2017 Paper 1

1a) $g(0) = 2\cos(0)$
$= 2(1)$
$= \underline{\underline{2}}$

$f(x) = 5x$
$f(g(0)) = 5(2)$
$= \underline{\underline{10}}$

b) $g(x) = 2\cos x$ $\quad f(x) = 5x$
$g(f(x)) = g(5x)$ $\quad g(5x) = \underline{\underline{2\cos 5x}}$

2. $x^2+y^2-8x-6y-15$ $\quad C(4,3)$ $\quad P(-2,1)$

$m = \dfrac{3-1}{4-(-2)}$
$= \dfrac{2}{6}$
$= \underline{\underline{1/3}}$

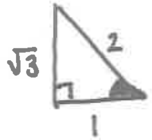
If $m_1 m_2 = -1$
$1/3 \times (-3) = -1$
$m_{tgt} = -3$
$y-1 = -3(x+2)$
$y-1 = -3x-6$
$\underline{\underline{y = -3x-5}}$

3. $y = (4x-1)^{12}$
$\dfrac{dy}{dx} = 12(4x-1)^{11} \times 4$
$= \underline{\underline{48(4x-1)^{11}}}$

4. $b^2-4ac = 0 \because$ equal roots
$4^2 - 4(1)(k-5) = 0$
$16 - 4k + 20 = 0$
$-4k + 36 = 0$
$4k = 36$
$\underline{\underline{k = 9}}$

5. $\underline{u}\cdot\underline{v} = (5\times3)+(1\times-8)+(-1\times6)$
$= 15 + (-8) + (-6)$
$= \underline{\underline{1}}$

b) $\underline{u}\cdot\underline{w} = |\underline{u}||\underline{w}|\cos\theta$

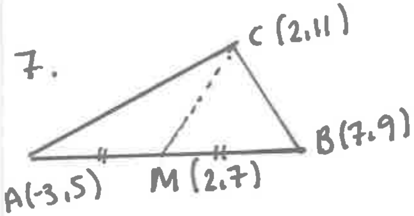


$= (3\sqrt{3})(\sqrt{3})\left(\tfrac{1}{2}\right)$
$= 3(3)(1/2)$
$= \underline{\underline{\dfrac{9}{2}}}$

$|\underline{u}| = \sqrt{5^2+1^2+(-1)^2}$
$= \sqrt{27}$
$= \underline{\underline{3\sqrt{3}}}$

6. $h(x) = x^3+7$
$y = x^3+7$
$y-7 = x^3$
$x = \sqrt[3]{y-7}$ $\quad \therefore \underline{\underline{h^{-1}(x) = \sqrt[3]{x-7}}}$

7.



$m_{cm} = \dfrac{y_c - y_m}{x_c - x_m}$
$= \dfrac{11-7}{2-2}$
$= \dfrac{4}{0}$
$\therefore$ undefined

Vertical line $\underline{\underline{\text{at } x=2}}$.

8. $d(t) = \frac{1}{2}t^{-1}$
$d'(t) = -\frac{1}{2}t^{-2}$
$= \dfrac{-1}{2t^2}$

$d'(5) = \dfrac{-1}{2(5)^2}$
$= \underline{\underline{\dfrac{-1}{50}}}$

9. $u_{n+1} = m u_n + 6$ $\quad u_1 = 28$ $\quad u_2 = 13$
$13 = 28m + 6$
$7 = 28m$
$\dfrac{7}{28} = m$
$\underline{\underline{m = 1/4}}$

since $-1 < 1/4 < 1$ limit exists.
$L = 1/4\,L + 6$
$3/4\,L = 6$
$3L = 24$
$\underline{\underline{L = 8}}$

10. $x^3-4x^2+3x+1-(x^2-3x+1)$
$x^3-4x^2+3x+1-x^2+3x-1$
$= x^3-5x^2+6x$.

