Higher 2016 Paper 1

1. $/\!/$ to $y + 4x = 7$

$\therefore y = -4x + 7$

$\therefore m = -4, \quad (a,b) = (-2,3)$

$y - b = m(x - a)$

$y - 3 = -4(x - (-2))$

$y - 3 = -4x - 8$

$\underline{\underline{y = -4x - 5}}$

2. $y = 2x^3 + 8\sqrt{x}$

$y = 2x^3 + 8x^{1/2}$

$\frac{dy}{dx} = 6x^2 + 4x^{-1/2}$

$\underline{\underline{= 6x^2 + \frac{4}{\sqrt{x}}}}$

3a) $u_{n+1} = \frac{1}{3}u_n + 10 \qquad u_3 = 6$

$\therefore u_4 = \frac{1}{3}(6) + 10$

$= 2 + 10$

$= \underline{\underline{12}}.$

b) For $u_{n+1} = au_n + b$, a limit exists if $-1 < a < 1$.

Since $-1 < \frac{1}{3} < 1$, the limit exists.

c) $L = \frac{1}{3}L + 10$

$\frac{2}{3}L = 10$

$2L = 30$

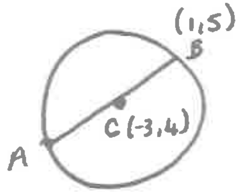
$L = \frac{30}{2} = \underline{\underline{15}}$

4. Centre = Midpoint AB:

$M_{AB} = \left(\frac{-7+1}{2}, \frac{3+5}{2}\right)$

$= \left(\frac{-6}{2}, \frac{8}{2}\right)$

$= \underline{(-3, 4)}$



Radius $= d_{BC} = \sqrt{(x_B - x_C)^2 + (y_B - y_C)^2}$

$= \sqrt{(1+3)^2 + (5-4)^2}$

$= \sqrt{4^2 + 1^2}$

$= \sqrt{17}.$

$\therefore$ eq^n circle: $(x+3)^2 + (y-4)^2 = 17.$

5. $\int 8\cos(4x+1)\,dx$

$= \frac{8}{4}\sin(4x+1) + C$

$\underline{\underline{= 2\sin(4x+1) + C}}$

6. a) $f(x) = 3x + 5$

$y = 3x + 5$

$y - 5 = 3x$

$\frac{y-5}{3} = x \qquad \therefore f^{-1}(x) = \underline{\frac{x-5}{3}}.$

b) Since $g(2) = 7$

$\underline{g^{-1}(7) = 2}$

7. $\vec{FH} = \vec{FG} + \vec{GH}$

$= \begin{pmatrix} -2 \\ -6 \\ 3 \end{pmatrix} + \begin{pmatrix} 3 \\ 9 \\ -7 \end{pmatrix}$

$= \begin{pmatrix} 1 \\ 3 \\ -4 \end{pmatrix}$

$\therefore \vec{FH} = \underline{i} + 3\underline{j} - 4\underline{k}$

b) $\vec{FE} = \vec{FH} + (-\vec{EH})$

$= \begin{pmatrix} 1 \\ 3 \\ -4 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} -1 \\ 0 \\ -5 \end{pmatrix}$

$\therefore \vec{FE} = -\underline{i} - 5\underline{k}$

2016 P1 Ctd.

8. $x^2+y^2+2x-4y-5=0$

when $y=3x-5$

$x^2+(3x-5)^2+2x-4(3x-5)-5=0$

$\therefore x^2+9x^2-30x+25+2x-12x+20-5=0$

$\therefore 10x^2-40x+40=0$

$10(x^2-4x+4)=0$

b^2-4ac

$(-4)^2-4(1)(4)$

$=16-16$

$=0$

$b^2-4ac=0$ ∴ repeated root $\Rightarrow$ tangent.

