2018 Paper 1

1. Midpoint $(1,2)$ $m = 2$

$$m_{mr} = \frac{y_m - y_r}{x_m - x_r}$$

$$= \frac{2-6}{1-3}$$

$$= \frac{-4}{-2}$$

$$= \underline{\underline{2}}$$

$$y - 2 = 2(x-1)$$

$$y - 2 = 2x - 2$$

$$\underline{\underline{y = 2x}}$$

2. $g(x) = \frac{1}{5}x - 4$

$$y = \frac{1}{5}x - 4$$

$$y + 4 = \frac{1}{5}x$$

$$5(y+4) = x$$

$$\underline{\underline{g^{-1}(x) = 5(x+4)}}$$

3. $h(x) = 3\cos 2x$

$$h'(x) = -3\sin 2x \,(\times 2)$$

$$= -6\sin 2x$$

$$h'\left(\tfrac{\pi}{6}\right) = -6\sin\left(\tfrac{2\pi}{6}\right)$$

$$= -6\sin(\pi/3)$$

$$= -6\left(\tfrac{\sqrt{3}}{2}\right) = -3\sqrt{3}.$$

4. Centre $(6,3)$

$$m_{CK} = \frac{y_C - y_K}{x_C - x_K}$$

$$= \frac{3-(-5)}{6-8}$$

$$= \frac{8}{-2}$$

$$= \underline{\underline{-4}}$$

If $\perp$, $m_1 m_2 = -1$

$$m_{\perp} = \frac{1}{4}$$

$$m = \frac{1}{4}$$

$$(a,b) = (8,-5)$$

$$y + 5 = \frac{1}{4}(x-8)$$

$$4(y+5) = x - 8$$

$$4y + 20 = x - 8$$

$$4y = x - 28$$

$$\underline{\underline{y = \frac{1}{4}x - 7}}$$

5a) $(-3,4,-7)$ $(5,t,5)$ $(7,9,8)$

A B C

$$\vec{AB} = \begin{pmatrix}5\\t\\5\end{pmatrix} - \begin{pmatrix}-3\\4\\-7\end{pmatrix} = \begin{pmatrix}8\\t-4\\12\end{pmatrix}$$

$$\vec{AC} = \begin{pmatrix}7\\9\\8\end{pmatrix} - \begin{pmatrix}-3\\4\\-7\end{pmatrix} = \begin{pmatrix}10\\5\\15\end{pmatrix}$$

$\Rightarrow \vec{AB} = \frac{4}{5}\vec{AC}$ so ratio is $4:1$

5b $\frac{4}{5}(5) = t - 4$

$$4 = t - 4$$

$$\underline{\underline{t = 8}}$$

6. $\log_5 250 - \frac{1}{3}\log_5 8$

$$= \log_5 250 - \log_5 8^{1/3}$$

$$= \log_5 250 - \log_5 2$$

$$= \log_5\left(\frac{250}{2}\right)$$

$$= \log_5(125)$$

$$= \underline{\underline{3}}$$

7a) On y-axis, $x = 0$

$$y = 0^3 - 3(0^2) + 2(0) + 5$$

$$y = 5 \qquad P(0,5).$$

b) $\frac{dy}{dx} = 3x^2 - 6x + 2$

$\frac{dy}{dx} = 2$ at $x = 0$

$m = 2$

$(a,b) = (0,5)$

$$y - 5 = 2(x - 0)$$

$$y - 5 = 2x$$

$$\underline{\underline{y = 2x + 5}}$$

7c) let $x^3 - 3x^2 + 2x + 5 = 2x + 5$

$\therefore x^3 - 3x^2 = 0$

$x^2(x-3) = 0$

$\underline{x=0}$ or $\underline{x=3}$

at $x = 3$

$y = 3^3 - 3(3^2) + 2(3) + 5$

$= 27 - 27 + 6 + 5$

$= 11$ $\underline{\underline{Q(3,11)}}$

8. $y - \sqrt{3}x + 5 = 0$

$y = \sqrt{3}x - 5$

$\tan\theta = m$

$\tan\theta = \sqrt{3}$

$\theta = \tan^{-1}(\sqrt{3})$

$\underline{\underline{\theta = 60°}}$

9a) $\vec{BC} = -\underline{t} + \underline{u}$ or $\underline{u} - \underline{t}$.

b) $\vec{MD} = \frac{1}{2}\vec{BC} + \vec{CA} + \vec{AD}$

$= \frac{1}{2}(\underline{u} - \underline{t}) - \underline{u} + \underline{v}$

$= \frac{1}{2}\underline{u} - \frac{1}{2}\underline{t} - \underline{u} + \underline{v}$

