2010 Paper 1

1. $3y = 2x - 6$ If ⊥, $m_1 m_2 = -1$
$y = \frac{2}{3}x - 2$ $m = -3/2$ (A)

2 $u_1 = 2(3) + 3 = 5$ (C)
$u_2 = 2(5) + 3 = 13$

3. $3\begin{pmatrix}2\\0\\1\end{pmatrix} - 2\begin{pmatrix}-1\\2\\4\end{pmatrix} = \begin{pmatrix}8\\-4\\-5\end{pmatrix}$ (D)

4. (A)

5. $(x+4)^2 - 13$ (B)

6. $b^2 - 4ac = 0$ ∵ equal roots
$(-3)^2 - 4(k)(2) = 0$
$9 - 8k = 0$
$-8k = -9$
$k = 9/8$ (D)

7. $L = \frac{1}{4}L + 7$
$\frac{3}{4}L = 7$
$3L = 28$
$L = \frac{28}{3}$ (C)

8. $(3, 5)$ (B)

9. $\int 2x^{-4} + \cos 5x \, dx$
$\frac{2x^{-3}}{-3} + \frac{1}{5}\sin 5x + c$ (C)
=

10. If ⊥, $\underline{a} \cdot \underline{b} = 0$
$-3x + 10 - 7 = 0$
$-3x + 3 = 0$
$-x = -1$
$x = 1$ (B)

11. $g(\pi/6) = \frac{2\pi}{6} = \pi/3$
$f(g(\pi/6)) = \cos \pi/3$
$= 1/2$ (D)

12. $f(x) = \frac{1}{x^{1/5}} = x^{-1/5}$
$f'(x) = -\frac{1}{5}x^{-6/5}$ (A)
=

13. If $a > 0$ then ∨
$b^2 - 4ac > 0$ then distinct roots (B)

14. $\int_{-2}^{2} (14 - x^2) - (2x^2 + 2) \, dx$
$\int_{-2}^{2} (12 - 3x^2) \, dx$ (C)

15. $f'(1) = 1^2 - 9 = -8$

$\therefore$ decreasing at $x = 1$

$f'(-3) = (-3)^2 - 9 = 0$

Stationary at $x = -3$

C

16. $y = k(x-1)^2(x-5)$

at $(0,10)$

$10 = k(-1)^2(-5)$

$10 = k(-5)$

$\underline{k = -2}$

A

17. $s'(t) = 2t - 5$

$s'(3) = 2(3) - 5$

$= 6 - 5$

$\underline{= 1}$

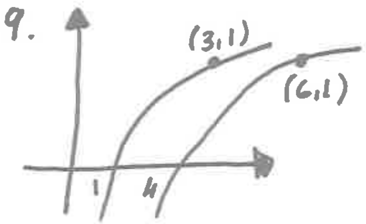
18. $x^2 + 4x > 0$

$x(x+4) > 0$

$\underline{x < -4}$ $\underline{x > 0}$

B

19.



C

20. ~~[illegible]~~ squash x coordinates

-3 from y-coords $(12,7)$

A

21. $M(11,10)$ $m = \frac{10-16}{11-(-4)} = \underline{\frac{-2}{5}}$

$y - 10 = -\frac{2}{5}(x - 11)$

$5y - 50 = -2x + 22$

$5y = -2x + 72$

$y = -\frac{2}{5}x + \frac{72}{5}$

b) at $x = 6$ $5y + (12) = 72$

$5y = 60$

$\underline{y = 12}$

$(6,12)$ lies $\underline{\text{on line}}$

c) $\vec{BT} = \underset{\sim}{t} - \underset{\sim}{b} = \begin{pmatrix}10\\-4\end{pmatrix} = 2\begin{pmatrix}5\\-2\end{pmatrix}$

$\vec{TQ} = \underset{\sim}{q} - \underset{\sim}{t} = \begin{pmatrix}5\\-2\end{pmatrix}$

$\vec{BT} = 2TQ$ $\therefore$ $\underline{2:1}$

2a)

$$\begin{array}{r|rrrr} 1 & 2 & 1 & -8 & 5 \\ & & 2 & 3 & -5 \\ \hline & 2 & 3 & -5 & \boxed{0} \end{array}$$

$\therefore$ factor

$(x-1)(2x^2 - 3x - 5)$

$(x-1)(2x+5)(x-1) = 0$

$\underline{x = 1}$ $2x = -5$ $\underline{x = 1}$

$\underline{x = -5/2}$

c) tangent at $x = 1$

(repeated roots)

$y = 2(1) - 3$

$= -1$

G $(1,-1)$

d) $y = 2\left(-\frac{5}{2}\right) - 3$

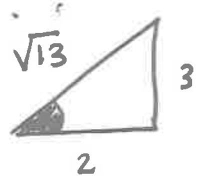
$= -5 - 3$

$= -8$

$H\left(-\frac{5}{2}, -8\right)$

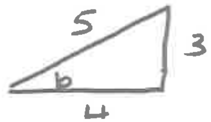
23a) $y = \frac{3}{2}x$

$\tan a = 3/2$



$\sin a = \frac{3}{\sqrt{13}}$

b)



$\sin b = \frac{3}{5}$

$\cos b = \frac{4}{5}$

c) $\sin(a-b) = \sin a \cos b - \cos a \sin b$

$= \frac{3}{\sqrt{13}} \times \frac{4}{5} - \frac{2}{\sqrt{13}} \times \frac{3}{5}$

$= \frac{12}{5\sqrt{13}} - \frac{6}{5\sqrt{13}}$

$= \underline{\frac{6}{5\sqrt{13}}}$

d $\sin(b-a) = \frac{-6}{5\sqrt{13}}$.

