# Higher 2015 Paper 1

If $\perp$, $\underline{a} \cdot \underline{b} = 0$

$$-24 + 2t + 6 = 0$$
$$2t - 18 = 0$$
$$2t = 18$$
$$t = 9$$

2. $y = 2x^3 + 3$

$$\frac{dy}{dx} = 6x^2$$

at $x = -2$

$$\frac{dy}{dx} = 6(-2)^2 = 24$$

at $x = -2$
$$y = 2(-2)^3 + 3 = -16 + 3 = -13$$

$m = 24$ $(a,b) = (-2,-13)$

$$y + 13 = 24(x+2)$$
$$y + 13 = 24x + 48$$
$$y = 24x + 35$$

3.

| | | | | |
|---|---|---|---|---|
| $-3$ | 1 | $-3$ | $-10$ | 24 |
| | 0 | $-3$ | 18 | $-24$ |
| | 1 | $-6$ | 8 | 0 ∴ factor |

$$(x+3)(x^2 - 6x + 8)$$
$$(x+3)(x-4)(x-2)$$

4) $\frac{4-(-2)}{3} = 3$. $4 \times \frac{\pi}{2} = 2\pi$

up by 1

∴ $p = 3$, $q = 4$, $r = 1$

5. $g(x) = 6 - 2x$

$$y = 6 - 2x$$
$$2x + y = 6$$
$$2x = 6 - y$$
$$x = \frac{6-y}{2}$$

$$\therefore g^{-1}(x) = \frac{6-x}{2}$$

b) $g(g^{-1}(x)) = 6 - 2\left(\frac{6-x}{2}\right)$

$$= 6 - (6 - x)$$
$$= x$$

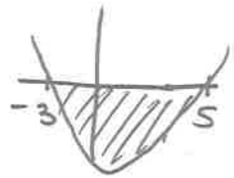
6. $\log_6 12 + \frac{1}{3}\log_6 27$

$$\log_6 12 + \log_6 27^{1/3}$$
$$\log_6 12 + \log_6 3$$
$$= \log_6(36) \qquad 6^2 = 36$$
$$= 2$$

7.
$$f(x) = x^{1/2}(3x - 2x^{-3/2})$$
$$= 3x^{3/2} - 2x^{-1}$$
$$f'(x) = \tfrac{9}{2}x^{1/2} + 2x^{-2}$$
$$= \frac{9}{2}\sqrt{x} + \frac{2}{x^2}$$
$$f'(4) = \frac{9}{2}\sqrt{4} + \frac{2}{4^2}$$
$$= 9 + \frac{2}{16} = 9\tfrac{1}{8}$$

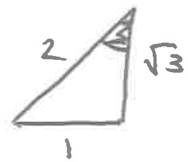
8. $x(x-2) = 15$

$$\therefore x^2 - 2x - 15 = 0$$
$$(x-5)(x+3) = 0$$
$$x = 5 \qquad x = -3$$




$$-3 < x < 5$$

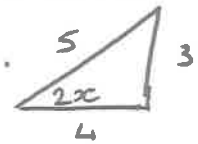
9. $m_{AB} = -\sqrt{3}$




$$\tan 150 = -\tan 30^\circ$$
$$= -\frac{1}{\sqrt{3}}$$

Since $m_{AB} \neq m_{BC}$, points are not collinear

10.




$$\cos 2x = \frac{4}{5}$$

$$2\cos^2 x - 1 = \frac{4}{5}$$
$$2\cos^2 x = \frac{9}{5}$$
$$\cos^2 x = \frac{9}{10}$$
$$\cos x = \frac{3}{\sqrt{10}}$$