$= -\frac{1}{2}\underline{u} - \frac{1}{2}\underline{t} + \underline{v}$

10. $y = \int 6x^2 - 3x + 4 \, dx$

$y = \frac{6x^3}{3} - \frac{3x^2}{2} + 4x + C$

$y = 2x^3 - \frac{3}{2}\cancel{3}x^2 + 4x + C$

at $(2,14)$

$14 = 2(2^3) - \frac{3}{2}(2^2) + 4(2) + C$

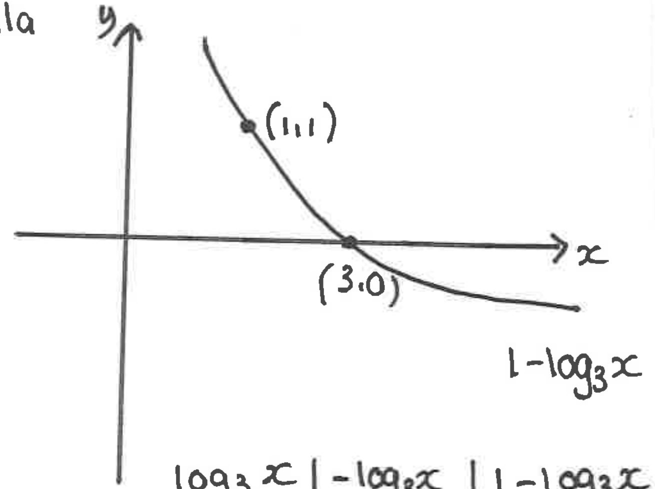
$14 = 16 - 6 + 8 + C$

$14 = 18 + C$

$\underline{\underline{C = -4}}$

$\underline{\underline{y = 2x^3 - \frac{3}{2}x^2 + 4x - 4}}$

11a



$\log_3 x$	$-\log_3 x$	$1-\log_3 x$
(1,0)	(1,0)	(1,1)
(3,1)	(3,-1)	(3,0)

b) $\log_3 x = 1 - \log_3 x$

$\log_3 x + \log_3 x = 1$

$\log_3 x^2 = 1$

$x^2 = 3^1$

$\underline{\underline{x = \sqrt{3}}}$

12a) $\underline{a} = \begin{pmatrix} 4 \\ -2 \\ 2 \end{pmatrix}$ $\underline{b} = \begin{pmatrix} -2 \\ 1 \\ p \end{pmatrix}$

$2\underline{a} + \underline{b} = 2\begin{pmatrix} 4 \\ -2 \\ 2 \end{pmatrix} + \begin{pmatrix} -2 \\ 1 \\ p \end{pmatrix}$

$= \begin{pmatrix} 8 \\ -4 \\ 4 \end{pmatrix} + \begin{pmatrix} -2 \\ 1 \\ p \end{pmatrix}$

$= \underline{\underline{\begin{pmatrix} 6 \\ -3 \\ p+4 \end{pmatrix}}}$

b) $|2\underline{a} + \underline{b}| = \sqrt{6^2 + (-3)^2 + (p+4)^2}$

$7 = \sqrt{36 + 9 + p^2 + 8p + 16}$

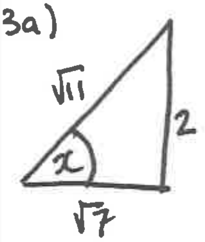
$49 = p^2 + 8p + 61$

$0 = p^2 + 8p + 12$

$0 = (p+6)(p+2)$

$\therefore \underline{\underline{p = -6}}$ and $\underline{p = 2}$

3a)



$$\sqrt{(\sqrt{11})^2 - 2^2}$$
$$= \sqrt{11-4}$$
$$= \underline{\underline{\sqrt{7}}}$$

