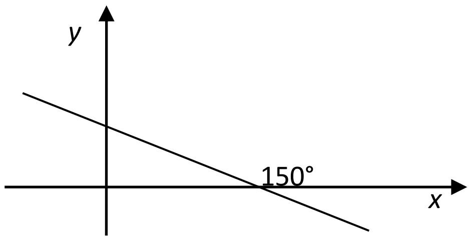
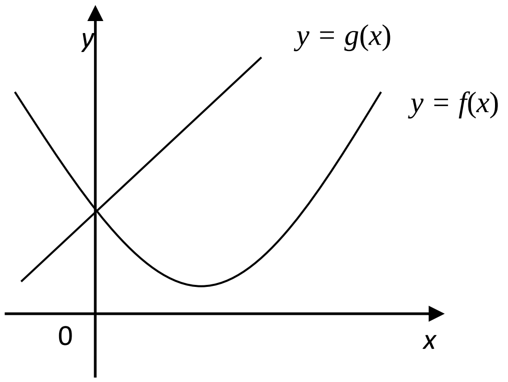
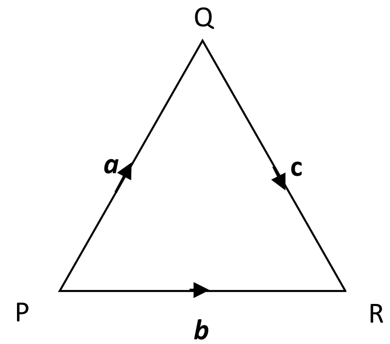
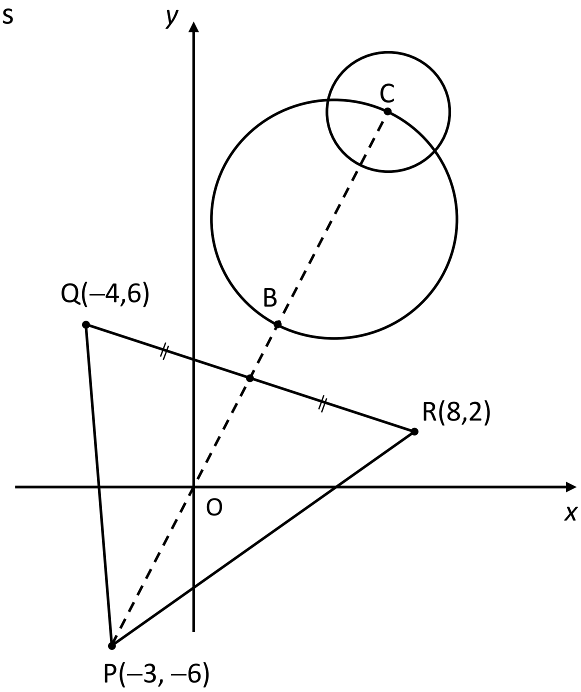
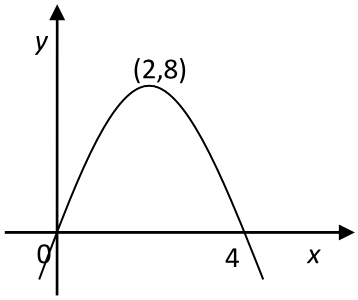
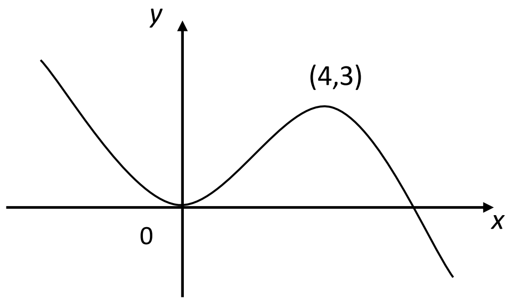
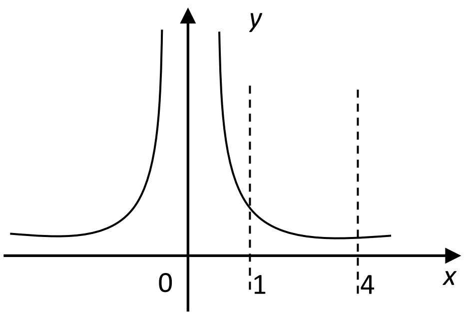
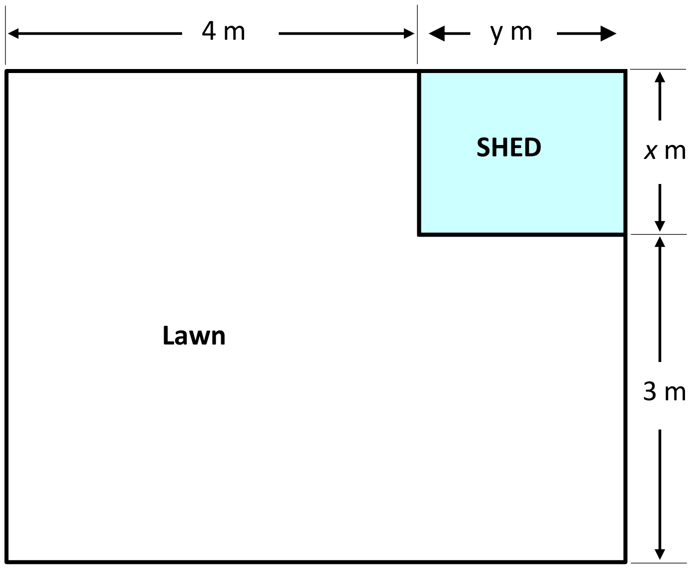
Easter School Prelim – Paper 1

