2007 Paper 1

1) $3x - y + 2 = 0$ $\quad m = 3$ $\quad (a,b) = (-1,4)$

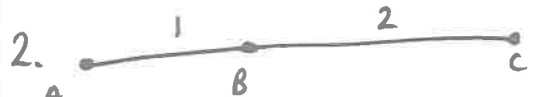
$y = 3x + 2$

$y - 4 = 3(x+1)$

$y - 4 = 3x + 3$

$y = 3x + 7$

2.



$\vec{AC} = 3\vec{AB}$

$\underline{c} - \underline{a} = 3\underline{b} - 3\underline{a}$

$\underline{c} = 3\underline{b} - 2\underline{a}$

$= 3\begin{pmatrix}1\\3\\2\end{pmatrix} - 2\begin{pmatrix}-2\\1\\-1\end{pmatrix}$

$= \begin{pmatrix}3\\9\\6\end{pmatrix} - \begin{pmatrix}-4\\2\\-2\end{pmatrix} = \begin{pmatrix}7\\7\\8\end{pmatrix}$ C (7,7,8)

3. $f(x) = x^2 + 1$ $\quad g(x) = 1 - 2x$

$g(f(x)) = g(x^2+1)$

$= 1 - 2(x^2+1)$

$= 1 - 2x^2 - 2$

$= -1 - 2x^2$

b) $g(g(x)) = g(1-2x)$

$= 1 - 2(1-2x)$

$= 1 - 2 + 4x$

$= -1 + 4x$

$= 4x - 1$

4. $b^2 - 4ac < 0$ since no real roots

$(-1)^2 - 4(k)(-1) < 0$

$1 + 4k < 0$

$4k < -1$

$k < -1/4$

5. C (7,8) $\quad r = \sqrt{49 + 64 - 77}$

$= \sqrt{36}$

$= 6$

radius D = 2

Centre B (7,8)

Centre D (15,8)

$(x-15)^2 + (x-8)^2 = 4$

6. $\sin 2x = 6\cos x$

$2\sin x \cos x = 6\cos x$

$2\sin x \cos x - 6\cos x = 0$

$2\cos x (\sin x - 3) = 0$

$\cos x = 0$ $\quad x = 90, 270°$ $\quad \sin x = 3$ no solutions

7 a $u_{n+1} = \frac{1}{4}u_n + 16$

$u_1 = \frac{1}{4}(0) + 16 = 16$

$u_2 = \frac{1}{4}(16) + 16 = 20$

$u_3 = \frac{1}{4}(20) + 16 = 21$

b) Since $-1 < \frac{1}{4} < 1$, limit exists.

$L = \frac{1}{4}L + 16$

$\frac{3}{4}L = 16$

$3L = 64$

$L = \frac{64}{3}$

8a)

3	1	-4	1	6
		3	-3	-6
	1	-1	-2	0 ∴ factor

b) $y = (x-3)(x^2 - x - 2)$

$= (x-3)(x-2)(x+1)$

on x axis, $y = 0$

$\therefore x = 3, \quad x = 2, \quad x = -1$

B (3,0) $\quad$ A (2,0) $\quad$ (-1,0)

