2008 Paper 1

$u_{11} = 0{\cdot}3 \times 10 + 6 = 9$

$u_{12} = 0{\cdot}3(9) + 6$

$= 8{\cdot}7$ (C)

2. $(x+7)^2 + (y-6)^2 = 36$ (D)

3. $\underline{u} \cdot \underline{v} = 0$

$0k - 4 + k = 0$

$\underline{\underline{k = 4}}$ (C)

4. $L = 0{\cdot}4L = 240$

$0{\cdot}6L = -240$

$L = \frac{-240}{0{\cdot}6}$

$L = \frac{-2400}{6} = -400$ (B)

5. $m_{radius} = \frac{9-5}{7-2} = \frac{4}{5}$

$m_{tgt} = -5/4$

$y - 9 = -5/4(x-7)$ (A)

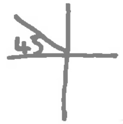
6. $\sin x = \frac{\sqrt{3}}{2}$ ✓ S | A ✓ / T | C

$x = \cancel{\frac{\pi}{3}}, \frac{2\pi}{3}$ (B)

7. $m = \tan 135$

$= -\tan 45°$

$= \underline{\underline{-1}}$ (C)



8. change signy
add 2 to x. (D)

9. $\sin x \cos a + \cos x \sin a$

$= \frac{4}{5}\sin x + \frac{3}{5}\cos x$ (B)

10. $\vec{EP} = \underline{p} - \underline{e} = \begin{pmatrix}3\\6\\3\end{pmatrix}$

$\vec{PF} = \underline{f} - \underline{p} = \begin{pmatrix}6\\12\\6\end{pmatrix} = 2\begin{pmatrix}3\\6\\3\end{pmatrix}$ (B)

$1:2$

12. $\underline{g} - \underline{f} - \underline{h}$ (C)

$= -\underline{f} + \underline{g} - \underline{h}$

13. $y = k(x-1)(x-4)$

at $(0,12)$

$12 = k(-1)(-4)$

$\underline{\underline{k = 3}}$ (A)

14. $-4\cos(2x+3) \div 2 + c$

$= \frac{-1}{2}\cos(2x+3) + c$ (B)

15. $f'(x) = 2(x^3+4) \times 3x^2$

$= 6x^2(x^3+4)$ (C)

16. $2x^2 + 4x + 7 = a(x+p)^2 + q$

$= ax^2 + 2apx + ap^2 + q$

$\underline{a = 2}$ $\quad 2(2)p = 4$ $\quad ap^2 + q = 7$

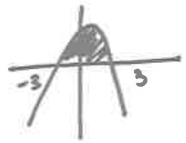
$\underline{p = 1}$ $\quad (2)(1) + q = 7$

$\underline{q = 5}$ (A)

$2(x+1)^2 + 5$

17. $9 - x^2 \geq 0$

$(3-x)(3+x) \geq 0$



$-3 < x < 3$ (C)

18. $\underline{q} \cdot (\underline{p} + \underline{q})$

$= \underline{q} \cdot \underline{p} + \underline{q} \cdot \underline{q}$

$= 10 + |q||q|\cos 0$

$= 10 + 16$

$= \underline{\underline{26}}$ (C)

19. $y = 2m^x$

$54 = 2m^3$

$m^3 = 27$

$\underline{\underline{m = 3}}$ (B)

20. $2 = \log_3(q-4)$

$q - 4 = 3^2$

$q - 4 = 9$

$\underline{\underline{q = 13}}$ (D)

21a) $f'(x) = 3x^2 - 3$

$= 0$ at SPs

$3(x^2 - 1) = 0$

$3(x-1)(x+1) = 0$

$x = 1 \quad x = -1$

$f(1) = 1^3 - 3(1) + 2$

$= 0$

$f(-1) = (-1)^3 - 3(-1) + 2$

$= -1 + 3 + 2$

$= 4$

$(1,0) \quad (-1,4)$

	→	−1	→	1	→
$f'(x)$	− −	0	− +	0	+ +
$(x-1)$	+		−		+
$(x+1)$	/	—	\	—	/

max at $\underline{\underline{(-1,4)}}$ $\quad$ min at $\underline{\underline{(1,0)}}$

b) $$\begin{array}{c|cccc} 1 & 1 & 0 & -3 & 2 \\ & & 1 & 1 & -2 \\ \hline & 1 & 1 & -2 & \boxed{0} \end{array}$$ $\therefore$ factor

$(x-1)(x^2+x-2) = 0$

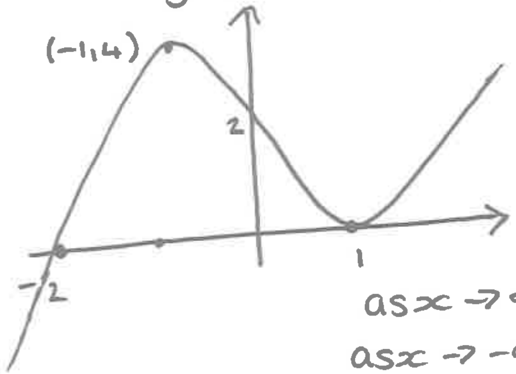
$(x-1)(x+2)(x-1) = 0$

$\underline{x=1}$ $\underline{x=2}$

$(1,0)$ $(2,0)$

on y-axis, $x=0$ $(0,2)$

$y=2$



as $x \to \infty$, $y \to \infty$

as $x \to -\infty$, $y \to -\infty$.