$$\sin x = \frac{2}{\sqrt{11}}$$

$$\cos x = \frac{\sqrt{7}}{\sqrt{11}}$$

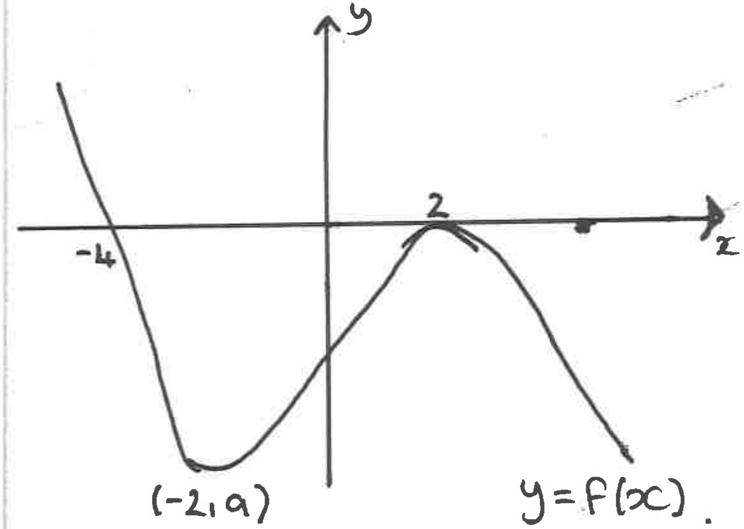
$$\sin 2x = 2\sin x\cos x$$
$$= 2\left(\frac{2}{\sqrt{11}}\right)\left(\frac{\sqrt{7}}{\sqrt{11}}\right)$$
$$= \underline{\underline{\frac{4\sqrt{7}}{11}}}$$

$$\cos 2x = \cos^2 x - \sin^2 x$$
$$= \left(\frac{\sqrt{7}}{\sqrt{11}}\right)^2 - \left(\frac{2}{\sqrt{11}}\right)^2$$
$$= \frac{7}{11} - \frac{4}{11}$$
$$= \underline{\underline{\frac{3}{11}}}$$

$$\sin 3x = \sin(2x + x)$$
$$= \sin 2x \cos x + \cos 2x \sin x$$
$$= \left(\frac{4\sqrt{7}}{11}\right)\left(\frac{\sqrt{7}}{\sqrt{11}}\right) + \left(\frac{3}{11}\right)\left(\frac{2}{\sqrt{11}}\right)$$
$$= \frac{4\times 7}{11\sqrt{11}} + \frac{6}{11\sqrt{11}}$$
$$= \frac{34}{11\sqrt{11}}$$

14. $$\int_{-4}^{9} (2x+9)^{-2/3}\, dx$$
$$= \left[\frac{(2x+9)^{1/3}}{1/3 \times 2}\right]_{-4}^{9}$$
$$= \left[\frac{\sqrt[3]{(2x+9)}}{2/3}\right]_{-4}^{9}$$
$$= \left[\frac{3\sqrt[3]{(2x+9)}}{2}\right]_{-4}^{9}$$
$$= \left[\frac{3\sqrt[3]{18+9}}{2}\right] - \left[\frac{3\sqrt[3]{-8+9}}{2}\right]$$
$$= \left[\frac{3(\sqrt[3]{27})}{2}\right] - \left[\frac{3\sqrt{1}}{2}\right]$$
$$= \frac{9}{2} - \frac{3}{2}$$
$$= \frac{6}{2}$$
$$= \underline{\underline{3}}$$

- $(x+4)$ is a factor
  $\Rightarrow$ root at $x = -4$
- TP (repeated root at $x = 2$
- grad at $x = -2$ is zero
  $\Rightarrow$ Stationary pt.
- grad is positive at intercept.



1. Area $= \int_{-1}^{3} 3 + 2x - x^2 \, dx$

$= \left[3x + x^2 - \frac{x^3}{3}\right]_{-1}^{3}$

$= \left[9 + 9 - \frac{27}{3}\right] - \left[-3 + 1 + \frac{1}{3}\right]$

$= 9 + 9 - 9 + 3 - 1 - \frac{1}{3}$

$= 11 - \frac{1}{3}$

$= 10\frac{2}{3}$ units^2

2. $\underline{u} \cdot \underline{v} = 7 + 32 - 15$
$= 24$

$\underline{u} = \begin{pmatrix} -1 \\ 4 \\ -3 \end{pmatrix}$

$\underline{v} = \begin{pmatrix} -7 \\ 8 \\ 5 \end{pmatrix}$

$\cos\theta = \frac{\underline{u} \cdot \underline{v}}{|\underline{u}||\underline{v}|}$

$= \frac{24}{\sqrt{26 \times 138}}$

$\theta = 66.4^\circ$

$|\underline{u}| = \sqrt{1 + 16 + 9}$
$= \sqrt{26}$

$|\underline{v}| = \sqrt{49 + 64 + 25}$
$= \sqrt{138}$

3. $f'(x) = 3x^2 - 7$

$f'(2) = 3(2^2) - 7$

$= 12 - 7$

$= 5$

Since $f'(2) > 0$, function is increasing.