1	Vectors ***p***, ***q***, and ***r*** are defined by ***p*** = ***i*** + ***j*** – ***k***, ***q*** = ***i*** + 4***k***, ***r*** = 4***i*** - 3***j*** (a) Express ***p*** – ***q*** + 2***r*** in component form (b) Find the components of the unit vector parallel to ***r***	 2 2
2	Find the equation of the perpendicular bisector of the straight line joining A(2,-1) and B(8,3)	4
3	State the rate of change of the function $f(x) = \sin(x) + 3x$, when $x = \frac{\pi}{3}$	3
4	Find k if $x - 2$ is a factor of $x^3 + kx^2 - 4x - 12$	2
5	State the gradient of the straight line own in the diagram 	2
6	Express $f(x) = 4x^2 + 8x - 5$ in the form $a(x+b)^2 + c$	3
7	Two functions *f* and g are defined on suitable domains by $f(x) = \frac{1}{x-4}$ and $g(x) = 2x + 3$ (a) Find the inverse function $g^{-1}(x)$ (b) Find an expression for *f*(g(*x*)). Give your answer in the simplest form (c) State any restrictions on the domain of *f*(g(*x*))	 1 2 1
8	In a right-angled triangle angle *x* is acute and is such that $\cos x = \frac{\sqrt{2}}{10}$. (a) Show clearly that the exact value of $\sin x$ is $\frac{7\sqrt{2}}{10}$. (b) Hence show that $\sin(x + 45)^\circ = 0 \cdot 8$.	 2 3

9	(a) Find the coordinates of the stationary points for the function $f(x)=x^3-12x+2$ and determine their nature (b) Hence determine the range of values for x for which the function *f*(*x*) is strictly increasing	7 2
10	Show that the straight line $y=10-2x$ is a tangent to the circle $x^2+y^2+2x-4y-15=0$	4
11	The graphs of $y=f(x)$ and $y=g(x)$ intersect at point A on the *y*-axis as shown in the diagram  If $g(x)=3x+4$ and the derived function $f'(x)=2x-3$, find $f(x)$	4
12	(a) Find the derivative of the function $f(x)=(8-x^3)^{1/2}$, $x<2$ (b) Hence integrate $\int \frac{x^2}{(8-x^3)^{1/2}}dx$	3 2
13	PQR is an equilateral triangle of side 2 units $\vec{PQ}=\boldsymbol{a}$, $\vec{PR}=\boldsymbol{b}$, $\vec{QR}=\boldsymbol{c}$, Evaluate $\boldsymbol{a}\bullet(\boldsymbol{b}+\boldsymbol{c})$ and hence identify two vectors which are perpendicular 	5
14	(a) Given that $3\log_x y = \log_x y^2 + 2$, find a relationship connecting *x* and *y*. (b) Hence find the two values of y when $x=y-2$.	4 2
	END OF PAPER 1	

