2005 Paper 1

1) $\tan 60 = \sqrt{3}$ $\quad m = \sqrt{3}$ $\quad (-2, 0)$

$$y - 0 = \sqrt{3}(x+2)$$
$$\underline{\underline{y = \sqrt{3}x + 2\sqrt{3}}}$$

2. $A(-3,-2)$ $\quad B(3,6)$

P is midpoint $\therefore$ $P(0,2)$

$$d_{AB} = \sqrt{(x_A - x_B)^2 + (y_A - y_B)^2}$$
$$= \sqrt{(-6)^2 + (-8)^2}$$
$$= \sqrt{100}$$
$$d_{AB} = \underline{10 \text{ units}}$$

3a $D(6,3,9)$ —2— F —1— $B(12,6,0)$

$$\frac{\vec{DF}}{\vec{FB}} = \frac{2}{1}$$
$$\therefore \vec{DF} = 2\vec{FB}$$
$$\underline{f} - \underline{d} = 2(\underline{b} - \underline{f})$$
$$\underline{f} - \underline{d} = 2\underline{b} - 2\underline{f}$$
$$3\underline{f} = 2\underline{b} + \underline{d}$$
$$3\underline{f} = \begin{pmatrix} 24 \\ 12 \\ 0 \end{pmatrix} + \begin{pmatrix} 6 \\ 3 \\ 9 \end{pmatrix}$$
$$3\underline{f} = \begin{pmatrix} 30 \\ 15 \\ 9 \end{pmatrix}$$
$$\underline{f} = \cancel{10}\begin{pmatrix} 10 \\ 5 \\ 3 \end{pmatrix} \quad F(10,5,3)$$

4a) $f(x) = 3x - 1$ $\quad g(x) = x^2 + 7$

$$g(f(x)) = g(3x-1) \qquad g(3x-1) = (3x-1)^2 + 7$$
$$= 9x^2 - 6x + 1 + 7$$
$$\underline{\underline{= 9x^2 - 6x + 8}}$$

b) TP $(1/3, 7)$

range ≥ 7 (as 7 is min TP)

5. $y = (1 + 2\sin x)^4$

$$\frac{dy}{dx} = 4(1 + 2\sin x)^3 \times 2\cos x$$
$$\underline{\underline{= 8\cos x\,(1 + 2\sin x)^3}}$$

6a) $U_{n+1} = kU_n + 5$

when $L = 4$

$$4 = 4k + 5$$
$$-1 = 4k$$
$$\underline{\underline{k = -1/4}}$$

b) $U_1 = 3m + 5$

$$U_2 = m(3m+5) + 5$$
$$= 3m^2 + 5m + 5$$

let $3m^2 + 5m + 5 = 7$

$$3m^2 + 5m - 2 = 0$$
$$(3m - 1)(m + 2) = 0$$
$$m = 1/3 \qquad m = -2$$

No limit when $m = -2$.

7 a) $f(x) = \log_b(x-a)$

right by 4 $\quad (x-4)$

$y = b^x$ at $(1, 5)$ $\quad (9-4) = 5$

$5 = b^1$

$\underline{b = 5}$

b) $\underline{\underline{x > 4}}$

8)

