frac{8}{3}$

•9 $y = -36$, and $y = \frac{400}{27}$ (14.8)

•10

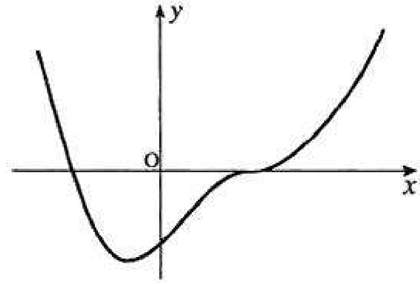
	$-\frac{8}{3}^-$	$-\frac{8}{3}$	$-\frac{8}{3}^+$	2^-	2	2^+
$f'(x)$	$+$	0	$-$	$-$	0	$+$
	⋰	⋯	⋱	⋱	⋯	⋰

•11 max at $\left(-\frac{8}{3}, \frac{400}{27}\right)$, min at $(2, -36)$

[SQA] 11. The function f, whose incomplete graph is shown in the diagram, is defined by

$f(x) = x^4 - 2x^3 + 2x - 1.$

Find the coordinates of the stationary points and justify their nature. **(8)**



Part	Marks	Level	Calc.	Content	Answer	U2 OC1
	8	C	CN	A21, C8		1993 P2 Q1

•[1] for knowing to differentiate

•[2] $f'(x) = 4x^3 - 6x^2 + 2$

•[3] for putting $f'(x) = 0$

•[4] for factorising or checking zeros

•[5] $x = -\frac{1}{2},\ x = 1$

•[6] $y = -\frac{27}{16},\ y = 0$

•[7] completed nature table

x	$< -\frac{1}{2}$	$-\frac{1}{2}$	$> -\frac{1}{2}$	< 1	1	> 1
$f'(x)$	$-ve$	0	$+ve$	$+ve$	0	$+ve$
	$\searrow$	—	$\nearrow$	$\nearrow$	—	$\nearrow$

•[8] $(1,0)$ is pt. of inflexion, $\left(-\frac{1}{2}, -1\frac{11}{16}\right)$ is min t.p.

[SQA] 12. A function f is defined by the formula $f(x) = 4x^2(x-3)$ where $x \in \mathbb{R}$.

(*a*) Write down the coordinates of the points where the curve with equation $y = f(x)$ meets the x- and y-axes. **2**

(*b*) Find the stationary points of $y = f(x)$ and determine the nature of each. **6**

(*c*) Sketch the curve $y = f(x)$. **2**

(*d*) Find the area completely enclosed by the curve $y = f(x)$ and the x-axis. **4**

Part	Marks	Level	Calc.	Content	Answer	U2 OC2
(*a*)	2	C	NC	A6		1989 P2 Q1
(*b*)	6	C	NC	C8		
(*c*)	2	C	NC	C10		
(*d*)	4	C	NC	C16		

(*a*) $\bullet^1$ (0,0)

$\bullet^2$ (3,0)

(*b*) $\bullet^3$ $f'(x) = 12x^2 - 24x$

$\bullet^4$ $f'(x) = 0$ stated explicitly

$\bullet^5$ $x = 0,\ x = 2$

$\bullet^6$

x	0^-	0	0^+	2^-	2	2^+
f'	$+$	0	$-$	$-$	0	$+$

$\bullet^7$ max at (0,0)

$\bullet^8$ min at (2,−16)

(*c*) $\bullet^9$ correct shape

$\bullet^{10}$ (0,0),(3,0),(2,−16) annotated

(*d*) $\bullet^{11}$ $\int_0^3 \left(4x^3 - 12x^2\right) dx$

$\bullet^{12}$ area $= -\int_0^3 \left(4x^3 - 12x^2\right) dx$

$\bullet^{13}$ $\left[-x^4 + 4x^3\right]_0^3$

$\bullet^{14}$ 27

[END OF QUESTIONS]