$\displaystyle\int_0^2 x^3-5x^2+6x\,dx = \left[\frac{x^4}{4} - \frac{5}{3}x^3 + 3x^2\right]_0^2$

$= \left[\dfrac{2^4}{4} - \dfrac{5}{3}(2^3) + 3(2^2)\right] - [0]$
$= \dfrac{16}{4} - \dfrac{40}{3} + 12$
$= 16 - \dfrac{40}{3}$
$= \dfrac{48}{3} - \dfrac{40}{3}$
$= \underline{\underline{8/3 \text{ units}^2}}$

2017 Paper 1

10b) $x^3 - 4x^2 + 3x + 1 - (1 - x)$

$= \int_0^2 x^3 - 4x^2 + 4x \, dx$

$= \left[\frac{x^4}{4} - \frac{4}{3}x^3 + 2x^2\right]_0^2$

$= \left[\frac{2^2}{4} - \frac{4}{3}(2^3) + 2(2^2)\right] - [0]$

$= \frac{16}{4} - \frac{32}{3} + 8$

$= 12 - \frac{32}{3}$

$= \frac{36}{3} - \frac{32}{3}$

$= 4/3$

$\therefore \frac{1}{2}$ of area in (a)

11. $m_{AB} = \frac{y_A - y_B}{x_A - x_B}$

$= \frac{2 - a}{-7 - 5}$

$= \frac{2 - a}{-12}$

$3y = 2x + 4$

$y = \frac{2}{3}x + \frac{4}{3}$.

Since //, $m = 2/3$

$\therefore \frac{2 - a}{-12} = \frac{2}{3}$

$2 - a = \frac{-24}{3}$

$2 - a = -8$

$-a = -10$

$a = 10$

12. $\log_a 36 - \log_a 4 = \frac{1}{2}$

$\log_a (9) = \frac{1}{2}$

$9 = a^{1/2}$ or $\sqrt{a}$

$a = 81$

13. $\int \frac{1}{(5 - 4x)^{1/2}} \, dx$

$= \int (5 - 4x)^{-1/2} \, dx$

$= \frac{(5 - 4x)^{1/2}}{1/2 \times -4} + C$

$= \frac{(5 - 4x)^{1/2}}{-2} + C$

$= -\frac{1}{2}(5 - 4x)^{1/2} + C$

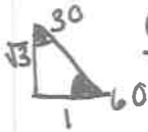
14. $\sqrt{3}\sin x - \cos x = k\sin x\cos a - k\cos x\sin a$

