vector length

[SQA] 1. The vectors $\boldsymbol{p}$, $\boldsymbol{q}$ and $\boldsymbol{r}$ are defined as follows:

$$\boldsymbol{p} = 3\boldsymbol{i} - 3\boldsymbol{j} + 2\boldsymbol{k},\ \boldsymbol{q} = 4\boldsymbol{i} - \boldsymbol{j} + \boldsymbol{k},\ \boldsymbol{r} = 4\boldsymbol{i} - 2\boldsymbol{j} + 3\boldsymbol{k}.$$

(*a*) Find $2\boldsymbol{p} - \boldsymbol{q} + \boldsymbol{r}$ in terms of $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$. **1**

(*b*) Find the value of $|2\boldsymbol{p} - \boldsymbol{q} + \boldsymbol{r}|$. **2**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	1	C	CN	G18		1989 P1 Q3
(*b*)	2	C	CN	G16		

•[1] $6i - 7j + 6k$

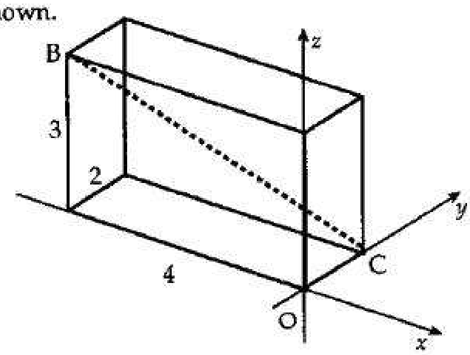
•[2] $\sqrt{6^2 + (-7)^2 + 6^2}$

•[3] 11

[SQA] 2. A cuboid crystal is placed relative to the coordinate axes as shown.

(*a*) Write down $\overrightarrow{BC}$ in component form.

(*b*) Calculate $|\overrightarrow{BC}|$. 2



Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	1	C	CN	G16		1990 P1 Q5
(*b*)	1	C	CN	G16		

•[1] $\overrightarrow{BC} = \begin{pmatrix} 4 \\ 2 \\ -3 \end{pmatrix}$

•[2] $\sqrt{29}$

[SQA] 3. A is the point $(-3,2,4)$ and B is $(-1,3,2)$. Find

(*a*) the components of vector $\overrightarrow{AB}$; 1

(*b*) the length of AB. 2

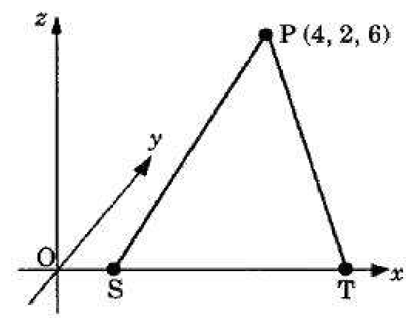
Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	1	C	CN	G16		1993 P1 Q1
(*b*)	2	C	CN	G16		

•1 $\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$

•2 $\sqrt{(-3+1)^2 + (2-3)^2 + (4-2)^2}$

•3 3

[SQA] 4. The diagram shows a point P with coordinates (4, 2, 6) and two points S and T which lie on the *x*-axis. If P is 7 units from S and 7 units from T, find the coordinates of S and T. 3



Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	3	A/B	CN	G16		1994 P1 Q18

•1 $(x,0,0)$ or equiv.	*OR*	•1	$PQ=\sqrt{40}$	*OR*	•1	$d^2=7^2-6^2-2^2$
•2 $(x-4)^2+4+36=49$ or equiv.		•2	$d=3$		•2	$d=3$
•3 $x=1,\ 7$		•3	(1,0,0), (7,0,0)		•3	(1,0,0), (7,0,0)

[SQA] 5. Vectors p, q and r are defined by

$$p = i + j - k, \quad q = i + 4k \quad \text{and} \quad r = 4i - 3j.$$

(a) Express $p - q + 2r$ in component form. 2

(b) Calculate $p.r$ 1

(c) Find $|r|$. 1

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(a)	2	C	CN	G16		1998 P1 Q3
(b)	1	C	CN	G26		
(c)	1	C	CN	G16		