$x^2-4x+4=0$

at $(x-2)(x-2)=0$

$x=2$ $\quad y=3x-5$

$y=3(2)-5$

$y=1$

∴ tangent at $(2,1)$

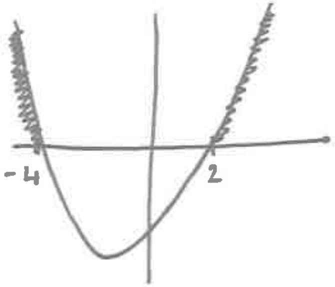
9. $f(x)=x^3+3x^2-24x$

$f'(x)=3x^2+6x-24=0$ at max/min

$3(x^2+2x-8)=0$

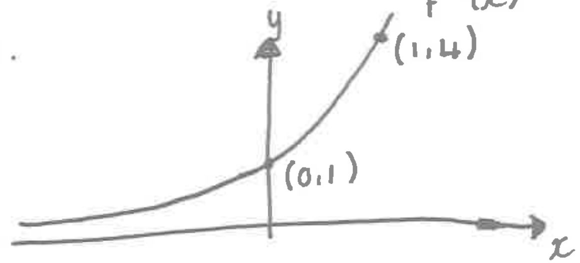
$3(x+4)(x-2)=0$

$x=-4$ or $x=2$



Increasing for: $x<-4$ and $x>2$

10.



11. $\dfrac{\overrightarrow{AB}}{\overrightarrow{BC}}=\dfrac{1}{2}$

$\therefore 2\overrightarrow{AB}=\overrightarrow{BC}$

$2(\underline{b}-\underline{a})=\underline{c}-\underline{b}$

$2\underline{b}-2\underline{a}=\underline{c}-\underline{b}$

$3\underline{b}=\underline{c}+2\underline{a}$

$3\underline{b}=\begin{pmatrix}4\\-3\\4\end{pmatrix}+2\begin{pmatrix}1\\3\\-2\end{pmatrix}$

11 ctd.

$3\underline{b}=\begin{pmatrix}4\\-3\\4\end{pmatrix}+\begin{pmatrix}2\\6\\-4\end{pmatrix}$

$3\underline{b}=\begin{pmatrix}6\\3\\0\end{pmatrix}$

$\therefore \underline{b}=\begin{pmatrix}2\\1\\0\end{pmatrix}$

$B(2,1,0)$

b. $\overrightarrow{AC}=\underline{c}-\underline{a}$

$=\begin{pmatrix}3\\-6\\6\end{pmatrix}$

$|\overrightarrow{AC}|=\sqrt{3^2+(-6)^2+6^2}$

$=\sqrt{9+36+36}$

$=\sqrt{81}$

$=9$

$|k||\overrightarrow{AC}|=1$ $\qquad \frac{1}{9}\times 9=1$

$\therefore k=\frac{1}{9}$

2016 P1 Ctd

12. $f(x) = 2x^2 - 4x + 5$

$g(x) = 3 - x.$

$h(x) = f(g(x))$

$f(x) = 2x^2 - 4x + 5$

$f(3-x) = 2(3-x)^2 - 4(3-x) + 5$

$= 2(9 - 6x + x^2) - 12 + 4x + 5$

$= 18 - 12x + 2x^2 - 12 + 4x + 5$

$= 11 - 8x + 2x^2$

b. $p(x+q)^2 + r$

$p(x^2 + 2qx + q^2) + r$

$px^2 + 2pqx + pq^2 + r.$

$+2x^2 \quad - 8x \quad + 11$

$p = 2$

$q = -2$

$2pq = -8$

$2(2)q = -8$

$q = -2.$

$pq^2 + r = 11$

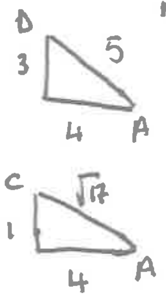
$2(-2^2) + r = 11$

$8 + r = 11$

$r = 3$

$h(x) = 2(x-2)^2 + 19$

13. $\cos(q-p) = \cos q \cos p + \sin q \sin p$



$= \left(\frac{4}{5}\right)\left(\frac{4}{\sqrt{17}}\right) + \left(\frac{3}{5}\right)\left(\frac{1}{\sqrt{17}}\right)$

