Higher Mathematics

September Assessment
Practice Questions

Name: .....

Written Solutions are available at https://www.maths.scot/higher-maths-past-papers

Video Solutions from youtube.com/dlbmaths :

2023 Paper

https://www.youtube.com/playlist?list=PL4RWqHp0H-2dkGd2JEzkbkS8DbwcEWR0m

2021 Paper

https://www.youtube.com/playlist?list=PL4RWqHp0H-2dDREQA0cPC2reIilsmo2_j

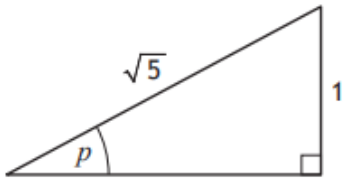
2015-2019 and 2022 paper

https://www.youtube.com/playlist?list=PLo_1r0X_Lv9CyILNIfNpRcajyfstuyARH

Study support is available at lunchtimes and after school in the Maths department

2024 Paper 1

6. The right-angled triangle in the diagram is such that $\sin p = \frac{1}{\sqrt{5}}$ and $0 < p < \frac{\pi}{4}$.



(a) Determine the value of:

(i) $\sin 2p$ **3**

(ii) $\cos 2p$. **1**

(b) Hence determine the value of $\sin 4p$. **1**

9. Express $\log_a 5 + \log_a 80 - 2\log_a 10$ in the form $\log_a k$ where k is a positive integer. **3**

11. (a) Express $\cos x° + \sqrt{3}\sin x°$ in the form $k\cos(x-a)°$, where $k > 0$ and $0 < a < 360$. **4**

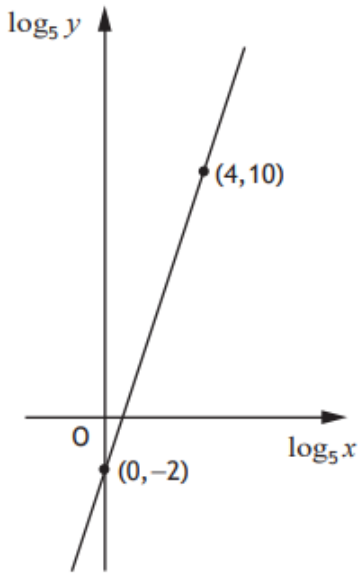
(b) Hence, or otherwise, sketch the graph with equation $y = \cos x° + \sqrt{3}\sin x°$, $0 \le x \le 360$.

Use the diagram provided in your answer booklet. **3**

2024 Paper 2

6. Two variables, x and y, are connected by the equation $y = ax^b$.

The graph of $\log_5 y$ against $\log_5 x$ is a straight line as shown.



Find the values of a and b. **5**

11. The number of electric vehicles worldwide can be modelled by

$$N = 6.8e^{kt}$$

where:

- N is the estimated number of vehicles in millions
- t is the number of years since the end of 2020
- k is a constant.

(a) Use the model to estimate the number of electric vehicles worldwide at the end of 2020. **1**

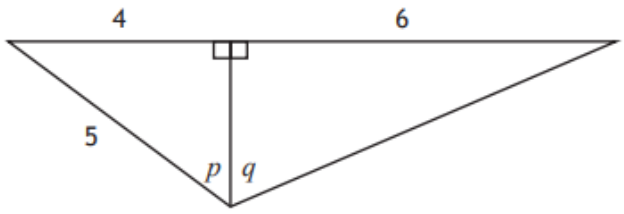
At the end of 2030, it is estimated there will be 125 million electric vehicles worldwide.

(b) Determine the value of k. **4**

2023 Paper 1

3. Solve $\log_5 x - \log_5 3 = 2$. **3**

4. The diagram shows two right-angled triangles with angles p and q as marked.



(a) Determine the value of:

(i) $\cos p$ **1**

(ii) $\cos q$. **1**

(b) Hence determine the value of $\cos(p+q)$. **3**

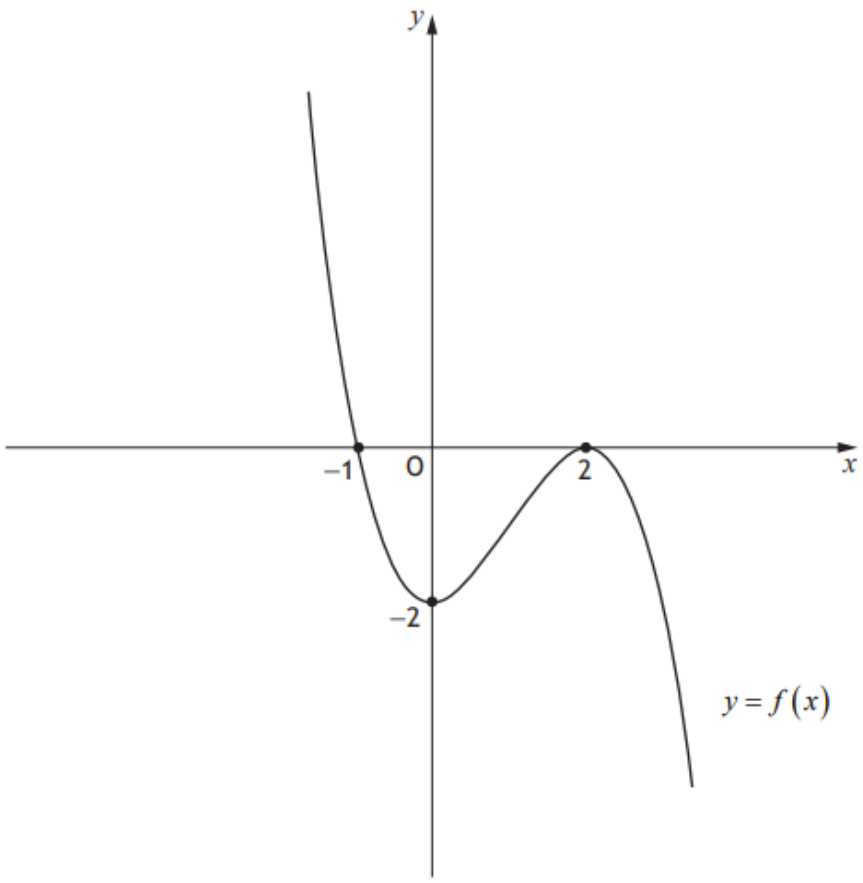
7. (a) Evaluate $\log_2 5 + \log_2 \frac{1}{40}$. **2**

(b) Given that $a \in \mathbb{R}$ and that $\log_8 a$ is negative, state the range of possible values of a. **1**

12. Express $-2x^2 - 12x + 7$ in the form $a(x+b)^2 + c$. **3**

2023 Paper 2

4. The diagram shows the cubic graph of $y = f(x)$, with stationary points at $(2, 0)$ and $(0, -2)$.



On the diagram in your answer booklet, sketch the graph of $y = 2f(-x)$.

9. (a) Express $7\cos x° - 3\sin x°$ in the form $k\sin(x + a)°$ where $k > 0$, $0 < a < 360$.

(b) Hence, or otherwise, find:

(i) the maximum value of $14\cos x° - 6\sin x°$

(ii) the value of x for which it occurs where $0 \leq x < 360$.

13. A patient is given a dose of medicine.

The concentration of the medicine in the patient's blood is modelled by

$$C_t = 11e^{-0.0053t}$$

where:

- t is the time, in minutes, since the dose of medicine was given
- C_t is the concentration of the medicine, in mg/l, at time t.

(a) Calculate the concentration of the medicine 30 minutes after the dose was given. 1

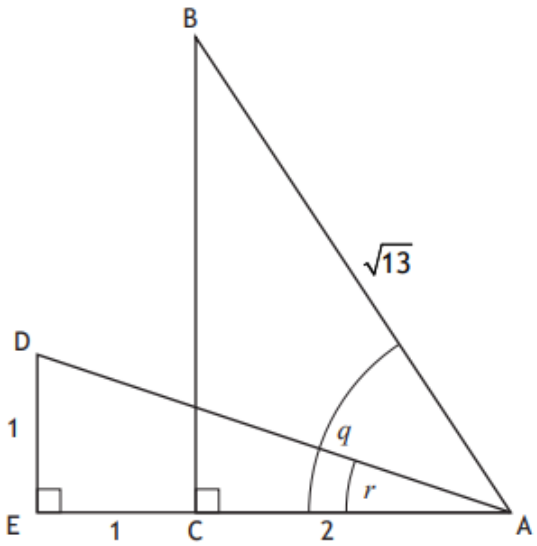
The dose of medicine becomes ineffective when its concentration falls to 0.66 mg/l.