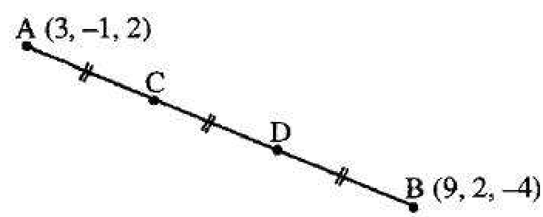
•[1] $p = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, q = \begin{pmatrix} 1 \\ 0 \\ 4 \end{pmatrix}, r = \begin{pmatrix} 4 \\ -3 \\ 0 \end{pmatrix}$ s/i by •[2]

•[2] $\begin{pmatrix} 8 \\ -5 \\ -5 \end{pmatrix}$

•[3] 1

•[4] 5

[SQA] 6. The line AB is divided into 3 equal parts by the points C and D, as shown. A and B have coordinates (3, –1, 2) and (9, 2, –4).



(a) Find the components of $\overrightarrow{AB}$ and $\overrightarrow{AC}$. 2

(b) Find the coordinates of C and D. 2

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(a)	2	C	CN	G16		1998 P1 Q5
(b)	2	C	CN	G16		

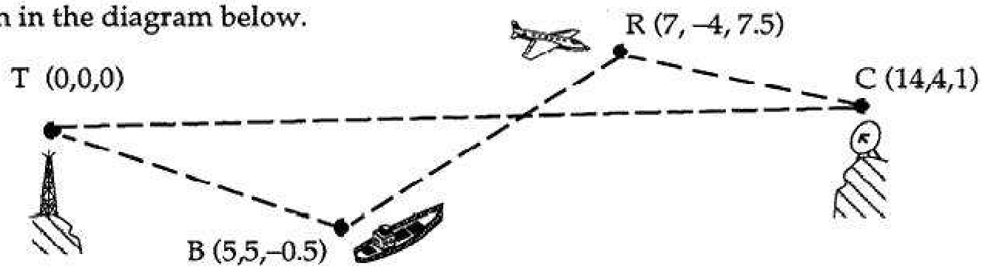
•[1] $\overrightarrow{AB} = \begin{pmatrix} 6 \\ 3 \\ -6 \end{pmatrix}$

•[2] $\overrightarrow{AC} = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$

•[3] $C = (5,0,0)$

•[4] $D = (7,1,-2)$

[SQA] 7. Relative to a suitable set of co-ordinate axes with a scale of 1 unit to 2 kilometres, the positions of a transmitter mast, ship, aircraft and satellite dish are shown in the diagram below.



The top T of the transmitter mast is the origin, the bridge B on the ship is the point (5, 5, –0·5), the centre C of the dish on the top of a mountain is the point (14, 4, 1) and the reflector R on the aircraft is the point (7, –4, 7·5).

(*a*) Find the distance from the bridge of the ship to the reflector on the aircraft. **(3)**

(*b*) Three minutes earlier the aircraft was at the point M(–2, 4, 8·5). Find the speed of the aircraft in kilometres per hour. **(2)**

(*c*) Prove that the direction of the beam TC is perpendicular to the direction of the beam BR. **(3)**

(*d*) Calculate the size of angle TCR. **(5)**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	3	C	CR	G16		1992 P2 Q2
(*b*)	2	C	CR	G16		
(*c*)	3	C	CR	G27		
(*d*)	5	C	CR	G28		

(*a*) •[1] Strategy: use vectors or 3-D distance formula

•[2] $\overrightarrow{BR} = \begin{pmatrix} 2 \\ 7 \\ 4 \end{pmatrix}$ *or* $BR^2 = 2^2 + 7^2 + 4^2$

•[3] answer

(*b*) •[4] $|\overrightarrow{MR}| = \sqrt{115.25}$ or equivalent

•[5] answer

(*c*) •[6] know to use a scalar product

•[7] $\overrightarrow{TC}.\overrightarrow{BR} = 0$

•[8] communication: 0 ⇔ perpendicularity

(*d*) •[9] Strategy: know to use

$\cos T\hat{C}R = \dfrac{\overrightarrow{TC}.\overrightarrow{RC}}{|TC||RC|}$ or equiv.