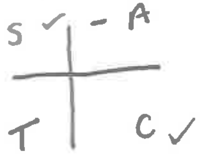
$k\sin a = -1$

$k\cos a = \sqrt{3}$

$\tan a = \frac{-1}{\sqrt{3}}$

$a = -30°$



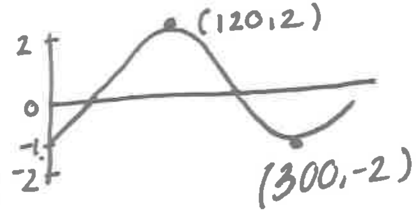


$k = \sqrt{(\sqrt{3})^2 + (1)^2}$

$= \sqrt{4}$

$= 2$

$2\sin(x - 30)°$



When $x = 0$

$y = 2\sin(-30)$

$= -1$

15a) $h(x)$: slide right 5: $f(x - 5)$

slide up 3 $f(x - 5) + 3$

b) $\int_1^3 f(x)\,dx = 4$ Area□ $= 3 \times 2 = 6$

$4 + 6 = 10$

$\int_6^8 h(x)\,dx = 10$

c) $f'(1) = 6$

$h'(8) = -6$

at $x = 1$, $m = 6$

$\therefore$ at $x = 8$, $m = -6$

2017 Paper 2

1) $M_{BC} = (6,-1)$ $\qquad m_{BC} = \frac{0+2}{3-9} = -1/3$

If ⊥, $m_1 m_2 = -1$

$-1/3 \times 3 = -1$

$y+1 = 3(x-6)$

$y+1 = 3x-18$

$\underline{\underline{y = 3x-19}}$

b) $m = \tan 45 = 1$ $\qquad (3,0)$

$y - 0 = 1(x-3)$

$\underline{y = x-3}$

c) $3x-19 = x-3$ $\qquad y = 8-3$

$2x = 16$ $\qquad \underline{y = 5}$

$\underline{x = 8}$ $\qquad \underline{\underline{(8,5)}}$

2a)

$$\begin{array}{r|rrrr} 1 & 2 & -5 & 1 & 2 \\ & 0 & 2 & -3 & -2 \\ \hline & 2 & -3 & -2 & \boxed{0} \end{array}$$ ∴ factor

$(x-1)(2x^2-3x-2) = 0$

$(x-1)(2x+1)(x-2) = 0$

$x = 1 \qquad x = -1/2 \qquad x = 2$

3) $(x-2)^2 + (3x-1)^2 = 25$

$x^2 - 4x + 4 + 9x^2 - 6x + 1 = 25$

$10x^2 - 10x + 5 = 25$

$10x^2 - 10x - 20 = 0$

$(x^2 - x - 2) = 0$

$(x-2)(x+1) = 0$

∴ $x = 2 \qquad x = -1$

$y = 3(2) \qquad y = 3(-1)$

$y = 6 \qquad y = -3$

$\underline{\underline{(2,6)}} \qquad \underline{\underline{(-1,-3)}}$

4a) $a(x^2 + 2bx + b^2) + c$

$= ax^2 + 2abx + ab^2 + c$

$= 3x^2 + 24x + 50$

$\underline{\underline{a=3}}$ $\qquad 2ab = 24$ $\qquad ab^2 + c = 50$

$2(3)b = 24$ $\qquad (3)(16) + c = 50$

$\underline{\underline{b = 4}}$ $\qquad 48 + c = 50$

$\underline{\underline{c = 2}}$

$\underline{\underline{3(x+4)^2 + 2}}$

$f'(x) = 3x^2 + 24x + 50$

$= 3(x+4)^2 + 2$

Since $(x+4)^2 > 0$,

$f'(x) > 0$ ∴ always increasing.

5a)

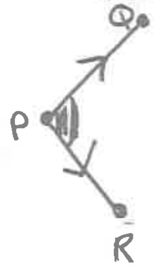
$\vec{PQ} = \vec{PR} + \vec{RQ}$

$= \begin{pmatrix} -3 \\ -4 \\ 5 \end{pmatrix} = -3\underline{i} - 4\underline{j} + 5\underline{k}$

b) $\vec{PS} = PQ + \frac{1}{3} QR$

$= \begin{pmatrix} -3 \\ -4 \\ 5 \end{pmatrix} + \frac{1}{3}\begin{pmatrix} 12 \\ 9 \\ -3 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix}$

$\underline{= 1\underline{i} - \underline{j} + 4\underline{k}}$



$|\vec{PQ}| = \begin{pmatrix} -3 \\ -4 \\ 5 \end{pmatrix}$ $\qquad \vec{PS} = \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix}$