Easter School Prelim – Paper 2		
1	(a) Given three points R(-1, -8, -2), S(2, -5, 4) and T(3, -4, 6)	
	(i) Show that R, S and T are collinear	3
	(ii) State the ratio in which S divides RT	1
	(b) Point F has the coordinates (5, -1, -1). Calculate angle FST	5
2	Two sequences are defined by these recurrence relations $U_{n+1} = 3U_n - 0.4,\ U_0 = 1$ and $V_{n+1} = 0.3V_n + 4,\ V_0 = 1$	
	(a) Explain why only one of these sequences approaches a limit as n → ∞	1
	(b) Find algebraically the exact value of the limit	2
	(c) For the other sequence, find the smallest value for *n* for which the *n*th term exceeds 1000 and the value of that term	1
3	In this diagram, the triangle has coordinates P(–3, –6), Q(–4,6) and R(8,2) 	
	(a) Find the equation of the median from point P	3
	(b) When this median is extended, it intersects with the larger circle at B and C. If the larger circle has as its equation $x^2 + y^2 - 10x - 20y + 105 = 0$, find the coordinates of C.	4
	(c) Given that C is the centre of the smaller circle and that its radius is **exactly half** of the larger circle, find the equation of the smaller circle.	3

4	The diagram shows a parabola passing through the points (0,0), (2,8) and (4,0). The equation of this parabola is in the form $y = ax(x - b)$. Find the values of a and b. 	3
5	(a) Express $2\cos x - 5\sin x$ in the form $k\cos(x+\alpha)^\circ$ stating the values of *k* and *a* where *k* >0 and $0 \le \alpha \le 360^0$ (b) Hence state the range of the function $y = 2\cos x - 5\sin x$	4 2
6	The diagram shows part of the graph of *y* = *f*(*x*) with turning points at (0,0) and (4,3)  Sketch the graph of (a) *y* = *f*(2*x*) (b) *y* = 1 - *f*(2*x*)	1 2
7	(a) Express $x^4 - x$ in its fully factorised form (b) Show that $x^4 - x = 0$ has only two real solutions and state the value of these two solutions	5 3
8	Algebraically solve the equation $\sin 2x = 2\cos^2 x$ for $0 \le x \le 2\pi$	5
9	The diagram shows the graph of the curve $y = \frac{1}{x^2}$ and the straight lines *x* = 1 and *x* = 4  (a) Find the area enclosed by the curve, the *x*-axis and the straight lines (b) Find the value of *k* such that the line *x* = *k* divides this area into 2 equal parts	5 4

10	The amount *A* grams of a radioactive substance at time *t* minutes is given by $A = A_0\, e^{-kt}$, where A_0 is the initial amount of the substance and *k* is a constant. (a) In 3 minutes, 10 grams of the substance Bismuth are reduced to 9 grams through radioactive decay. Find the value of *k* (b) The half-life of substance is the length of time in which half of the substance decays. Find the half-life of Bismuth	3 2
11	A plan for a garden is shown below. All dimensions are in metres. The gardener places a rectangular wooden storage shed, with length *y* metres and width *x* metres, in one corner.  (a) Given that the **area of the shed** is 3 square metres, show clearly that the area of the lawn, A square metres, is given in terms of x as $A(x) = 12 + 4x + \frac{9}{x}$. *(b)* Hence find the value of *x* which **minimises** the area of the lawn, **justify your answer**.	3 5
	END OF PAPER 2	

FORMULAE LIST

Circle

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\boldsymbol{a} \cdot \boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}|\cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$.

Or $\boldsymbol{a} \cdot \boldsymbol{b} = \boldsymbol{a}_1\boldsymbol{b}_1 + \boldsymbol{a}_2\boldsymbol{b}_2 + \boldsymbol{a}_3\boldsymbol{b}_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

Trigonometric formulae:

$$\begin{aligned}
\sin(A \pm B) &= \sin A\cos B \pm \cos A\sin B \\
\cos(A \pm B) &= \cos A\cos B \mp \sin A\sin B \\
\sin 2A &= 2\sin A\cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + c$
$\cos ax$	$\frac{1}{a}\sin ax + c$