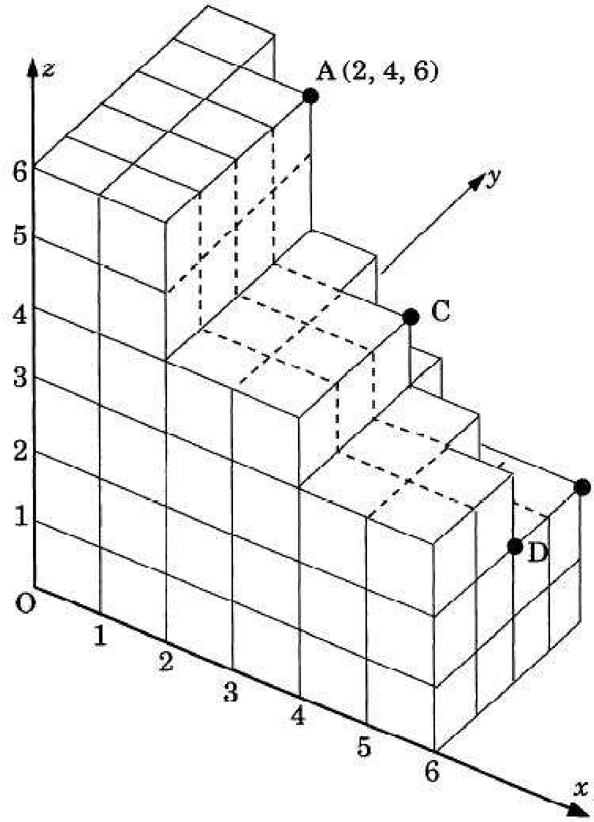
•[10] $\overrightarrow{TC} = \begin{pmatrix} 12 \\ -4 \\ 1 \end{pmatrix}$ and $\overrightarrow{RC} = \begin{pmatrix} 5 \\ -6 \\ -2 \end{pmatrix}$

•[11] $\sqrt{161}$ and $\sqrt{65}$

•[12] $\overrightarrow{TC}.\overrightarrow{RC} = 82$

•[13] 36.7°

[SQA] 8. With coordinate axes as shown, the point A is (2,4,6).



(*a*) Write down the coordinates of B,C and D. **(3)**

(*b*) Show that C is the midpoint of AD. **(1)**

(*c*) By using the components of the vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$, calculate the size of angle AOB, where O is the origin. **(4)**

(*d*) Hence calculate the size of angle OAB. **(2)**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	3	C	CR	G16		1994 P2 Q3
(*b*)	1	C	CR	G25		
(*c*)	4	C	CR	G28		
(*d*)	2	C	CR	CGD		

(a) •[1] One of B, C or D

•[2] Remaining two of B, C and D

•[3] B (6,4,2), C (4,3,4), D (6,2,2)

(b) •[4] $\left(\frac{2+6}{2}, \frac{4+2}{2}, \frac{6+2}{2}\right)$

(c) •[5] $\cos A\hat{O}B = \frac{\overrightarrow{OA}.\overrightarrow{OB}}{|\overrightarrow{OA}||\overrightarrow{OB}|}$ *or* $\frac{OA^2 + OB^2 - AB^2}{2 \times OA \times OB}$ or equivalents

•[6] $\overrightarrow{OA}.\overrightarrow{OB} = 40$ *or* $AB^2 = 32$

•[7] $OA = \sqrt{56} = OB$

•[8] $44°$

(d) •[9] strategy: e.g. use isosceles Δ

•[10] $68°$

[SQA] 9. The first four levels of a stepped pyramid with a square base are shown in Diagram 1.