2010 Paper 2

1a) $M(0,1,0)$ $\quad N(4,2,2)$

$\vec{VM} = \underline{m} - \underline{v} = \begin{pmatrix} 0 \\ -1 \\ -3 \end{pmatrix}$ $\quad |\vec{VM}| = \sqrt{\cancel{10}} = \underline{\underline{\sqrt{10}}}$

$\vec{VN} = \underline{n} - \underline{v} = \begin{pmatrix} 4 \\ 0 \\ -1 \end{pmatrix}$ $\quad |\vec{VN}| = \underline{\underline{\sqrt{17}}}$

$\cos\theta = \dfrac{0 - 0 + 3}{\sqrt{10}\sqrt{17}}$

$= \dfrac{3}{\sqrt{10}\sqrt{17}}$

$\underline{\underline{\theta = 76.7^\circ}}$

2a) $12\cos x - 5\sin x = k\cos(x+a)$

$= k\cos x\cos a - k\sin x\sin a$

$k\sin a = 5$ $\quad k = \sqrt{5^2 + 12^2}$

$k\cos a = 12$ $\quad = \underline{\underline{13}}$

$\tan a = 5/12$

S | A
T | C

$a = 22.6^\circ$

$13\cos(x + 22.6)^\circ$

2b) $max = 13$ $\quad min = -13$

$x + 22.6 = 0$ $\quad x + 22.6 = 360$

$\cancel{x = -22.6}$ $\quad x = 337.4^\circ$

min: $180 - 22.6 = \underline{\underline{157.4}}$

3a)

$x^2 + (3-x)^2 + 14x + 4(3-x) - 19 = 0$

$x^2 + 9 - 6x + x^2 + 14x + 12 - 4x - 19 = 0$

$2x^2 + 4x + 2 = 0$

$2(x + 2x + 1) = 0$

$2(x+1)(x+1) = 0$

$x = -1$ $\quad \therefore$ tangent.

$y = 3 - (-1)$ $\quad \underline{\underline{P(-1, 4)}}$

$= 4$

3b) $C(1,6)$ $\quad P(-1,4)$

$r = \sqrt{(6-4)^2 + (1+1)^2}$

$= \sqrt{4+4}$

$= \underline{\underline{\sqrt{8}}}$

$(x-1)^2 + \underline{\underline{(y-6)^2}} = 8$

4. $2\cos 2x - 5\cos x - 4 = 0$

$2(2\cos^2 x - 1) - 5\cos x - 4 = 0$

$4\cos^2 x - 2 - 5\cos x - 4 = 0$

$4\cos^2 x - 5\cos x - 6 = 0$

$(4\cos x + 3)(\cos x - 2) = 0$

$\cos x = -3/4$ $\quad \cos x = 2$

No sol^ns.

$\cos^{-1}(3/4) = 0.72$

S | A
T | C

$x = 2.42, 3.86$

$\underline{\underline{\text{radians}}}$

5a) when $x = 0$ $\quad y = \frac{2}{5}(10 - 0^2)$

$= \underline{\underline{4}}$ $\quad T(0,4)$

$PQ = 10 - x^2 - 4$

$= 6 - x^2$

$A(x) = 2x(6 - x^2) = 12x - 2x^3$

$A'(x) = 12 - 6x^2 = 0$ at max/min

$6x^2 = 12$

$x = \pm\sqrt{2}$

	$\to -\sqrt{2}$	$\to \sqrt{2}$	$\to$
$2-x^2$	$-$ $\quad 0$	$+$ $\quad 0$	$-$

$A(\sqrt{2})$

$= 12\sqrt{2} - 2(\sqrt{2})^3$

$= 12\sqrt{2} - 4\sqrt{2}$

$= \underline{\underline{8\sqrt{2}}}$

6a) $y = (18-9)^{1/2}$

$= 9^{1/2}$

$= \sqrt{9}$

$= 3$ $\quad$ $(9,3)$

$\frac{dy}{dx} = \frac{1}{2}(2x-9)^{-1/2} \cdot 2$

$= (2x-9)^{-1/2}$

$= \frac{1}{\sqrt{2x-9}}$

at $x=9$,

$m = \frac{1}{\sqrt{18-9}}$

$= \frac{1}{\sqrt{9}}$

$= \frac{1}{3}$

$y - 3 = \frac{1}{3}(x-9)$

$3y - 9 = x - 9$

$3y = x$

$y = \frac{1}{3}x$

b) When $y = 0$

$(2x-9)^{1/2} = 0$

$2x - 9 = 0$

$2x = 9$

$x = 9/2$ $\quad$ $A(9/2, 0)$

c) $A_{tri} = \frac{1}{2}(9)(3) = \frac{27}{2}$

$\text{Area}_{\text{curve}} = \int_{9/2}^{9} (2x-9)^{1/2}\,dx$

$= \left[\frac{(2x-9)^{3/2}}{3/2 \times 2}\right]_{9/2}^{9}$

$= \left[\frac{\sqrt{(2x-9)^3}}{3}\right]_{9/2}^{9}$

$= \left[\frac{\sqrt{9}^3}{3}\right] - \left[\frac{0}{3}\right]$

$= 9$

shaded area $= \frac{27}{2} - 9$

$= 13 \cdot 5 - 9$

$= 4 \cdot 5$

7a) $\log_4 x = p$

$x = 4^p$

$\log_{16} x = \log_{16} 4^p$

$= p \log_{16} 4$

$= p\left(\frac{1}{2}\right)$

$= \frac{1}{2}p$

b) $\log_3 x + \log_9 x = 12$

$\therefore$ $p + \frac{1}{2}p = 12$

$\frac{3}{2}p = 12$

$3p = 24$

$p = 8$

$\log_3 x = 8$

$x = 3^8$

$x = 6561$