HIGHER PAPER I 2019

1. FOR S.P $\frac{dy}{dx} = 0$

$y = \frac{1}{2}x^4 - 2x^3 + 6$

$\frac{dy}{dx} = 2x^3 - 6x^2$

$2x^3 - 6x^2 = 0$

$2x^2(x - 3) = 0$

$\underline{x = 0} \quad \underline{x = 3}$

2. FOR EQUAL ROOTS

$b^2 - 4ac = 0$

$a = 1 \quad b = (k-5) \quad c = 1$

$b^2 - 4ac = 0$

$(k-5)^2 - 4(1)(1) = 0$

$k^2 - 10k + 25 - 4 = 0$

$k^2 - 10k + 21 = 0$

$(k-3)(k-7) = 0$

$\underline{k = 3} \quad \underline{k = 7}$

3. C_1 CENTRE $(3, 1)$

RADIUS $= \sqrt{9 + 1 + 26}$

$= 6$

C_2 CENTRE $(4, -2)$

RADIUS $= 6$

EQN $\underline{(x-4)^2 + (y+2)^2 = 36}$

4. $9 - 6m + c$

$11 = 9m + c$

$2 = 3m$

$\underline{m = \frac{2}{3}}$

$9 = 6(\frac{2}{3}) + c$

$\underline{c = 5}$

b) $u_4 = \frac{2}{3}u_3 + 5$

$= \frac{2}{3}(11) + 5$

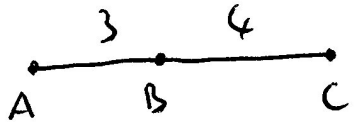
$= \underline{12\frac{1}{3}}$

5. $\vec{AB}\begin{pmatrix}3\\-6\\3\end{pmatrix}$ $\vec{BC}\begin{pmatrix}4\\-8\\-4\end{pmatrix}$

$4\vec{AB} = 3\vec{BC}$

As $\vec{AB}$ is a multiple of $\vec{BC}$ they are parallel and as they share a common point 'B' they are collinear

b)



AB : BC

3 : 4

6. $y = (1-3x)^{-5}$

$\frac{dy}{dx} = -5(1-3x)^{-6} \times (-3)$

$= 15(1-3x)^{-6}$

$= \frac{15}{(1-3x)^6}$

7. TAN 30 = m

$m = \frac{1}{\sqrt{3}}$ $m_1 = -\sqrt{3}$

$y + 4 = -\sqrt{3}(x-0)$

$y + \sqrt{3}x = 4$

8. $\int_{-1}^{2} (x^2+2x+3) - (2x^2+x+1)\,dx$

$= \int_{-1}^{2} -x^2 + x + 2 \; dx$

b) $\left[-\frac{1}{3}x^3 + \frac{1}{2}x^2 + 2x\right]_{-1}^{2}$

$= \left(-\frac{1}{3}(2)^3 + \frac{1}{2}(2)^2 + 2(2)\right) - \left(-\frac{1}{3}(-1)^3 + \frac{1}{2}(-1)^2 + 2(-1)\right)$