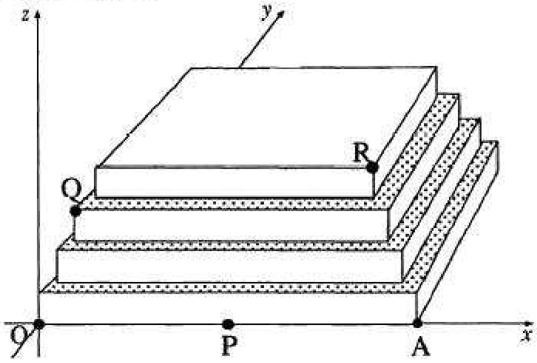


Diagram 1

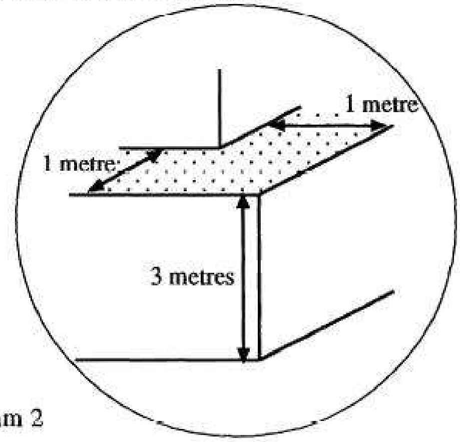


Diagram 2

Each level is a square-based cuboid with a height of 3 m. The shaded parts indicate the steps which have a "width" of 1 m.
The height and "width" of a step at a corner are shown in the enlargement in Diagram 2.
With coordinate axes as shown and 1 unit representing 1 metre, the coordinates of P and A are (12, 0, 0) and (24, 0, 0).

(*a*) Find the coordinates of Q and R. **(2)**

(*b*) Find the size of angle QPR. **(7)**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	2	C	CR	G16		1996 P2 Q3
(*b*)	7	C	CR	G28		

(a) •[1] $Q = (2,2,9)$

•[2] $R = (21,3,12)$

(b) •[3] $\cos\theta = \frac{a.b}{|a||b|}$ *with some subsequent use*

eg $\cos Q\hat{P}R = \frac{\overrightarrow{PQ}.\overrightarrow{PR}}{|\overrightarrow{PQ}||\overrightarrow{PR}|}$

•[4] $\overrightarrow{PQ} = \begin{pmatrix} -10 \\ 2 \\ 9 \end{pmatrix}$ •[5] $\overrightarrow{PR} = \begin{pmatrix} 9 \\ 3 \\ 12 \end{pmatrix}$

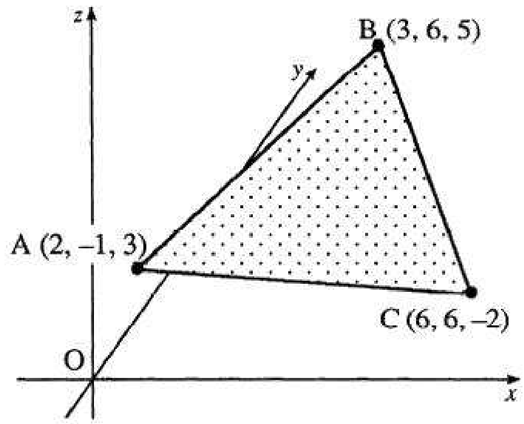
•[6] $|\overrightarrow{PQ}| = \sqrt{185}$

•[7] $|\overrightarrow{PR}| = \sqrt{234}$

•[8] $\overrightarrow{PQ}.\overrightarrow{PR} = 24$

•[9] $Q\hat{P}R = 83 \cdot 4^\circ$

[SQA] 10. A triangle ABC has vertices A (2, –1, 3), B(3, 6, 5) and C (6, 6, –2).



(*a*) Find $\overrightarrow{AB}$ and $\overrightarrow{AC}$. **(2)**

(*b*) Calculate the size of angle BAC. **(5)**

(*c*) Hence find the area of the triangle. **(2)**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	2	C	CR	G16		1998 P2 Q1
(*b*)	5	C	CR	G28		
(*c*)	2	C	CR	CGD		

(*a*)

•[1] $\overrightarrow{AB} = \begin{pmatrix} 1 \\ 7 \\ 2 \end{pmatrix}$

•[2] $\overrightarrow{AC} = \begin{pmatrix} 4 \\ 7 \\ -5 \end{pmatrix}$

(*b*)