(b) Calculate the time taken for this dose of the medicine to become ineffective. 3

2022 Paper 1

2. Evaluate $2\log_3 6 - \log_3 4$. **3**

7. Triangles ABC and ADE are both right angled.
Angle BAC $= q$ and angle DAE $= r$ as shown in the diagram.



(a) Determine the value of:

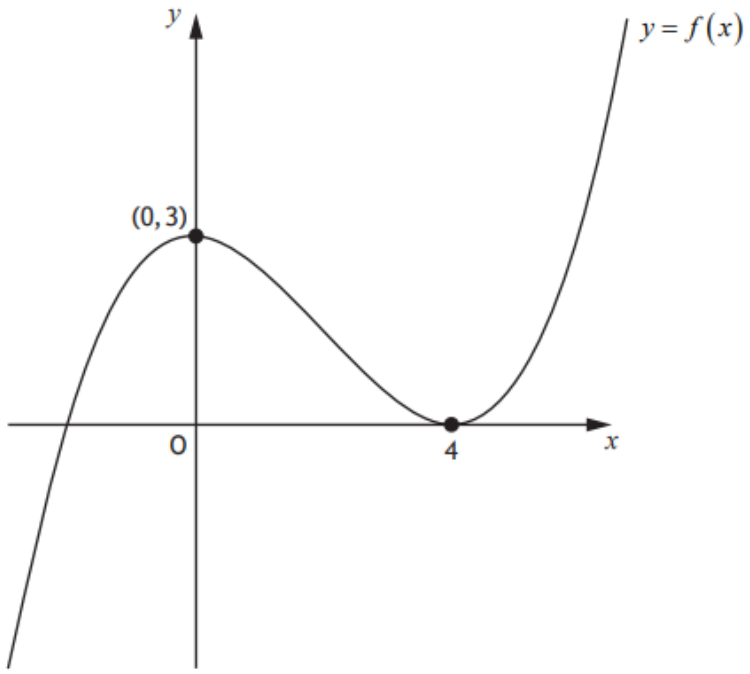
(i) $\sin r$ **1**

(ii) $\sin q$. **1**

(b) Hence determine the value of $\sin(q - r)$. **3**

8. Solve $\log_6 x + \log_6 (x+5) = 2$, where $x > 0$. **4**

10. The diagram shows the graph of a cubic function with equation $y = f(x)$.
The curve has stationary points at $(0, 3)$ and $(4, 0)$.



(a) Sketch the graph of $y = 2f(x) + 1$. **3**
Use the diagram provided in the answer booklet.

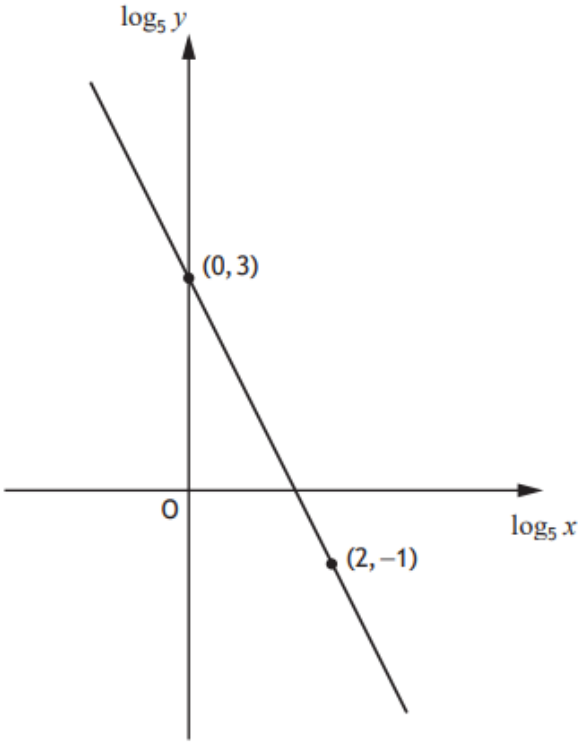
(b) State the coordinates of the stationary points on the graph of $y = f\left(\frac{1}{2}x\right)$. **1**

11. Express $2x^2 + 12x + 23$ in the form $p(x + q)^2 + r$. **3**

2022 Paper 2

3. (a) Express $4\sin x + 5\cos x$ in the form $k\sin(x+a)$ where $k > 0$ and $0 < a < 2\pi$. **4**

7. Two variables, x and y, are connected by the equation $y = kx^n$.
The graph of $\log_5 y$ against $\log_5 x$ is a straight line as shown.