$= 10/3 - (-7/6)$

$= \frac{9}{2}$ units^2

9. $u.v = p(2p+16) + ((-2)\times(-3)) + (4\times 6)$

$= 2p^2 + 16p + 6 + 24$

$= 2p^2 + 16p + 30$

b) FOR $\perp$ $u.v = 0$

$2p^2 + 16p + 30 = 0$

$2(p^2 + 8p + 15) = 0$

$2(p+3)(p+5) = 0$

$\underline{p = -3} \quad \underline{p = -5}$

c) FOR PARALLEL $3\underline{u} = 2\underline{v}$

$3p = 2(2p+16)$

$3p = 4p + 32$

$\underline{p = -32}$

10. a) $a = 3$

b) $k = -2$

11. $\int_0^{\pi/4} \cos(3x - \pi/6)\,dx$

$= \left[\tfrac{1}{3}\sin(3x - \pi/6)\right]_0^{\pi/4}$

$= \tfrac{1}{3}\sin(\pi/3 - \pi/6) - \tfrac{1}{3}\sin(-\pi/$

$= \tfrac{1}{3}\sin \pi/6 - \tfrac{1}{3}\sin(-\pi/6)$

$= \tfrac{1}{3} \times \tfrac{1}{2} - \tfrac{1}{3}(-\tfrac{1}{2})$

$= \tfrac{1}{6} - (-\tfrac{1}{6})$

$= \underline{\tfrac{1}{3} \text{ units}^2}$

12. $f(5-x)$

$= \frac{1}{\sqrt{(5-x)}}$

b) UNDEFINED FOR $x \geq 5$

13. a) i) $\cos P = \frac{2}{\sqrt{5}}$

ii) $\cos Q = \frac{3}{\sqrt{10}}$

b) $\sin(P+Q) = \sin P \cos Q + \cos P \sin$

$= \frac{1}{\sqrt{5}} \times \frac{3}{\sqrt{10}} + \frac{2}{\sqrt{5}} \times$

$= \frac{3}{\sqrt{50}} + \frac{2}{\sqrt{50}}$

$= \frac{5}{\sqrt{50}} = \frac{1}{\sqrt{2}}$

14. $\log_{10} 4 + 2\log_{10} 5$

$= \log_{10} 4 + \log_{10} 5^2$

$= \log_{10} 4 + \log_{10} 25$

$= \log_{10} 100$

$= 2$

b) $\log_2(7x-2) - \log_2 3 = 5$

$\log_2 \frac{7x-2}{5} = 5$

$\frac{7x-2}{5} = 32$

$7x - 2 = 94$

$x = 14$

15. $\sin 2x + 6\cos x = 0$

$2\sin x \cos x + 6\cos x =$

$2\cos x (\sin x + 3) =$

$\cos x = 0 \quad \sin x = -3$

$x = 90, 270$ NO Solu

b) $\sin 4x + 6\cos 2x = 0$

$2x = 90, 270, 450, 63$

$x = 45, 135, 225, 315$

16. C $(1, -2)$

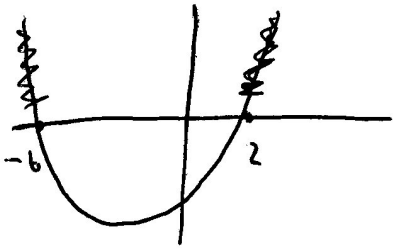
DISTANCE $\sqrt{3^2 + (k+2)^2}$

$= \sqrt{9 + k^2 + 4k + 4}$

$= \sqrt{k^2 + 4k + 13}$

b) $k^2 + 4k + 13 > 25$

$k^2 + 4k - 12 > 0$



$(k+6)(k-2)$

$-6 \qquad 2$

$x < -6$ AND $x > 2$

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17. $(\sin x - \cos x)^2 = \sin^2 x - 2\cos x \sin x + \cos^2 x$

$= 1 - 2\sin x \cos x$

$= 1 - \sin 2x$

b) $\int 1 - \sin 2x \; dx$

$= x - \frac{1}{2}\cos 2x + C$

HIGHER PAPER II 2014

1. Midpt AC $(-4, -3)$

$M_{BD} = -\frac{1}{3}$

$y + 8 = -\frac{1}{3}(x - 11)$

$3y = -x - 13$

b) $M_{BC} = -\frac{14}{14} = -1$

$M_{\perp} = 1$

$y + 12 = x + 5$

$y = x - 7$

c) $4y = -20$

$y = -5$

$x = 2$

$(2, -5)$

2. $\int 6x^{1/2} - 4x^{-3} + 5 \, dx$

$= 4x^{3/2} + 2x^{-2} + 5x + C$

3. a) $\vec{BE} = -\underline{p} + \underline{r} \quad (\underline{r} - \underline{p})$

b) $\vec{EF} = -\underline{r} + \underline{p} + \frac{3}{4}\underline{q} \quad (\frac{3}{4}\underline{q} + \underline{p} - \underline{r})$

4. $u_{n+1} = 0.973 u_n + 30$

$a = 0.973 \quad b = 30$

AS $-1 < 0.973 < 1$ THE POPULATION WILL STABILISE IN THE LONG TERM.

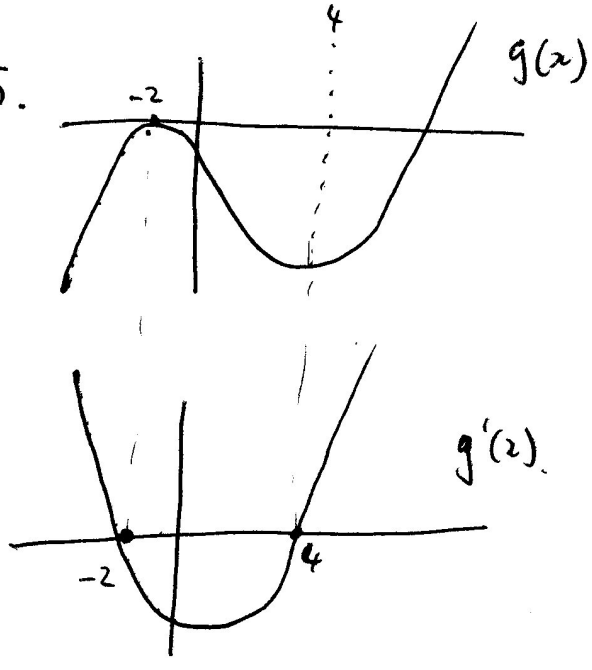
ii) $0.027L = 30$

$L = \frac{30}{0.027}$

$= 1111.111\ldots$

$= 1100$ TO NEAREST HUNDRED.

5.