4. $ax^2 + 2abx + ab^2 + c$

$-3x \quad -6x \quad +7$

$a = -3$

$2ab = -6$
$ab = -3$
$(-3)b = -3$
$b = \cancel{4}1$

$ab^2 + c = 7$

$-3(1) + c = 7$

$-3 + c = 7$

$c = 10$

$-3(x+1)^2 + 10$

5a $M(6, 1)$

$m_{PQ} = \frac{y_P - y_Q}{x_P - x_Q}$

$= \frac{-2 - 4}{9 - 3}$

$= \frac{-6}{6}$

$= -1$

If $\perp$, $m_1 m_2 = -1$

$-1 \times 1 = -1$

$(a, b) = (6, 1)$

$m = 1$

$y - 1 = 1(x - 6)$

$y - 1 = x - 6$

$y = x - 5$

b) $3y + x = 25$
$y - x = -5$
$4y = 20$
$y = 5$

$y - x = -5$
$5 - x = -5$
$-x = -10$
$x = 10$

$(10, 5)$

c) $r = |\overrightarrow{CP}|$

$= \sqrt{(-7)^2 + (-1)^2}$

$= \sqrt{49 + 1}$

$= \sqrt{50}$

$= 5\sqrt{2}$

$|\overrightarrow{CP}| = \begin{pmatrix} 3 \\ 4 \end{pmatrix} - \begin{pmatrix} 10 \\ 5 \end{pmatrix}$

$= \begin{pmatrix} -7 \\ -1 \end{pmatrix}$

$(x - 10)^2 + (y - 5)^2 = 50$

6a) $f(g(x)) = f(2x)$

$= 3 + \cos 2x$

b) $g(f(x)) = g(3+\cos x)$

$= 2(3+\cos x)$

$= 6 + 2\cos x$

c) $3 + \cos 2x = 6 + 2\cos x$

$\cos 2x - 2\cos x - 3 = 0$

$(2\cos^2 x - 1) - 2\cos x - 3 = 0$

$2\cos^2 x - 2\cos x - 4 = 0$

$2(\cos^2 x - \cos x - 2) = 0$

$2(\cos x - 2)(\cos x + 1) = 0$

$\cos x = 2$ $\quad\quad$ $\cos x = -1$

$\therefore$ no solutions $\quad\quad$ $x = \pi$

7.

$$\begin{array}{c|cccc} 2 & 2 & -3 & -3 & 2 \\ & \downarrow & 4 & 2 & -2 \\ \hline & 2 & 1 & -1 & \boxed{0} \end{array}$$

$0 \therefore$ factor.

$(x-2)(2x^2+x-1) = 0$

$(x-2)(2x-1)(x+1) = 0$

b) $u_6 = a(2a-3) - 1$

$= 2a^2 - 3a - 1$

$u_7 = a(2a^2 - 3a - 1) - 1$

$= 2a^3 - 3a^2 - a - 1$

c) $2a^3 - 3a^2 - a - 1 = 2a - 3$

$2a^3 - 3a^2 - 3a + 2 = 0$

$(a-2)(2a-1)(a+1) = 0$

$a = 2$ $\quad\quad$ $a = \frac{1}{2}$ $\quad\quad$ $a = -1$

Since limit exists, $a = \frac{1}{2}$.

$L = \frac{1}{2}L - 1$

$\frac{1}{2}L = -1$

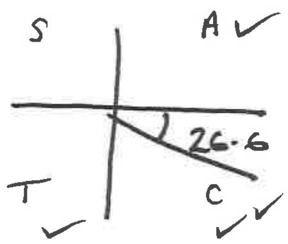
$L = -2$

8. $\underbrace{2\cos x}$ $- \sin x = k\cos x \cos a + k\sin x \sin a$

$k\sin a = -1$

$k\cos a = 2$

$\tan a = -\frac{1}{2}$



$\tan^{-1}(\frac{1}{2}) = 26.6$

$a = 360 - 26.6$

$= 333.4°$

$k = \sqrt{2^2 + (-1)^2}$

$= \sqrt{4+1}$

$= \sqrt{5}$

$2\cos x - \sin x = \sqrt{5}\cos(x - 333.4)°$

$2\cos x - \sin x$ has min at $-\sqrt{5}$

so

$6\cos x - 3\sin x$ has min at $-3\sqrt{5}$

let $x - 333.4 = 180$

$x = 513.4 \quad (-360)$

$x = 153.4°$

9. $P = 2x + 128x^{-1}$

$P'(x) = 2 - 128x^{-2} = 0$ at max/min

$2 - \frac{128}{x^2} = 0$

$2x^2 - 128 = 0$

$2(x^2 - 64) = 0$

$2(x+8)(x-8) = 0$

$x = -8 \quad x = 8$

x	$\rightarrow$	-8	$\rightarrow$	8	$\rightarrow$
$f'(x)$ $2(x-8)(x+8)$	$+--$ $+$	0	$+-+$ $-$	0	$+++$ $+$
	/	— max at $x=-8$	\	— min at $x=8$.	/