Find the values of k and n. **5**

10. The heptathlon is an athletics contest made up of seven events.

Athletes score points for each event.

In the 200 metres event, the points are calculated using the formula

$$P = 4.99087\left(42.5 - T\right)^{1.81}$$

where P is the number of points awarded, and T is the athlete's time, in seconds.

(a) Calculate how many points would be awarded for a time of 24.55 seconds in the 200 metres event. **1**

In the long jump event, the points are calculated using the formula

$$P = 0.188807\left(D - 210\right)^{k}$$

where P is the number of points awarded, D is the distance jumped, in centimetres, and k is a constant.

(b) Given that 850 points are awarded for a jump of 600 cm, calculate the value of k. **4**

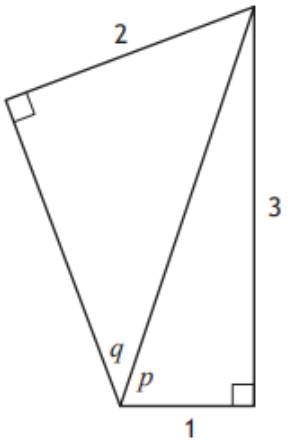
2021 Paper 1

3. A function $f(x)$ is defined on $\mathbb{R}$, by

$$f(x)=\frac{x+3}{2}.$$

Find the inverse function, $f^{-1}(x)$. **3**

5. Two right-angled triangles are shown below.



(a) Determine the value of

(i) $\sin p$ **1**

(ii) $\cos q$. **2**

(b) Find the exact value of $\cos(p+q)$. **3**

11. A function, f, defined on $\mathbb{R}$, is such that

- the maximum value of f is 8
- the maximum occurs when $x=6$.

The function g is given by $g(x)=2f(x)-9$.

(a) State the maximum value of g. **1**

The function h is given by $h(x)=f(x-4)+5$.

(b) (i) State the maximum value of h. **1**

(ii) State the value of x when the maximum value of h occurs. **1**

16. Evaluate $\log_2 6+\log_2 12-2\log_2 3$. **4**

2021 Paper 2

5. (a) Express $3\cos t° + 5\sin t°$ in the form $k\sin(t+a)°$, $k > 0$, $0 < a < 360$. **4**

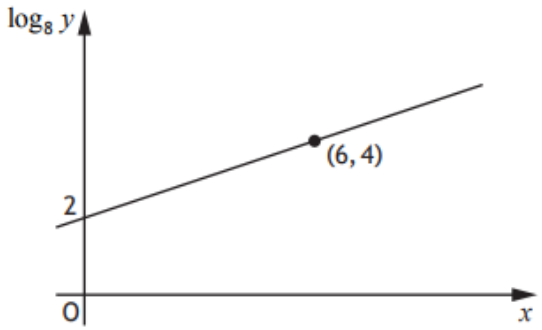
(b) A function, f, is defined by $f(t) = 3\cos t° + 5\sin t°$, $0 \leq t < 360$.

(i) State the minimum value of $f(t)$. **1**

(ii) Determine the value of t where this minimum occurs. **1**

16. Two variables, x and y, are connected by the equation $y = ab^x$.

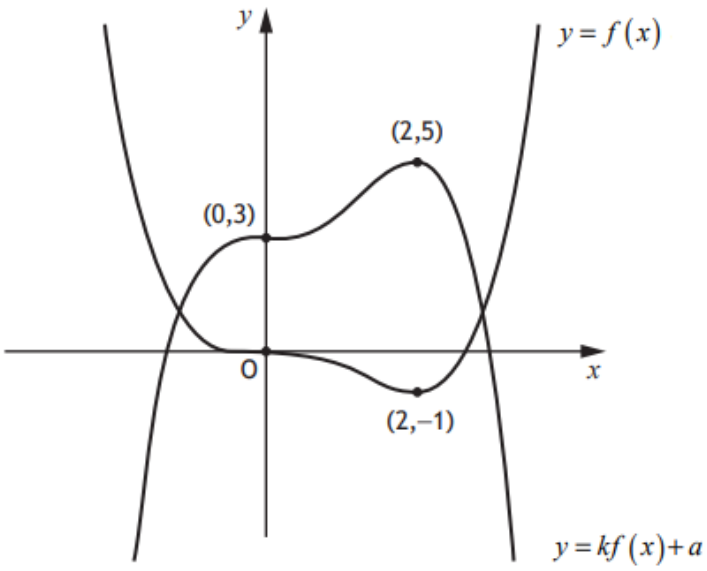
The graph of $\log_8 y$ against x is a straight line as shown.