3	2	-7	0	9
	0	6	-3	-9
	2	-1	-3	0 ∴ factor

$f(x) = (x-3)(2x^2 - x - 3)$

$= (x-3)(2x-3)(x+1)$

on x axis, $y=0$

$\underline{x = 3}$, $\underline{x = 3/2}$, $\underline{x = -1}$

$(3,0)$ $\quad (3/2, 0)$ $\quad (-1, 0)$

on y axis, $x = 0$

$y = (-3)(-3)(1)$

$\underline{y = 9}$

$\underline{\underline{(0,9)}}$

$f(x) = 2x^3 - 7x^2 + 9$

$f'(x) = 6x^2 - 14x = 0$ at max/min

$2x(3x-7) = 0$

$x = 0 \qquad x = 7/3$

$f'(x)$	$\to$	0	$\to$	$7/3$	$\to$
$2x(3x-7)$	+	0	−	0	+
	/	—	\	—	/

max $= (0, 9)$

∴ $\underline{\text{greatest value} = 9.}$

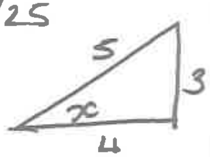
9. $\cos 2x = \frac{7}{25}$

$2\cos^2 x - 1 = 7/25$

$2\cos^2 x = 32/25$

$\cos^2 x = \frac{32}{50} = 16/25$

$\underline{\underline{\cos x = 4/5}}$



∴ $\underline{\underline{\sin x = 3/5}}$

10) $\sin x - \sqrt{3}\cos x = k\sin x\cos a - k\cos x \sin a$

$-k\sin a = -\sqrt{3}$

$k\cos a = 1$

$\tan a = \frac{\sqrt{3}}{1}$

$k = \sqrt{(\sqrt{3})^2 + 1^2}$

$= \sqrt{4}$

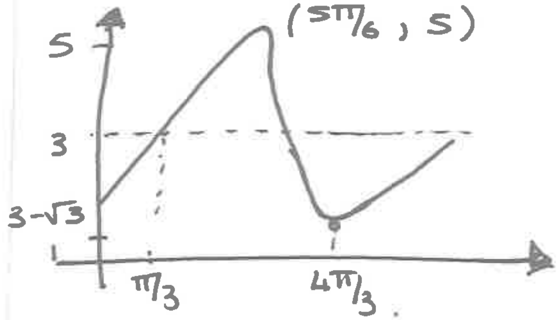
$\underline{\underline{= 2}}$

$a = \pi/3$

∴ $\sin x - \sqrt{3}\cos x = 2\sin\left(x - \frac{\pi}{3}\right)$

b) $y = 2\sin\left(x - \frac{\pi}{3}\right) + 3$

- max min $2/-2$ - up 3 $\quad$ max 5, min 1
- Right by $\pi/3$
- On y axis, $x = 0$ $\quad y = 2\left(\frac{-\sqrt{3}}{2}\right) + 3$
  $= 3 - \sqrt{3}$



11 a) $(x-t)^2 + y^2 = 4$

b) $y = 2x$

$(x-t)^2 + (2x)^2 = 4$

$x^2 - 2tx + t^2 + 4x^2 = 4$

$5x^2 - 2tx + t^2 - 4 = 0$

If tangent, $b^2 - 4ac = 0$

$(-2t)^2 - 4(5)(t^2 - 4) = 0$

$4t^2 - 20t^2 + 80 = 0$

$-16t^2 + 80 = 0$

$t^2 = 5$

$t = \pm\sqrt{5}$

∴ $\underline{\underline{t = \sqrt{5}}}$

2005 Paper 2

$$\int \frac{4x^3}{x^2} - \frac{1}{x^2}\,dx$$

$$= \int 4x - x^{-2}\,dx$$

$$= \frac{4}{2}x^2 \cancel{\mp} \frac{x^{-1}}{-1} + C$$

$$= 2x^2 + x^{-1} + C$$

$$= 2x^2 + \frac{1}{x} + C$$

2a) $\sin(p+q) = \sin p \cos q + \cos p \sin q$

$$= \frac{15}{17} \times \frac{18}{10} + \frac{8}{17} \times \frac{6}{10}$$

$$= \frac{168}{170} \cancel{\neq} \frac{84}{85}$$

b) $\cos(p+q) = \cos p \cos q - \sin p \sin q$

$$= \frac{8}{17} \times \frac{8}{10} - \frac{15}{17} \times \frac{6}{10}$$

$$= \frac{-26}{170} = \frac{-13}{85}$$

$$\tan(p+q) = \frac{\sin(p+q)}{\cos(p+q)} = \frac{84}{85} \div \cancel{8}\frac{-13}{85}$$

$$= \frac{-84}{13}$$

3a) $M\,(3,2)$ $\quad m_{AB} = 1$

$m_2 = -1$

$$y - 2 = -1(x-3)$$

$$y - 2 = -x + 3$$

$$y = -x + 5 \Rightarrow x + y = 5$$

b) $x + 3y = 1$ $\quad$ $m\,tgt = -1/3$

$3y = -x + 1$ $\quad$ $\therefore m\,radius = 3$

$y = \frac{-1}{3}x + \frac{1}{3}$

at $(1,0)$

$$y - 0 = 3(x-1)$$

$$y = 3x - 3$$

c) let $3x - 3 = -x + 5$

$$4x = 8$$

$$x = 2$$

$$y = 3(2) - 3$$

$$y = 3$$

centre $(2,3)$

radius $= d_{AC} = \sqrt{(x_A - x_C)^2 + (y_A - y_C)^2}$

$$= \sqrt{1^2 + 3^2}$$

$$= \sqrt{10}$$

$$(x-2)^2 + (x-3)^2 = 10$$

4a)