6. $k\cos(x+a) = k\cos x \cos a - k\sin x \sin a$

$k\cos a = 2$

$k\sin a = 3$

$\tan a = \frac{3}{2} \quad a = 56.3$

$k = \sqrt{2^2 + 3^2}$

$= \sqrt{13}$

$2\cos x - 3\sin x = \sqrt{13}\cos(x + 56.3)$

b) $\sqrt{13}\cos(x+56.3) = 3$

$\cos(x+56.3) = 3/\sqrt{13}$

$x+56 = 33.7, 326.3, 393.7$

$x = \underline{270°}, \underline{337.4°}$

7. $-6\left(x^2 - 4x + \frac{25}{6}\right)$

$= -6(x-2)^2 - 1$

b) $f'(x) = -6x^2 + 24x - 25$

$= -6(x-2)^2 - 1$ (From pt (a))

ALWAYS NEGATIVE FOR ALL VALUES OF x

8. $y = \sqrt[3]{x} + 8$

$y - 8 = \sqrt[3]{x}$

$(y-8)^3 = x$

$f^{-1}(x) = (x-8)^3$

b) $9 \leq x \leq 18$

9. a) 120 WATTS

b) [redacted]

$102 = 120e^{-0.0074t}$

$0.85 = e^{-0.0074t}$

$\ln 0.85 = \ln e^{-0.0074t}$

$\ln 0.85 = -0.0074t \ln e$

$t = \dfrac{\ln 0.85}{-0.0074}$

$t = 20.57$ YEARS

10.

-3	3	10	1	-8	-6
	0	-9	-3	6	6
	3	1	-2	-2	0

SINCE $r=0$ $(x+3)$ IS A FAC

b) $(x+3)(3x^3+x^2-2x-2)$

1	3	1	-2	-2
	0	3	4	2
	3	4	2	0

SINCE $r=0$ $(x-1)$ IS A FACTOR.

$(x+3)(x-1)(3x^2+4x+2)$

FOR $3x^2+4x+2$

b^2-4ac

$= (4)^2 - 4(3)(2)$

$= 16 - 24$

$= -8$

$b^2-4ac < 0$: NO REAL ROOTS.

11. $SA = 4(3x h) + 2(3x)^2 + 4xh - 2x^2$

$= 12xh + 18x^2 + 4xh - 2x^2$

$= 16xh + 16x^2$

$V = (3x)^2 h - x^2 h$

$2000 = 9x^2 h - x^2 h$

$2000 = 8x^2 h$

$h = \frac{2000}{8x^2}$

$h = \frac{250}{x^2}$

$SA = 16x\left(\frac{250}{x^2}\right) + 16x^2$

$= \frac{4000}{x} + 16x^2$

$SA = 16x^2 + \frac{4000}{x}$ AS REQ

b) FOR MAX/MIN $\frac{dA}{dx} = 0$

$\frac{dA}{dx} = 32x - 4000x^{-2}$

$= 32x - \frac{4000}{x^2}$

	→ 5 →
$\frac{dA}{dx}$	− 0 +
	\ − /

MINIMUM AT $x = 5$

$32x - \frac{4000}{x^2} = 0$

$32x = \frac{4000}{x^2}$

$32x^3 = 4000$

$x^3 = \frac{4000}{32} = 125$ $x = 5$

AT $x = 5$

$A = 16(5)^2 + \frac{4000}{5}$

$= 400 + 20$

$= 600$

12. $y = ab^x$

$\log_4 y = \log_4 ab^x$

$\log_4 y = \log_4 a + \log_4 b^x$

$\log_4 y = x \log_4 b + \log_4 a$

$\log_4 b = m \qquad \log_4 a = c$

$m = 3 \qquad c = -1$

$\log_4 b = 3 \qquad \log_4 a = -1$

$b = 4^3 \qquad a = 4^{-1}$

$\underline{b = 64} \qquad \underline{a = \frac{1}{4}}$

13. $\int 3x^2 - 16x + 11 \, dx$

$= x^3 - 8x^2 + 11x + C$

AT $x = 7 \quad y = 0$

$0 = 7^3 - 8(7)^2 + 11(7) + C$

$\underline{C = -28}$

$f(x) = x^3 - 8x^2 + 11x - 28$

14. $u.(u+v) = 21$

$u.u + u.v = 21$

$|4|^2 + 4 \times 5 \times \cos\theta = 21$

$16 + 20\cos\theta = 21$

$20\cos\theta = 5$

$\cos\theta = \frac{1}{4}$

$\theta = 75.522°\ldots$

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15. $m_{RADIUS} = -\frac{1}{3}$ $\quad m_{TANGENT} = 3$

$y - 13 = 3(x-5)$

$y - 13 = 3x - 15$

$y = 3x - 2$

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b) i) $T(0, -2)$

ii) As $TPC = 90°$ TC is a diameter.

Midpt $TC = (4, 5)$

[redacted]

Radius $= \sqrt{4^2 + 7^2}$

$= \sqrt{65}$

Eqn of circle $(x-4)^2 + (y-5)^2 = 65$