$= \frac{16}{5\sqrt{17}} + \frac{3}{5\sqrt{17}}$

$= \frac{19}{5\sqrt{17}}$

$= \frac{19}{5\sqrt{17}} \times \frac{\sqrt{17}}{\sqrt{17}}$

$= \frac{19\sqrt{17}}{5 \times 17}$

$= \frac{19\sqrt{17}}{85}$

14a $\log_5 25 = 2 \qquad (5^2 = 25)$

b. $\log_4 x + \log_4 (x-6) = \log_5 25$

$\log_4 x(x-6) = 2$

$x(x-6) = 4^2$

$x^2 - 6x - 16 = 0$

$(x-8)(x+2) = 0$

$x = 8$ ✓ $\quad \cancel{x = -2}$ ✗

15. $f(x) = k(x-a)(x-b)^2$

$y = k(x-4)(x+5)(x+5)$

at $(1,9)$

$9 = k(-3)(6)(6)$

$9 = k(-108)$

$36 \times 3 = 108$

$k = \frac{-1}{12} \qquad a = 4 \qquad b = -5$

b) $d > 9$

(graph slides down 9 or more leaving only 1 root)

2016 Paper 2

1. $M(2,4)$ $\quad m_{pm} = \frac{(-4-4)}{0-2} = \underline{\underline{4}}$

$y-4 = 4(x-2)$

$y-4 = 4x-8$

$\underline{\underline{y = 4x-4}}$

b) $m_{PR} = \frac{-4-6}{0-10} = 1$

If $\perp$, $m_1 m_2 = -1$ $\quad P(2,4)$

$1 \times (-1) = -1$

$y-4 = -1(x-2)$

$y-4 = -x+2$

$\underline{\underline{y = -x+6}}$

c) $m_{PR} = (5,1)$

at $x=5$, $y = -(5)+6$

$= -5+6$

$y = 1$

$\therefore$ $(5,1)$ satisfies equation $\therefore$ lies on line.

3a

-1	2	-9	3	14
	0	-2	11	-14
	2	-11	14	$\boxed{0}$ $\therefore$ factor

$(x+1)(2x^2-11x+14) = 0$

$(x+1)(2x-7)(x-2) = 0$

$\underline{x=-1}$ $\quad \underline{x = 7/2}$ $\quad \underline{x=2}$

b) $A(-1,0)$ $\quad B(2,0)$ $\quad C(7/2, 0)$

$$\int_{-1}^{2} 2x^3 - 9x^2 + 3x + 14 \, dx$$

$$\left[\frac{x^4}{2} - 3x^3 + \frac{3}{2}x^2 + 14x\right]_{-1}^{2}$$

$$= \left[\frac{16}{2} - 24 + \frac{12}{2} + 28\right] - \left[\frac{1}{2} + 3 + \frac{3}{2} - 14\right]$$

$$= 8 - 24 + 6 + 28 - [-9]$$

$= 18 + 9$

$= \underline{\underline{27 \text{ units}^2}}$

4. $C_A(-5,6)$ $\quad r_A = 3$

$C_B(3,0)$ $\quad r_B = 5$

$r_A + r_B = 8$

$d_{AB} = \sqrt{(-5-3)^2 + (6-0)^2}$

$= \sqrt{(-8)^2 + 6^2}$

$= \underline{\underline{10}}$

Circles do not intersect as $r_A + r_B <$ distance between centres.