Find the values of a and b. **5**

2019 Paper 1

10. The diagram shows the graphs with equations $y = f(x)$ and $y = kf(x) + a$.

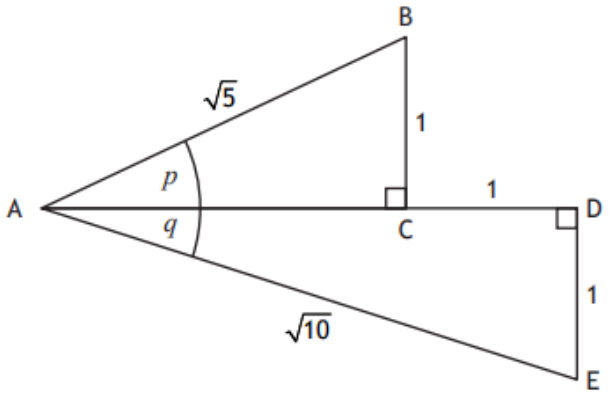


(a) State the value of a. **1**

(b) Find the value of k. **1**

13. Triangles ABC and ADE are both right angled.

Angles p and q are as shown in the diagram.



(a) Determine the value of

(i) $\cos p$ **1**

(ii) $\cos q$. **1**

(b) Hence determine the value of $\sin(p+q)$. **3**

14. (a) Evaluate $\log_{10} 4 + 2\log_{10} 5$. **3**

(b) Solve $\log_2(7x-2) - \log_2 3 = 5, \quad x \geq 1$. **3**

2019 Paper 2

6. (a) Express $2\cos x° - 3\sin x°$ in the form $k\cos(x+a)°$ where $k>0$ and $0 \le a < 360$. **4**

9. Electricity on a spacecraft can be produced by a type of nuclear generator.
The electrical power produced by this generator can be modelled by

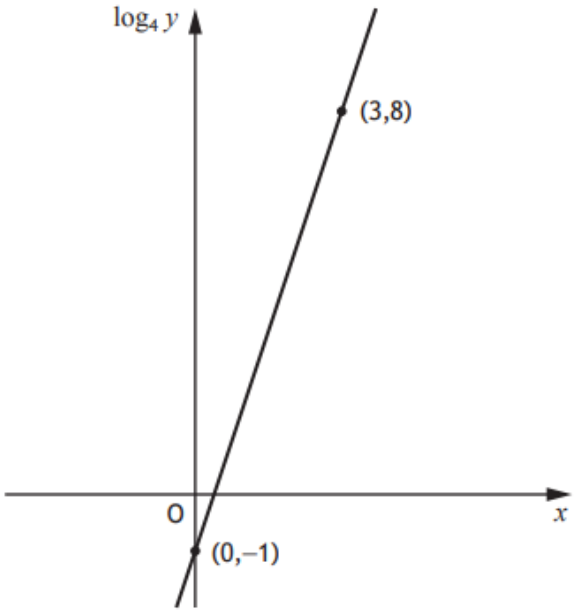
$$P_t = 120e^{-0.0079t}$$

where P_t is the electrical power produced, in watts, after t years.

(a) Determine the electrical power initially produced by the generator. **1**

(b) Calculate how long it takes for the electrical power produced by the generator to reduce by 15%. **4**

12. Two variables, x and y, are connected by the equation $y = ab^x$.
The graph of $\log_4 y$ against x is a straight line as shown.