22. $\frac{dy}{dx} = 3x^2 - 12x + 8 = -1$

$3x^2 - 12x + 9 = 0$

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$x=3$ $x=1$

$y = 1^3 + 6(1^2) + 8$

$= 3$ $(1,3)$

$y = 3^3 - 6(3^2) + 8(3)$

$= 27 - 54 + 24$

$= -3$ $(3,-3)$

b) $$\begin{array}{c|cccc} 1 & 1 & -6 & 9 & -4 \\ & & 1 & -5 & 4 \\ \hline & 1 & -5 & 4 & \boxed{0} \end{array}$$ $\therefore$ factor

$(x-1)(x^2-5x+4) = 0$

$(x-1)(x-4)(x-1) = 0$

tangent at $x=1$

$\underline{\underline{A(1,3)}}$

23a) $h(f(x)) = \log_2(x^2-x+10)$

$h(g(x)) = \log_2(5-x)$

$h(f(x)) - h(g(x)) = 3$

$\log_2(x^2-x+10) - \log_2(5-x) = 3$

$\log_2\left(\frac{x^2-x+10}{5-x}\right) = 3$

$\frac{x^2-x+10}{5-x} = 2^3$

$x^2 - x + 10 = 8(5-x)$

$x^2 - x + 10 = 40 - 8x$

$x^2 + 7x - 30 = 0$

$(x+10)(x-3) = 0$

$\underline{\underline{x=-10}}$ $\underline{\underline{x=3}}$

2008 Paper 2

1a) $m_{BC} = \dfrac{-5+1}{5+3} = \dfrac{-4}{8} = -\dfrac{1}{2}$

If ⊥, $m_1 m_2 = -1$

$-\frac{1}{2} \times 2 = -1$ $\quad M_{BC}(1,-3)$

$y + 3 = 2(x-1)$

$y + 3 = 2x - 2$

$\underline{\underline{y = 2x - 5}}$

b) $M_{AB}(2,4)$

$m_{cm} = \dfrac{4+5}{2-5} = \dfrac{9}{-3} = \underline{\underline{-3}}$

$y - 4 = -3x + 6$

$\underline{\underline{y = -3x + 10}}$

c) $2x - 5 = -3x + 10$

$5x - 5 = 10$

$5x = 15$

$\underline{\underline{x = 3}}$

$y = 2(3) - 5$

$= 6 - 5$

$\underline{\underline{= 1}}$

Intersect at $(3,1)$

2a) $P(8,0,4)$

$Q(0,4,3)$

$\vec{PQ} = \underline{q} - \underline{p} = \begin{pmatrix} -8 \\ 4 \\ -1 \end{pmatrix}$

$|\vec{PQ}| = \sqrt{64+16+1}$

$= \sqrt{81} = 9$

$\vec{PA} = \underline{a} - \underline{p} = \begin{pmatrix} 0 \\ 0 \\ -4 \end{pmatrix}$

$|\vec{PA}| = \sqrt{16}$

$= 4$.

$\cos\theta = \dfrac{(0) + (0) + (4)}{4 \times 9}$

$= \dfrac{4}{36}$

$\underline{\underline{\theta = 83.6°}}$

3a $f(x) = \sqrt{7}\cos x \quad g(x) = -3\sin x$

$k\cos(x+a)$

$= k\cos x\cos a - k\sin x\sin a$

$k\sin a = 3$

$k\cos a = \sqrt{7}$

$k = \sqrt{(\sqrt{7})^2 + 3^2}$

$= \sqrt{16}$

$= 4$

$\tan a = \dfrac{3}{\sqrt{7}}$

S A

T C

$a = 0.85$ radians

$f(x) + g(x) = 4\cos\underline{\underline{(x + 0.85)}}$

c) $-4\sin\underline{\underline{(x + 0.85)}}$

4a) $C(-4,-2) \quad r = \sqrt{4^2 + 2^2 + 38}$

$= \underline{\underline{\sqrt{58}}}$

b) $C(4,6) \quad r = \sqrt{26}$

$d_{c_1c_2} = \sqrt{(-4-4)^2 + (-2-6)^2}$

$= \sqrt{64+64}$

$= \underline{\underline{\sqrt{128}}}$

$\sqrt{26} + \sqrt{58} = 12.7$

$\sqrt{128} = 11.3$

Since $d_{c_1c_2}$ is less than the sum of the radii, the circles must intersect.