min $P = 2(8) + \frac{128}{8}$

$= 16 + 16$

$= \underline{\underline{32}}$

10. $a = 1$

$b = m-3$

$c = m$

$b^2 - 4ac > 0$ since real & distinct roots.

$(m-3)^2 - 4(1)(m) > 0$

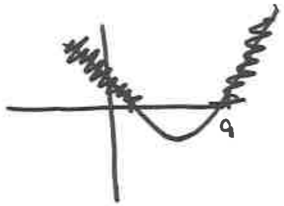
$m^2 - 6m + 9 - 4m > 0$

$m^2 - 10m + 9 > 0$

$(m-9)(m-1) > 0$

$\underline{\underline{m < 1}}$

$\underline{\underline{m > 9}}$



11a) $P = 100(1 - e^{kt})$

$50 = 100(1 - e^{kt})$

$0.5 = 1 - e^{3k}$

$-0.5 = -e^{3k}$

$e^{3k} = 0.5$

$3k = \ln 0.5$

$k = \frac{\ln 0.5}{3}$

$= \underline{\underline{-0.231}}$

b) $P = 100(1 - e^{-0.231(5)})$

$= 100(0.6849\ldots)$

$= 68.5\%$

$100 - 68.5$

$= 31.5\%$ wait more than 5 minutes.

12a $C_1\ (13, -4)$

$x^2 + y^2 + 14x - 22y + c = 0$

$169 + 16 + 182 + 88 + c = 0$

$455 + c = 0$

$c = -455$.

b) $C_2\ (-7, 11)$

$\overrightarrow{C_1C_2} = \begin{pmatrix} -7 \\ 11 \end{pmatrix} - \begin{pmatrix} 13 \\ -4 \end{pmatrix}$

$= \begin{pmatrix} -20 \\ 15 \end{pmatrix}$

$|\overrightarrow{C_1C_2}| = \sqrt{400 + 225}$

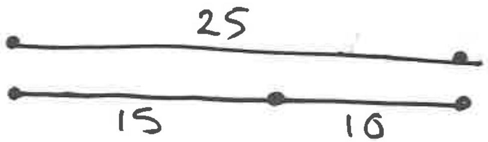
$= \sqrt{625}$

$= 25$

Distance between centres: 25

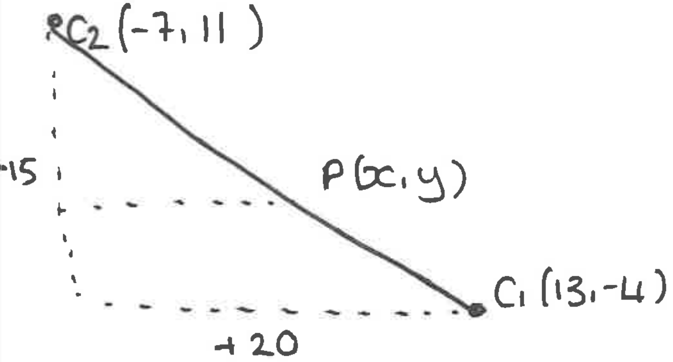
$R_1 = 10$

$R_2 = 15$



$= 15 : 10$

$= \underline{\underline{3 : 2}}$



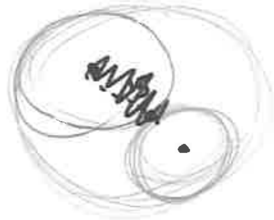
P_x = add $\frac{3}{5}$ of 20 to C_{2x}

$\therefore\ P_x = -7 + 12 = 5$

P_y = add $\frac{3}{5}$ of (-15) to C_{2y}

$P_y = 11 + (-9) = 2$

$\underline{\underline{P\,(5, 2)}}$



radius $C_2 = 25$

distance C_2 to $P = 15$

radius $C_3 = 40$

C_3: $(x-5)^2 + (y-2)^2 = 1600$.