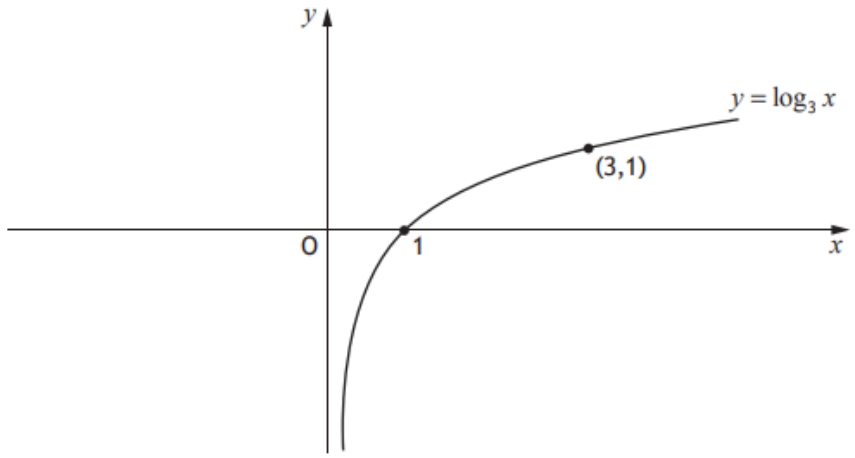


Find the values of a and b. **5**

2018 Paper 1

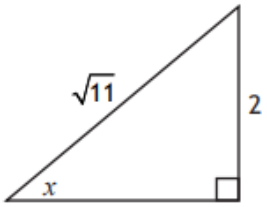
6. Find the value of $\log_5 250 - \frac{1}{3}\log_5 8$. **3**

11. The diagram shows the curve with equation $y = \log_3 x$.



(a) On the diagram in your answer booklet, sketch the curve with equation $y = 1 - \log_3 x$. **2**

13. The right-angled triangle in the diagram is such that $\sin x = \frac{2}{\sqrt{11}}$ and $0 < x < \frac{\pi}{4}$.



(a) Find the exact value of:

(i) $\sin 2x$ **3**

(ii) $\cos 2x$. **1**

(b) By expressing $\sin 3x$ as $\sin(2x + x)$, find the exact value of $\sin 3x$. **3**

2018 Paper 2

8. (a) Express $2\cos x° - \sin x°$ in the form $k\cos(x-a)°$, $k>0$, $0<a<360$. **4**

(b) Hence, or otherwise, find

(i) the minimum value of $6\cos x° - 3\sin x°$ and **1**

(ii) the value of x for which it occurs where $0 \le x < 360$. **2**

11. A supermarket has been investigating how long customers have to wait at the checkout.

During any half hour period, the percentage, $P\%$, of customers who wait for less than t minutes, can be modelled by

$$P = 100\left(1-e^{kt}\right), \text{ where } k \text{ is a constant.}$$

(a) If 50% of customers wait for less than 3 minutes, determine the value of k. **4**

(b) Calculate the percentage of customers who wait for 5 minutes or longer. **2**

2017 Paper 1

12. Given that $\log_a 36 - \log_a 4 = \frac{1}{2}$, find the value of a. **3**

14. (a) Express $\sqrt{3}\sin x° - \cos x°$ in the form $k\sin(x-a)°$, where $k > 0$ and $0 < a < 360$. **4**

(b) Hence, or otherwise, sketch the graph with equation $y = \sqrt{3}\sin x° - \cos x°$, $0 \le x \le 360$. **3**

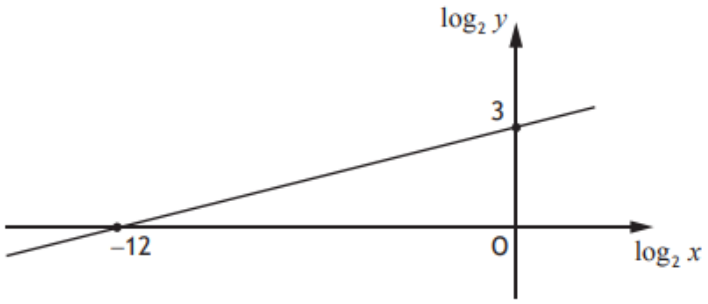
Use the diagram provided in the answer booklet.