$|\vec{PQ}| = \sqrt{50}$ $\qquad |\vec{PS}| = \sqrt{18}$

$\cos QPS = \dfrac{-3 + 4 + 20}{\sqrt{50}\sqrt{18}}$

$\underline{\underline{QPS = 45.6°}}$

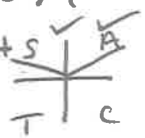
6. $5\sin x - 4 = 2(1 - 2\sin^2 x)$

$5\sin x - 4 = 2 - 4\sin^2 x$

$4\sin^2 x + 5\sin x - 6 = 0$

$(4\sin x - 3)(\sin x + 2) = 0$

$\sin x = 3/4$ $\qquad \sin x = -2$ No solutions as $-1 < \sin x < 1$



$0.84, 2.29$ radians

7a) $y = 6x - 2x^{3/2}$

$\frac{dy}{dx} = 6 - 3x^{1/2} = 0$ at max/min

$3\sqrt{x} = 6$

$\sqrt{x} = 2$

$x = 4$

at $x=4$

$y = 6(4) - 2\sqrt{(4^3)}$

$= 24 - 2\sqrt{4}^3$

$= 24 - 2(8)$

$= 8$ $(4,8)$

at $x=1$

$y = 6(1) - 2\sqrt{1}^3$ $(1,4)$

$= 6 - 2(1)$

$= 4$

at $x=9$ $(9,0)$

$y = 6(9) - 2\sqrt{9}^3$

$= 54 - 54$

$= 0$

max value 9
min value 0.

8) $u_0 = 5$ $u_{n+1} = k u_n - 20$

$u_1 = 5k - 20$

$u_2 = k(5k-20) - 20$

$= 5k^2 - 20k - 20$

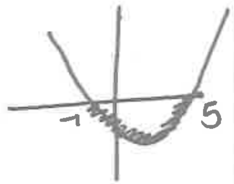
$5k^2 - 20k - 20 < 5$

$5k^2 - 20k - 25 < 0$

$k^2 - 4k - 5 < 0$

$(k-5)(k+1) < 0$

$k=5$ $k=-1$



$-1 \leq x \leq 5$

9) $\log_2 y = \frac{1}{4}\log_2 x + 3\log_2 2$

$\log_2 y = \log_2 x^{1/4} + \log_2 2^3$

$\log_2 y = \log_2 x^{1/4} + \log_2 8$

$\log_2 y = \log_2 8x^{1/4}$

$\therefore y = \log_2 8x^{1/4}$.

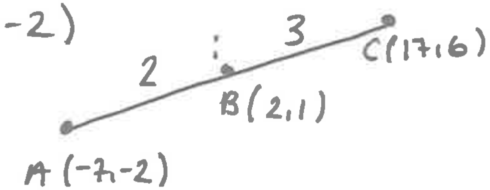
10a) $\vec{AB} = \underline{b} - \underline{a} = \begin{pmatrix}3\\9\end{pmatrix} = 3\begin{pmatrix}1\\3\end{pmatrix}$

$\vec{BC} = \underline{c} - \underline{b} = \begin{pmatrix}5\\15\end{pmatrix} = 5\begin{pmatrix}1\\3\end{pmatrix}$

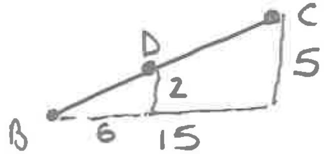
$\vec{AB} \parallel \vec{BC}$ and B's common point

$\therefore$ collinear.

b) $A(-7,-2)$



radius $= 6\sqrt{10}$.



$\frac{2}{5}(\vec{BC})$

$= \frac{2}{5}\begin{pmatrix}15\\5\end{pmatrix}$

$= \begin{pmatrix}6\\2\end{pmatrix}$

$D(8,3)$

$(x-8)^2 + (y-3)^2 = (6\sqrt{10})^2$

$(x-8)^2 + (y-3)^2 = 36\times10$

$(x-8)^2 + (y-3)^2 = 360$.

11a) $\frac{\sin 2x}{2\cos x} - \sin x\cos^2 x$

$= \frac{2\sin x\cos x}{2\cos x} - \sin x\cos^2 x$

$= \sin x - \sin x\cos^2 x$

$= \sin x(1 - \cos^2 x)$

$= \sin^3 x$

b) $y = \sin^3 x$ or $(\sin x)^3$

$\frac{dy}{dx} = 3\sin^2 x \cdot \cos x$

$= 3\sin^2 x\cos x$.