11. Centre $(-8,-2)$ T $(-2,-5)$

$$m_{CT} = \frac{-2+5}{-8+2} = \frac{3}{-6} = \frac{-1}{2}$$

If $\perp$, $m_1 m_2 = -1$

$\therefore m = 2$

$(a,b) = (-2,-5)$

$$y + 5 = 2(x-2)$$
$$y + 5 = 2x - 4$$
$$y = 2x - 9$$

b) $2x-1=-2x^2+px+1-p$

$2x^2+(2-p)x+(p-2)=0$

since tgt, $b^2-4ac=0$

$(2-p)^2-4(2)(p-2)=0$

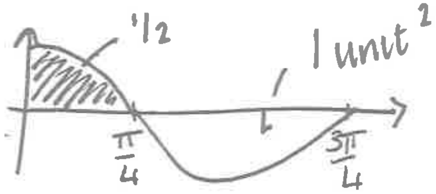
$4-4p-p^2-8p+16=0$

$20-12p-p^2=0$

$(2-p)(10-p)=0$

$\cancel{p=2}$ $\quad p=10$

since $p>3$, $\underline{\underline{p=10}}$

12. 

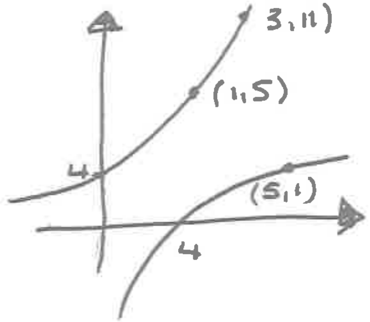

$$\int_0^{\frac{3\pi}{4}} a\cos bx\,dx$$

$$=\int_{\pi/4}^{3\pi/4} a\cos bx\,dx+\int_0^{\pi/4} a\cos bx\,dx$$

13. $f(x)=2^x+3$

$b=2^1+3$

$b=5$




for $4-f(x+1)$:

- reflect in $x$-axis (change $y$ ±)
- slide left by 1 (subtract 1)
- slide up 4

$\therefore R\ (3,11)$

$(3,-11)$

$(2,-11)$

$\underline{\underline{(2,-7)}}$

14. $x^2+y^2-12x-10y+k=0$

at $(0,5)$

$0+25-12(0)-10(5)+k=0$

$25-50+k=0$

$\underline{\underline{k=25}}$

15. $\dfrac{dT}{dt}=\dfrac{1}{25}t-k$

$T=\int \frac{1}{25}t-k\ dt$

$=\frac{1}{50}t^2-kt+C$

at $t=0$, $C=100$

$t_0=100$

$t_{10}=82$

at $t=10$

$\frac{1}{50}(100)-k(10)+100=82$

$2-10k+100=82$

$-10k+102=82$

$-10k=-20$

$\underline{k=2}$

$\therefore \underline{\underline{T=\frac{1}{50}t^2-2t+100}}$

# 2015 Paper 2

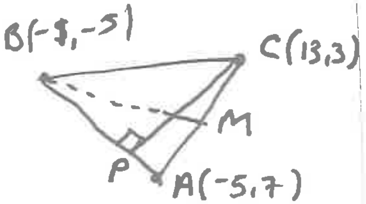
1. $m_{AB} = \frac{7+5}{-5+1}$

$= \frac{12}{-4}$

$= -3$

If ⊥, $m_1 m_2 = -1$

$-3 \times \frac{1}{3} = -1$




CP: $y-3 = \frac{1}{3}(x-13)$

$3(y-3) = 1(x-13)$

$3y-9 = x-13$

$3y = x-4$

$y = \frac{x}{3} - \frac{4}{3}$

b) $M_{AC}$ $(4, 5)$

$m_{BM} = \frac{-5-5}{-1-4}$

$= \frac{-10}{-5}$

$= 2$

$y-5 = 2(x-4)$

$y-5 = 2x-8$

$y = 2x-3$

c) $y = 2x-3$ $\quad y = \frac{x}{3} - \frac{4}{3}$

let $2x-3 = \frac{x}{3} - \frac{4}{3}$

$6x - 9 = x - 4$

$6x = x + 5$

$5x = 5$

$x = 1$

$y = 2(1) - 3$

$= 2-3$

$= -1$

pt of contact $(1,-1)$

2. $f(x) = 10+x$ $\quad g(x) = (1+x)(3-x)+2$

$f(g(x)) = 10 + (1+x)(3-x)+2$

$= 10 + 3 - x + 3x - x^2 + 2$

$= 15 + 2x - x^2$

b) $a(x+p)^2+q$

$ax^2 + 2apx + ap^2 + q = 15 + 2x - x^2$

$a = -1$, $\quad 2ap = 2$ $\quad ap^2+q = 15$

$2(-1)p = 2$ $\quad (-1)(-1^2)+q = 15$

$-2p = 2$ $\quad -1+q = 15$

$p = -1$ $\quad q = 16$

$f(g(x)) = -1(x-1)^2+16$

c) $f(g(x)) \neq 0$

$15 + 2x - x^2 \neq 0$

$(5-x)(3+x) \neq 0$

$x \neq 5$ $\quad x \neq -3$

3a) $t_2 = \frac{3}{4}(13)+13$

$= \frac{39}{4} + \frac{52}{4}$

$= \frac{91}{4}$ m.

b) do they reach 50m? Investigate limits.

$L_F = \frac{1}{3}L_F + 32$

$\frac{2}{3}L_F = 32$

$2L_F = 96$

$L_F = 48$

Frog only reaches 48m ∴ will not escape.

$L_T = \frac{3}{4}L_T + 13$

$\frac{1}{4}L_T = 13$

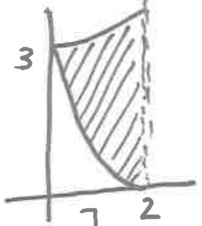
$L_T = 52$

Toad will escape as $52 > 50$m

4. $f(x) = g(x)$

$\frac{1}{4}x^2 - \frac{1}{2}x + 3 = \frac{1}{4}x^2 - \frac{3}{2}x + 5$

$x = 2$




$f(x) - h(x)$

$= \frac{1}{4}x^2 - \frac{1}{2}x + 3 - \left[\frac{3}{8}x^2 - \frac{9}{4}x + 3\right]$

$= -\frac{1}{8}x^2 + \frac{7}{4}x$

$\int_0^2 -\frac{1}{8}x^2 + \frac{7}{4}x \, dx$

$\left[-\frac{1}{24}x^3 + \frac{7}{8}x^2\right]_0^2$

$= \left[\frac{-8}{24} + \frac{28}{8}\right] - [0]$

$= \frac{-8}{24} + \frac{84}{24} = \frac{76}{24} = \frac{19}{6}$

Total area $= \frac{19}{6} \times 2$

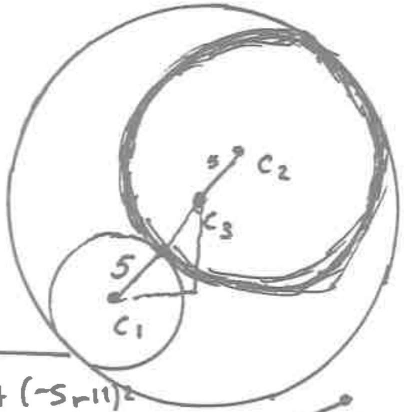
$= \frac{19}{3}$ units$^2$

5. $C_1\ (-3,-5)$

$\underline{r_1 = 5}$

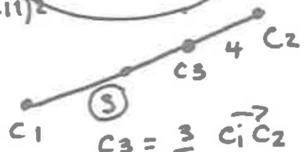
$C_2\ (9,11)$

$r_2 = 15$




$d_{C_1C_2} = \sqrt{(-3-9)^2 + (-5-11)^2}$

$\underline{= 20}$




$c_3 = \frac{3}{4}\vec{c_1c_2}$

$C_3 = (6,7)$

$(x-6)^2 + (y-7)^2 = 400.$

6. $\underline{p}\cdot(\underline{q}+\underline{r})$

$= \underline{p}\cdot\underline{q} + \underline{p}\cdot\underline{r}$

$= |p||q|\cos 60 + |p||r|\cos 90$

$= \frac{9}{2} + 0$

$= \underline{9/2}$

b) $\vec{EC} = \vec{ED} + \vec{DC}$

$= \underline{r} + (-\underline{q} + \underline{p})$

$= \underline{\underline{r} - \underline{q} + \underline{p}}$

c) $-\underline{q}\cdot\underline{q} + \underline{q}\cdot\underline{p} + \underline{q}\cdot\underline{r} = 9\sqrt{3} - \frac{9}{2}$