c) $\int_0^2 x^3 - 4x^2 + x + 6 \, dx$

$\left[\frac{x^4}{4} - \frac{4x^3}{3} + \frac{x^2}{2} + 6x\right]_0^2$

$= \left[4 - \frac{32}{3} + 2 + 12\right] - [0]$

$= 18 - \frac{32}{3}$

$= \frac{54}{3} - \frac{32}{3}$

$= \underline{\underline{\frac{22}{3} \text{ units}^2}}$

9a) $f(x) = 3x - x^3$

on x axis, $y = 0$

$3x - x^3 = 0$

$x(3 - x^2) = 3$

$x = 0, \; x = \sqrt{3}, \; x = -\sqrt{3}$

$(0,0), (\sqrt{3}, 0) \; (-\sqrt{3}, 0)$

on y axis, $x = 0$

$(0,0)$

$f'(x) = 3 - 3x^2 = 0$ at SPs

$3(1 - x^2) = 0$

$3(1-x)(1+x) = 0$

$\underline{x = 1}, \; \underline{x = -1}$

$f(1) = 3(1) - 1^3 = 3 - 1 = 2$

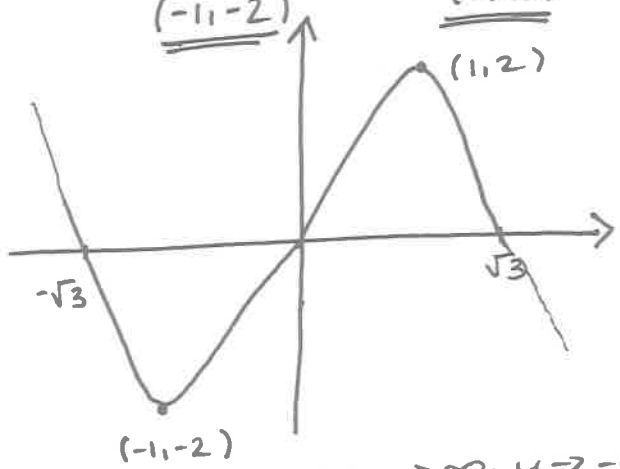
$f(-1) = 3(-1) - (-1)^3 = -3 + 1 = -2$

$(1, 2) \qquad (-1, -2)$

	→	−1	→	1	→
$3(1-x)(1+x)$	−	0	+	0	−
	↘	—	↗	—	↘

min at $(-1, -2)$

max at $(1, 2)$



as $x \to \infty$, $y \to -\infty$

as $x \to -\infty$, $y \to \infty$

10. $y = (3x^2 + 2)^{1/2}$

$\frac{dy}{dx} = \frac{1}{2}(3x^2 + 2)^{-1/2} \cdot 6x$

$= \frac{6x}{2}(3x^2 + 2)^{-1/2}$

$= \frac{3x}{\sqrt{3x^2 + 2}}$

11a) $\sqrt{3}\cos x + \sin x$

$k\cos(x - \alpha) = k\cos x \cos\alpha + k\sin x \sin\alpha$

$k\sin\alpha = 1$

$k\cos\alpha = \sqrt{3}$

$\tan\alpha = \frac{1}{\sqrt{3}}$

$\alpha = \pi/6$

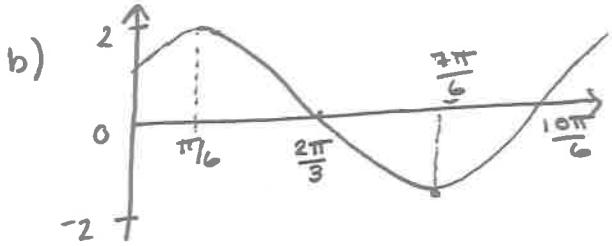
$R = \sqrt{(\sqrt{3}^2) + 1^2}$

$= 2$

S	A
T	C

$f(x) = 2\cos\left(x - \frac{\pi}{6}\right)$

b)



$\pi + \frac{\pi}{6} = \frac{7\pi}{6}$

$\frac{3\pi}{2} + \frac{\pi}{6} = \frac{9\pi}{6} + \frac{\pi}{6} = \frac{10\pi}{6}$.

$\frac{\pi}{2} + \frac{\pi}{6} = \frac{3\pi}{6} + \pi/6 = \frac{2}{3}\pi$

2007 Paper 2

1. $Q(0,2,2)$

$\underline{p} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ $\underline{q} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$

$\underline{p} \cdot \underline{q} = 0(1) + 1(2) + 1(1)$

$= 3$

$|\underline{p}| = \sqrt{2}$ $|\underline{q}| = \sqrt{1+4+1} = \sqrt{6}$

$\cos POQ = \frac{3}{\sqrt{2}\sqrt{6}} = \frac{3}{\sqrt{12}}$

$\angle POQ = 30°$

2a)

$\sin(c+d) = \sin c \cos d + \cos c \sin d$

$= \frac{1}{\sqrt{5}} \times \frac{3}{\sqrt{10}} + \frac{2}{\sqrt{5}} \times \frac{1}{\sqrt{10}}$

$= \frac{3}{\sqrt{50}} + \frac{2}{\sqrt{50}}$

$= \frac{5}{\sqrt{50}}$

$= \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}}$

b) $\sin 2c = 2 \sin c \cos c$

$= 2\left(\frac{1}{\sqrt{5}}\right)\left(\frac{2}{\sqrt{5}}\right)$

$= \frac{4}{5}$

$\cos 2d = 1 - 2\sin^2 \theta$

$= 1 - 2 \cdot \left(\frac{1}{\sqrt{10}}\right)^2$

$= 1 - \frac{2}{10}$

$= \frac{8}{10}$

$= \frac{4}{5}$

3.