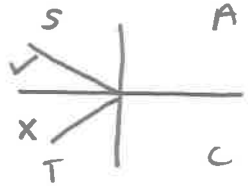
5. $\vec{AB} = \underline{b} - \underline{a} = \begin{pmatrix} -8 \\ 16 \\ 2 \end{pmatrix}$ $\quad |\vec{AB}| = 18$

$\vec{AC} = \underline{c} - \underline{a} = \begin{pmatrix} -2 \\ -8 \\ 16 \end{pmatrix}$ $\quad |\vec{AC}| = 18$

$$\cos\theta = \frac{\vec{AB} \cdot \vec{AC}}{|\vec{AB}||\vec{AC}|}$$

$$= \frac{16 - 128 + 32}{18 \times 18}$$

$$= \frac{-80}{324}$$



$\cos^{-1}(-80 \div 324) = 75.7$

$\theta = 180 - 75.7$

(Rule out T as < 180).

$\underline{\underline{\theta = 104.3^\circ}}$

6a) $B(t) = 200e^{0.107t}$

$B(0) = 200e^0$

$= \underline{\underline{200}}$

b) $400 = 200e^{0.107t}$

$2 = e^{0.107t}$

$\ln 2 = \ln e^{0.107t}$

$\ln 2 = 0.107t$

$t = \frac{\ln 2}{0.107} = 6.478\ldots$

$= \underline{\underline{6.5 \text{ hours}}}$

7a) $L \times b = 108m^2$

$3x(2y) = 108m^2$

$6xy = 108$

$y = \dfrac{108}{6x}$

$y = \dfrac{18}{x}$

$L(x) = 9x + 8.y$

$= 9x + 8\left(\dfrac{18}{x}\right)$

$= 9x + \dfrac{144}{x}$

$L(x) = 9x + 144x^{-1}$

$L'(x) = 9 - 144x^{-2} = 0$ at max/min

$9 - \dfrac{144}{x^2} = 0$

$9 = \dfrac{144}{x^2}$

$9x^2 = 144$

$9x^2 - 144 = 0$

$9(x^2 - 16) = 0$

$9(x-4)(x+4) = 0$

$x = 4 \quad \cancel{x = -4}$

$L'(x)$	-7	4	-7
$9(x-4)(x+4)$	$-$	0	$+$
	\	—	/

min at $x = 4$

8) $5\cos x - 2\sin x$

$= k\cos x\cos a - k\sin x\sin a$

$\Rightarrow -k\sin a = -2$ $\quad k = \sqrt{5^2 + 2^2}$

$k\cos a = 5$ $\quad = \sqrt{29}$

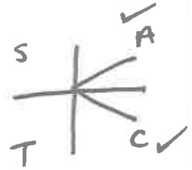
$\tan a = 2/5$

$a = 0.38$ radians

$\therefore 5\cos x - 2\sin x = \sqrt{29}\cos(x + 0.38)$

b) $\sqrt{29}\cos(x + 0.38) = 12 - 10$

$\cos(x + 0.38) = \dfrac{2}{\sqrt{29}}$



$x + 0.38 = 1.19,\ 5.09$

$x = 0.81,\ 4.71$ radians

9. $\displaystyle\int \frac{2x}{x^{1/2}} + \frac{1}{x^{1/2}}\, dx$

$= \displaystyle\int 2x^{1/2} + x^{-1/2}\, dx$

$= \dfrac{2x^{3/2}}{3/2} + \dfrac{x^{1/2}}{1/2} + c$

$= \dfrac{4}{3}\sqrt{x}^3 + 2\sqrt{x} + c$

at $x = 9$

$\frac{4}{3}(27) + 2(3) + c = 40$

$36 + 6 + c = 40$

$c = -2$

$f(x) = \frac{4}{3}\sqrt{x}^3 + 2\sqrt{x} - 2$

b) $(4x^2 + 7)^{1/2} + C$.

11) $\sin 2x \tan x = 1 - \cos 2x$

$2\sin x\cos x\tan x$

$= 2\sin x\cos x\left(\dfrac{\sin x}{\cos x}\right)$

$= 2\sin^2 x$

$= 1 - \cos 2x$

$f(x) = 1 - \cos 2x$

$f'(x) = 2\sin 2x$

(or) $2(2\sin x\cos x)$

$= 4\sin x\cos x$