2017 Paper 2

4. (a) Express $3x^2+24x+50$ in the form $a(x+b)^2+c$. **3**

9. Two variables, x and y, are connected by the equation $y=kx^n$.

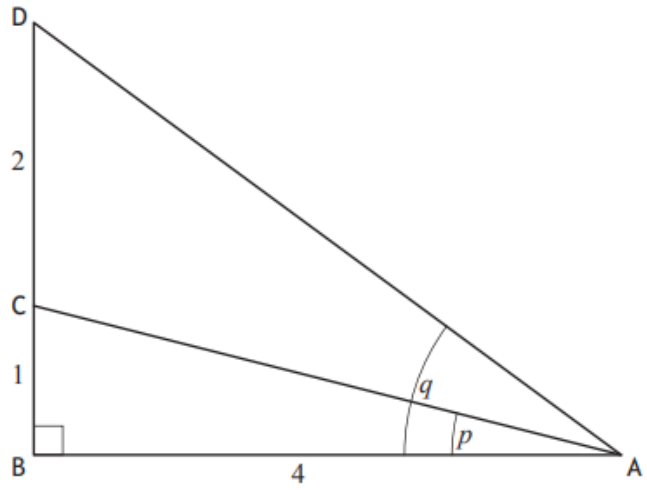
The graph of $\log_2 y$ against $\log_2 x$ is a straight line as shown.



Find the values of k and n. **5**

2016 Paper 1

13. Triangle ABD is right-angled at B with angles BAC = p and BAD = q and lengths as shown in the diagram below.



Show that the exact value of $\cos(q-p)$ is $\frac{19\sqrt{17}}{85}$. **5**

14. (a) Evaluate $\log_5 25$. **1**

(b) Hence solve $\log_4 x + \log_4 (x-6) = \log_5 25$, where $x > 6$. **5**

2016 Paper 2

6. Scientists are studying the growth of a strain of bacteria. The number of bacteria present is given by the formula

$$B(t) = 200e^{0.107t},$$

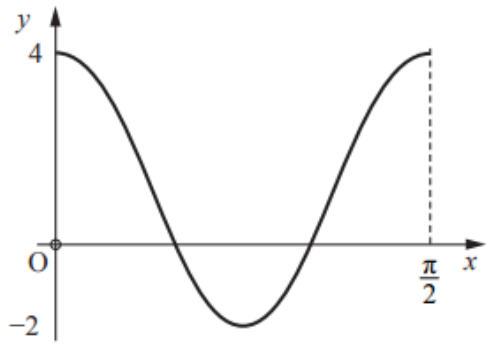
where t represents the number of hours since the study began.

(a) State the number of bacteria present at the start of the study. **1**

(b) Calculate the time taken for the number of bacteria to double. **4**

2015 Paper 1

4. The diagram shows part of the graph of the function $y = p\cos qx + r$.



Write down the values of p, q and r. **3**

6. Evaluate $\log_6 12 + \frac{1}{3}\log_6 27$. **3**

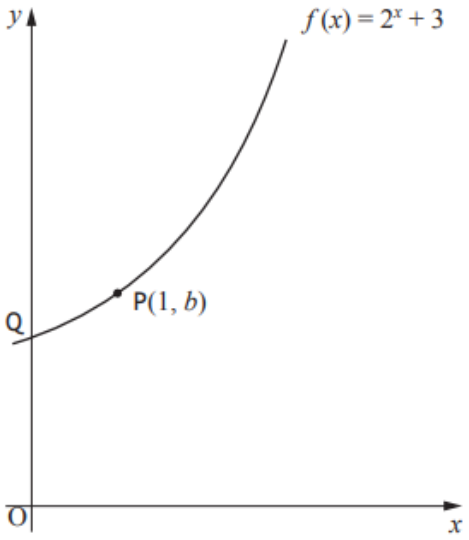
10. Given that $\tan 2x = \frac{3}{4}$, $0 < x < \frac{\pi}{4}$, find the exact value of

(a) $\cos 2x$ **1**

(b) $\cos x$. **2**

13. The function $f(x) = 2^x + 3$ is defined on $\mathbb{R}$, the set of real numbers.

The graph with equation $y = f(x)$ passes through the point P$(1, b)$ and cuts the y-axis at Q as shown in the diagram.



(a) What is the value of b? **1**

(b) (i) Copy the above diagram.

On the same diagram, sketch the graph with equation $y = f^{-1}(x)$. **1**

(ii) Write down the coordinates of the images of P and Q. **3**

(c) R $(3,11)$ also lies on the graph with equation $y = f(x)$.

Find the coordinates of the image of R on the graph with equation $y = 4 - f(x + 1)$. **2**

2015 Paper 2

No questions