•[3] $\cos B\hat{A}C = \dfrac{\overrightarrow{AB}.\overrightarrow{AC}}{|\overrightarrow{AB}|\,|\overrightarrow{AC}|}$ *stated or implied by responses to •[4] to •[7]*

•[4] $\overrightarrow{AB}.\overrightarrow{AC} = 4 + 49 - 10$

•[5] $\overrightarrow{AB} = \sqrt{54}$

•[6] $\overrightarrow{AC} = \sqrt{90}$

•[7] $B\hat{A}C = 51 \cdot 9°$

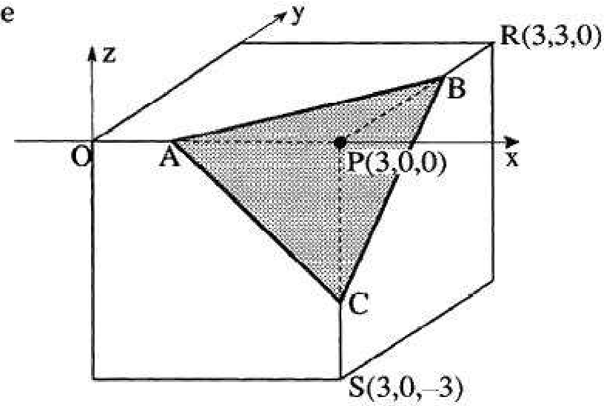
(*c*)

•[8] **identify** 2 sides and included angle *e.g.* $\sqrt{54}$, $\sqrt{90}$, $B\hat{A}C$

•[9] $27 \cdot 4$

[SQA] 11. A model of a crystal was made from a cube of side 3 units by slicing off the corner at P to leave a triangular face ABC. Coordinate axes have been introduced as shown in the diagram.

The point A divides OP in the ratio 1:2. Points B and C similarly divide RP and SP respectively in the ratio 1:2.



(*a*) Find the coordinates of A, B and C. **(3)**

(*b*) Calculate the area of triangle ABC. **(4)**

(*c*) Calculate the percentage increase or decrease in the surface area of the crystal compared with the cube. **(5)**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	3	C	CR	G25		1993 P2 Q5
(*b*)	4	C	CR	G16, CGD		
(*c*)	5	C	CR	CGD		

(*a*)
- •[1] $A(1,0,0)$
- •[2] $B(3,2,0)$
- •[3] $C(3,0,-2)$

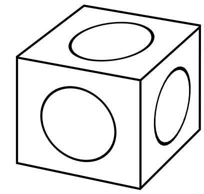
(*b*)
- •[4] strategy for area of triangle and attempt to calculate parts
- •[5] 60° or altitude $= \sqrt{6}$
- •[6] side $= 2\sqrt{2}$
- •[7] using chosen formula correctly

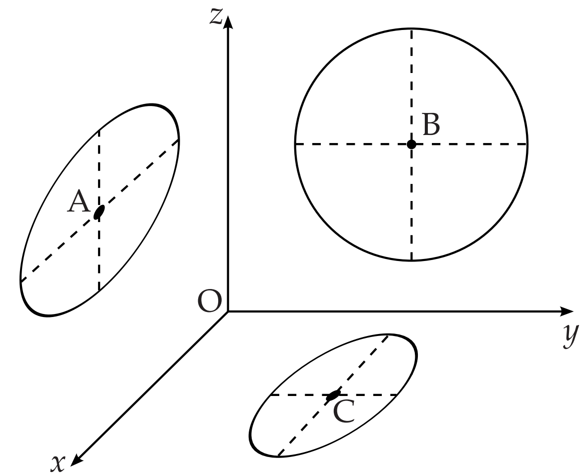
(*c*)
- •[8] 54 unit^2 for cube
- •[9] know how to calculate s.a of crystal
- •[10] area of 1 pentagonal face = 7 unit^2
- •[11] 51.5 unit^2 for crystal $(48 + 2\sqrt{3})$
- •[12] strategy for finding % decrease

[SQA] 12. A box in the shape of a cuboid is designed with **circles** of different sizes on each face.