$$\vec{TA} = \underline{a} - t = \begin{pmatrix} -5 \\ 15 \\ 1 \end{pmatrix}$$

$$\vec{TB} = \underline{b} - t = \begin{pmatrix} -40 \\ 15 \\ 2 \end{pmatrix}$$

$$|\vec{TA}| = \sqrt{(-5)^2 + 15^2 + 1^2}$$

$$= \sqrt{251}$$

$$|\vec{TB}| = \sqrt{(-40)^2 + 15^2 + 2^2}$$

$$= \sqrt{1829}$$

$$\vec{TA} \cdot \vec{TB} = (200) + (225) + (2)$$

$$= 427$$

$$\cos\theta = \frac{427}{\sqrt{1829}\sqrt{251}}$$

$$\theta = 50.9^\circ$$

5. pts of intersection

$$2x^2 - 9 = x^2$$

$$x^2 - 9 = 0$$

$$(x-3)(x+3) = 0$$

$$x = 3 \quad x = -3$$

$$\therefore \text{Area} = \int_{-3}^{3} x^2 - (2x^2 - 9)\,dx$$

$$= \int_{-3}^{3} -x^2 + 9\,dx$$

$$\left[-\frac{1}{3}x^3 + 9x\right]_{-3}^{3}$$

$$\left[-\frac{1}{3}(27) + 9(3)\right] - \left[-\frac{1}{3}(-27) + 9(-3)\right]$$

$= -9 + 27 - 9 + 27$

$= 18 + 18$

$= \underline{\underline{36 \text{ units}^2}}$

6). $y = \frac{24}{\sqrt{x}}$ at $x = 4$, $y = \frac{24}{\sqrt{4}} = 12$

$(4, 12)$. $y = 24x^{-1/2}$

$\frac{dy}{dx} = -12x^{-3/2}$

$= -\frac{12}{x^{3/2}}$

$= \frac{-12}{\sqrt{x^3}}$

at $x=4$ $\frac{dy}{dx} = \frac{-12}{\sqrt{4}^3}$

$= \frac{-12}{8}$

$= -3/2$

$y - 12 = -3/2(x - 4)$

$2y - 24 = -3x + 12$

$2y = -3x + 36$

$y = -3/2x + 18$

7) $\log_4(5-x) - \log_4(3-x) = 2$

$\log_4\left(\frac{5-x}{3-x}\right) = 2$

$\frac{5-x}{3-x} = 4^2$

$5 - x = 16(3-x)$

$5 - x = 48 - 16x$

$15x = 43$

$x = \underline{\underline{\frac{43}{15}}}$

8) $k\sin 2x = \sin x$

$2k\sin x\cos x = \sin x$

$2k\sin x\cos x - \sin x = 0$

$\sin x(2k\cos x - 1) = 0$

$\sin x = 0$ $\quad 2k\cos x - 1 = 0$

$x = 0, \pi, 2\pi$ $\quad 2k\cos x = 1$

$\underline{\underline{\cos x = 1/2k}}$

9) $V = 252e^{-0.06335t}$

when $t = 0$

$\underline{V = £252 \text{ million}}$

b) $252e^{-0.06335t} = 20$

$e^{-0.06335t} = \frac{20}{252}$

$-0.06335t = \ln\left(\frac{20}{252}\right)$

$t = 39.99\ldots \approx 40$ years

10. $\underline{a} \cdot (\underline{a} + \underline{b} + \underline{c})$

$= \underline{a}\cdot\underline{a} + \underline{a}\cdot\underline{b} + \underline{a}\cdot\underline{c}$

$= 3(3\cos 0) + 3(2\cos 90) + 3(3\cos 60)$

$= 9 + 0 + 4.5$

$= \underline{\underline{13.5}}$

11a) Let $f(x) = x^3 + px^2 + px + 1$

$f(-1) = (-1)^3 + p(-1)^2 + p(-1) + 1$

$= -1 + p - p + 1$

$= 0$ ∴ -1 is a solution

-1	1	p	p	1
		-1	$1-p$	-1
	1	$p-1$	1	$\boxed{0}$ ∴ factor

$f(x) = (x-1)(x^2 + (p-1)x + 1)$

for real roots

$b^2 - 4ac \geq 0$

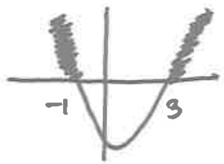
$(p-1)^2 - 4(1)(1) \geq 0$

$p^2 - 2p + 1 - 4 \geq 0$

$p^2 - 2p - 3 \geq 0$ - graph!

$(p - 3)(p + 1)$

$p = 3$ $\quad p = -1$



$p \leq -1$

$p \geq 3$