$(-3)(3)\cos 0 + 3(3\cos 60) + 3|r|\cos 30$

$= 9\sqrt{3} - \frac{9}{2}$

$\Rightarrow \frac{9}{2} - 9 + \frac{3\sqrt{3}}{2}|r| = 9\sqrt{3} - \frac{9}{2}$

$9 - 18 + 3\sqrt{3}|r| = 18\sqrt{3} - 9$

$3\sqrt{3}|r| = 18\sqrt{3}$

$|r| = \frac{18\sqrt{3}}{3\sqrt{3}}$

$\underline{|r| = 6}$

7 a) $\int 3\cos 2x + 1\ dx$

$= \underline{\frac{3}{2}\sin 2x + x + C}$

b) $3\cos 2x + 1 = 3(\cos^2 x - \sin^2 x) + 1$

$= 3\cos^2 x - 3\sin^2 x + (\sin^2 x + \cos^2 x)$

$= \underline{4\cos^2 x - 2\sin^2 x}$

c) $-\frac{1}{2}\int 4\cos^2 x - 2\sin^2 x\ dx$

$= -\frac{1}{2}\int 3\cos 2x + 1\ dx$

$= -\frac{1}{2}\left[\frac{3}{2}\sin 2x + x\right] + C$

$= -\frac{3}{4}\sin 2x - \frac{x}{2} + C$

8) $T(20) = 5\sqrt{36+20^2} + 0$

$= 5\times\sqrt{436}$

$= \underline{104.4}$

$T(0) = 5\sqrt{36} + 80$

$= 5\times 6 + 80$

$= 30 + 80$

$= \underline{110}$

b) $T(x) = 5(36+x^2)^{1/2} + 80 - 4x$

$T'(x) = 0$ at max/min

$T'(x) = \frac{5}{2}(36+x^2)^{-1/2} \times 2x - 4$

$= \frac{10x}{2}(36+x^2)^{-1/2} - 4$

$= \frac{5x}{\sqrt{(36+x^2)}} - 4 = 0$

$\frac{5x}{\sqrt{36+x^2}} = 4$

$\frac{25x^2}{36+x^2} = 16$

$25x^2 = 16(36+x^2)$

$25x^2 = 576 + 16x^2$

$9x^2 = 576$

$x^2 = 64$

$\underline{x = \pm 8}$

$T(8) = 5\sqrt{36+64} + 4(12)$

$= 5(10) + 48$

$= \underline{98}$

9. $h = 36\sin(1{\cdot}5t) - 15\cos(1{\cdot}5t) + 65$

$= k\sin 1{\cdot}5t\cos a - k\cos 1{\cdot}5t\sin a$

$=$

$-k\sin a = -15$

$k\cos a = 36$

$\tan a = \frac{15}{36} = \frac{5}{12}$

$k = \sqrt{36^2 + (-15)^2}$

$= \underline{\underline{39}}$

$a = 0{\cdot}394\ldots$

$a = 0{\cdot}4$ radians.

$\therefore\ 39\sin(1{\cdot}5t - 0{\cdot}4) + 65.$

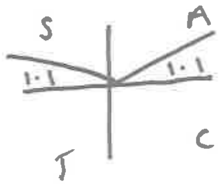
b) $39\sin(1{\cdot}5t - 0{\cdot}4) + 65 = 100$

$39\sin(1{\cdot}5t - 0{\cdot}4) = 35$

$\sin(1{\cdot}5t - 0{\cdot}4) = \frac{35}{39}$

$1{\cdot}5t - 0{\cdot}4 = 1{\cdot}113\ldots$

$= 1{\cdot}11,\ 2{\cdot}04$




$1{\cdot}5t = 1{\cdot}51,\ 2{\cdot}44$

$t = 1{\cdot}006,\ \cancel{1{\cdot}627}$

$\left(\text{since } \frac{\pi}{2} = 1{\cdot}57\right)$