The diagram shows three of the circles, where the origin represents one of the corners of the cuboid. The centres of the circles are $A(6, 0, 7)$, $B(0, 5, 6)$ and $C(4, 5, 0)$.

Find the size of angle ABC. **7**





Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	5	C	CR	G17, G16, G22		2001 P2 Q4
	2	A/B	CR	G26, G28	71·5°	

•[1] ss: use $\frac{\overrightarrow{BA}.\overrightarrow{BC}}{\lvert\overrightarrow{BA}\rvert\lvert\overrightarrow{BC}\rvert}$	•[1] use $\frac{\overrightarrow{BA}.\overrightarrow{BC}}{\lvert\overrightarrow{BA}\rvert\lvert\overrightarrow{BC}\rvert}$ *stated or implied by* •[7]
•[2] ic: state vector e.g. $\overrightarrow{BA}$	•[2] $\overrightarrow{BA} = \begin{pmatrix} 6 \\ -5 \\ 1 \end{pmatrix}$
•[3] ic: state a consistent vector e.g. $\overrightarrow{BC}$	•[3] $\overrightarrow{BC} = \begin{pmatrix} 4 \\ 0 \\ -6 \end{pmatrix}$
•[4] pd: process $\lvert\overrightarrow{BA}\rvert$	•[4] $\lvert\overrightarrow{BA}\rvert = \sqrt{62}$
•[5] pd: process $\lvert\overrightarrow{BC}\rvert$	•[5] $\lvert\overrightarrow{BC}\rvert = \sqrt{52}$
•[6] pd: process scalar product	•[6] $\overrightarrow{BA}.\overrightarrow{BC} = 18$
•[7] pd: find angle	•[7] $A\hat{B}C = 71{\cdot}5^\circ$

[SQA] 13. The position vectors of the points P and Q are $\boldsymbol{p} = -\boldsymbol{i} + 3\boldsymbol{j} + 4\boldsymbol{k}$ and $\boldsymbol{q} = 7\boldsymbol{i} - \boldsymbol{j} + 5\boldsymbol{k}$ respectively.

(*a*) Express $\overrightarrow{PQ}$ in component form. **2**

(*b*) Find the length of PQ. **1**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	2	C	CN	G18, G16		1997 P1 Q4
(*b*)	1	C	CN	G16		

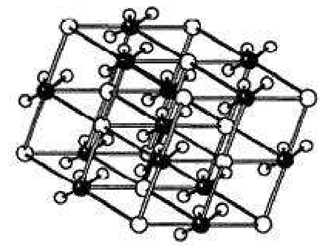
•[1] $q - p = 8i - 4j + k$

or $p = \begin{pmatrix} -1 \\ 3 \\ 4 \end{pmatrix}$, $q = \begin{pmatrix} 7 \\ -1 \\ 5 \end{pmatrix}$

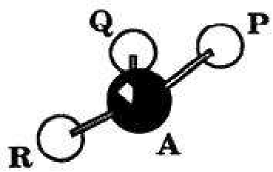
•[2] $\overrightarrow{PQ} = \begin{pmatrix} 8 \\ -4 \\ 1 \end{pmatrix}$

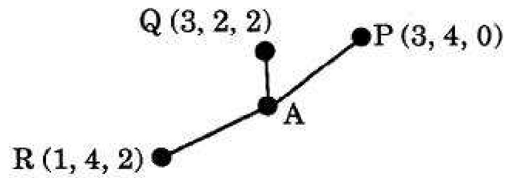
•[3] 9

[SQA] 14. The diagram shows the rhombohedral crystal lattice of calcium carbonate.



The three oxygen atoms P, Q and R around the carbon atom A have coordinates as shown below.





(*a*) Calculate the size of angle PQR. **(4)**

(*b*) M is the midpoint of QR and T is the point which divides PM in the ratio 2:1.

(i) Find the coordinates of T.

(ii) Show that P, Q and R are equidistant from T. **(6)**

(*c*) The coordinates of A are (2, 3, 1).