c) $y = 4 - x$

$x^2 + (4-x)^2 + 8x + 4(4-x) - 38 = 0$

$x^2 + 16 - 8x + x^2 + 8x + 16 - 4x - 38 = 0$

$2x^2 - 4x - 6 = 0$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$\underline{x = 3} \quad \underline{x = -1}$

$(3,1)$

$\underline{\underline{(-1,5)}}$

5. $\cos 2x + 2\sin x = \sin^2 x$

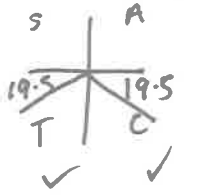
$1 - 2\sin^2 x + 2\sin x = \sin^2 x$

$3\sin^2 x - 2\sin x - 1 = 0$

$(3\sin x + 1)(\sin x - 1) = 0$

$\sin x = -1/3$ $\qquad$ $\sin x = 1$

$\underline{\underline{x = 90^\circ}}$



$\underline{\underline{x = 199.5^\circ, 340.5^\circ}}$

6.

$SF = \dfrac{3-t}{3}$

$PQR = 6\left(\dfrac{3-t}{3}\right)$

$QR = 2(3-t)$

$\underline{\underline{= 6-2t}}$

$Q(t, 6-2t)$

$A(t) = l \times b$

$A(t) = t(6-2t)$

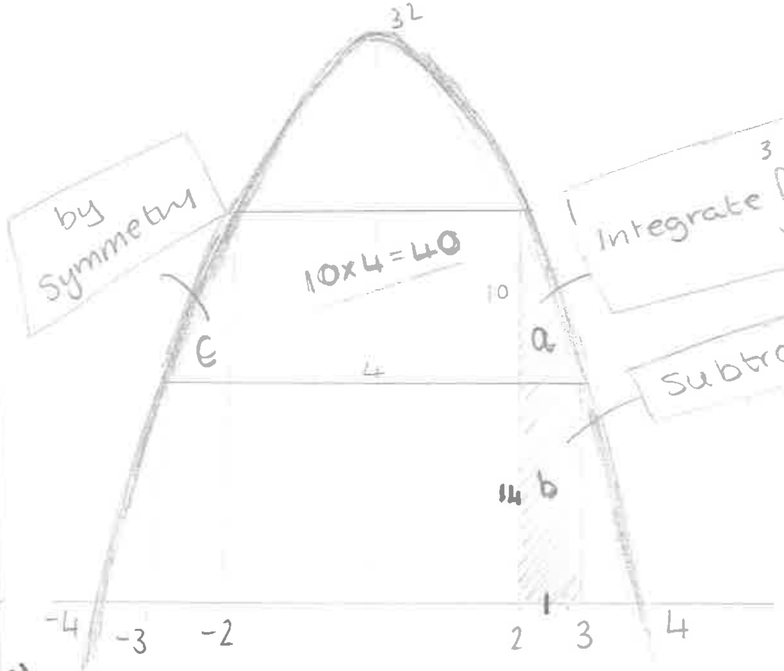
$= 6t - 2t^2$

$A'(t) = 6 - 4t = 0$ at max/min

$4t = 6$

$t = 3/2$

$\underline{\underline{Q(3/2, 3)}}$



Integrate $\int_2^3 32 - 2x^2$

Subtract A rectangle from (a)

roots: $32 - 2x^2 = 0$

$2(4-x)(4+x) = 0$

$\underline{x = 4}$ $\quad$ $\underline{x = -4}$

$32 - 2x^2 = 14$

$18 - 2x^2 = 0$

$2(3-x)(3+x) = 0$

$\underline{x = 3}$ $\quad$ $\underline{x = -3}$

$32 - 2x^2 = 24$

$8 - 2x^2 = 0$

$2(2-x)(2+x) = 0$

$\underline{x = 2}$ $\quad$ $\underline{x = -2}$

a)

$$\int_2^3 32 - 2x^2 \, dx$$

$$\left[32x - \frac{2}{3}x^3\right]_2^3$$

$$= \left[96 - \frac{2}{3}(27)\right] - \left[64 - \frac{2}{3}(8)\right]$$

$$= 96 - 18 - 64 + \frac{16}{3}$$

$$= 14 + \frac{16}{3}$$

$$= \underline{\underline{19\tfrac{1}{3}}}$$

b) $19\frac{1}{3} - 14 = 5\frac{1}{3}$.

Shaded area $= 40 + 10\frac{2}{3}$

$= \underline{\underline{50\frac{2}{3} \text{ units}^2}}$