$x^2 + (6-2x)^2 + 6x - 4(6-2x) - 7 = 0$

$x^2 + 36 - 24x + 4x^2 + 6x - 24 + 8x - 7 = 0$

$5x^2 - 10x + 5 = 0$

$x^2 - 2x + 1 = 0$

If tangent, $b^2 - 4ac = 0$

$(-2)^2 - 4(1)(1) -$

$= 4 - 4$

$= 0 \therefore$ tangent

$(x-1)(x-1) = 0$

$x = 1$

$y = 6 - 2(1)$

$y = 4$

prof contact $(1,4)$

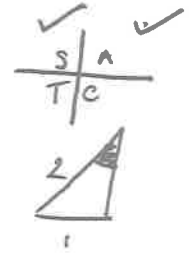
4a) $y = 2\sin 3x - 1$

$a = 2, \quad b = 3, \quad c = -1$

b) $2\sin 3x - 1 = 0$

$2\sin 3x = 1$

$\sin 3x = 1/2$



$3x = 30, 150°,$

$x = 10°, 50°.$

$P(50°, 0)$

5a) $y = \frac{1}{2}x^2 - 8x + 34$

$\frac{dy}{dx} = x - 8 = 4$

$x = 12$

$y = \frac{1}{2}(12^2) - 8(12) + 34$

$= 72 - 96 + 34$

$= 10$ $(12, 10)$

b) $\frac{1}{2}x^2 - 8x + 34 = 10$

$\frac{1}{2}x^2 - 8x + 24 = 0$

$x^2 - 16x + 48 = 0$

$(x-4)(x-12) = 0$

$x = 4$ $x = 12$

$P(4, 10)$

C (x_c, y_c)

$x_c = \frac{1}{2}(x_P + x_Q)$

$= \frac{1}{2}(16)$ $\qquad (8, y_c)$

$= 8$

at P, $m_{tgt} = -4$, $\therefore m_{CP} = 1/4$

$m_{CP} = \frac{y_c - 10}{8 - 4} = \frac{1}{4}$

$\Rightarrow \frac{y_c - 10}{4} = \frac{1}{4}$

$y_c - 10 = 1$

$y_c = 11$

C (8, 11)

6a) $ST = \sqrt{100 + 100}$

$= \sqrt{200}$

$= 10\sqrt{2}\,m$

$l = 10\sqrt{2} - 2x$

$A(x) = x(10\sqrt{2} - 2x)$

$= 10\sqrt{2}x - 2x^2$

$A'(x) = 10\sqrt{2} - 4x = 0$

$4x = 10\sqrt{2}$

$x = \frac{10\sqrt{2}}{4}$

$x = \frac{5\sqrt{2}}{2}$

$l = 10\sqrt{2} - 2\left(\frac{5\sqrt{2}}{2}\right)$

$= 10\sqrt{2} - 5\sqrt{2}$

$= 5\sqrt{2}\,m$

max: length $5\sqrt{2}$

breadth $\frac{5\sqrt{2}}{2}$

7. $\int_0^2 \sin(4x+1)\,dx$ Radians!!

$= \left[-\frac{\cos(4x+1)}{4}\right]_0^2$

$= \left(\frac{-1}{4}\cos(9)\right) - \left(-\frac{1}{4}\cos(1)\right)$

$= 0.363$

8. $\log_3(a-1) - 2 \cdot 2 = 0$

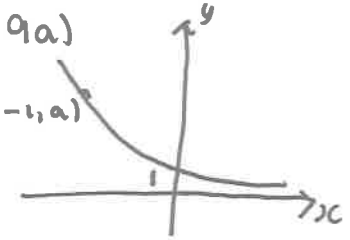
$\log_3(a-1) = 2 \cdot 2$

$a - 1 = 3^{2 \cdot 2}$

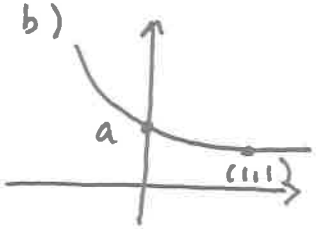
$a = 3^{2 \cdot 2} + 1$

$a = 12.2$ to 1dp

9a)



b)



b) add 1 to x coord.

10 a) $f'(x) = k(x-2)(x-4)$

$6 = k(-2)(-4)$

$6 = 8k$

$k = 3/4$

b) $\int 3/4\,(x-2)(x-4)$

$= \int \frac{3}{4}(x^2 - 6x + 8)\,dx$

$= \frac{3}{4}x^3 - \frac{9x^2}{4} + 6x + C$

$f(0) = 6$

$\therefore c = 6$

$f(x) = \frac{x^3}{4} - \frac{9x^2}{4} + 6x + 6$

a) $6 = 3 \times 4^a$

$2 = 4^a$

$a = 1/2$

$b = 3 \times 4^{-1/2}$

$= \frac{3}{2}$

c) $\log_{10} y = \log_{10} 3 + \log_{10} 4^x$

$\log_{10} y = x \log_{10} 4 + \log_{10} 3$

$(y = mx + c)$.

$m = \log_{10} 4$