(i) Show that P, Q and R are also equidistant from A

(ii) Explain why T, and not A, is the centre of the circle through P, Q and R. **(2)**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	4	C	CN	G28		1995 P2 Q5
(*b*)	6	C	CN	G25, G16		
(*c*)	1	C	CN	G16, G1		
(*c*)	1	A/B	CN	G16, CGD		

(a)

•1 $PQ = \sqrt{8},\ RQ = \sqrt{8},$

•2 Use s.p.: $\vec{PQ}.\vec{RQ} = |\vec{PQ}|.|\vec{RQ}|\cos\theta$

•3 $\begin{pmatrix}0\\-2\\2\end{pmatrix}.\begin{pmatrix}2\\-2\\0\end{pmatrix} = 4$

•4 60°

(b)

•5 $M = (2,3,2)$

•6 $\vec{PT} = \frac{2}{3}\vec{PM}$ or equivalent

•7 $\vec{PT} = \frac{2}{3}\begin{pmatrix}-1\\-1\\2\end{pmatrix}$ or equiv.

•8 $T = \left(\frac{7}{3}, \frac{10}{3}, \frac{4}{3}\right)$

•9 $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2}$ stated or implied

•10 $PT = 2\sqrt{\frac{2}{3}}, QT = 2\sqrt{\frac{2}{3}}, RT = 2\sqrt{\frac{2}{3}}$ or equivalent

(c)

•11 $PA = QA = RA = \sqrt{3}$

•12 A is in a different plane

[SQA] 15. Diagram 1 shows a christmas tree decoration which is made of coloured glass rods in the shape of a square-based prism topped by a square pyramid. Diagram 2 shows the decoration relative to the origin and rectangular coordinate axes OX, OY and OZ.

The vertex F has position vector $\begin{pmatrix} 2 \\ 2 \\ -7 \end{pmatrix}$

and the vertex V has position vector $\begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}$

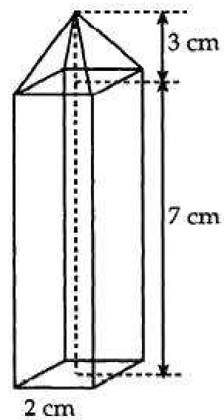


Diagram 1

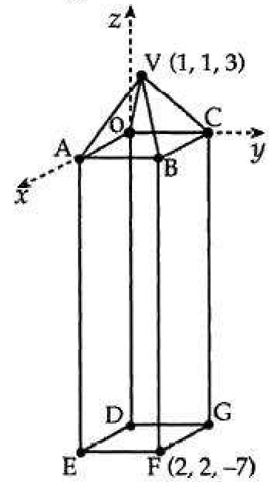


Diagram 2

(*a*) Find

(i) the components of the vectors represented by $\overrightarrow{VF}$ and $\overrightarrow{VE}$;

(ii) the size of angle EVF. **(7)**

(*b*) To make the decoration more attractive, triangular sheets of coloured glass VEF and VDG are added to it.

Calculate the area of the glass triangle VEF. **(3)**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	7	C	CR	G28, G16		1991 P2 Q5
(*b*)	3	C	CR	CGD		

(*a*)

•[1] $\overrightarrow{VF} = \begin{pmatrix} 1 \\ 1 \\ -10 \end{pmatrix}$

•[2] $E = (2, 0, -7)$

•[3] $\overrightarrow{VE} = \begin{pmatrix} 1 \\ -1 \\ -10 \end{pmatrix}$

•[4] $\cos E\hat{V}F = \dfrac{\overrightarrow{VE}.\overrightarrow{VF}}{|\overrightarrow{VE}|\,|\overrightarrow{VF}|}$ **This may appear as $\frac{100}{102}$ after the completion of •[5] and •[6].**

•[5] $\overrightarrow{VE}.\overrightarrow{VF} = 100$

•[6] $|\overrightarrow{VE}|\,|\overrightarrow{VF}| = 102$

•[7] $11.4°$

(*b*)

•[8] $\frac{1}{2} VE \times VF \sin E\hat{V}F$

•[9] $\frac{1}{2} \times 102 \times \sin 11.4°$

•[10] 10.02

[END OF QUESTIONS]