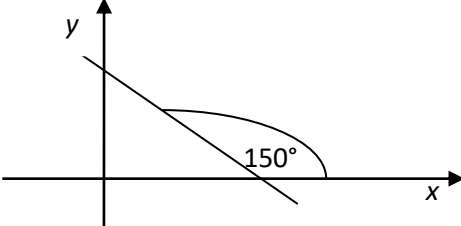
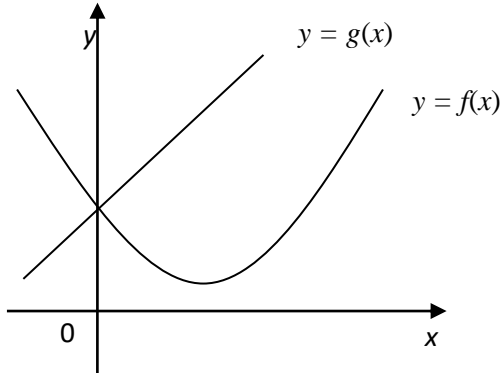
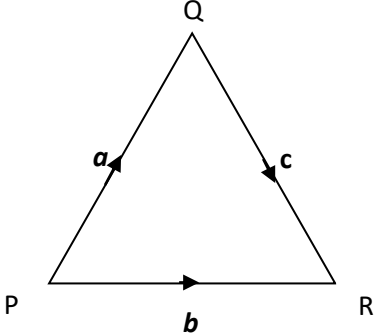
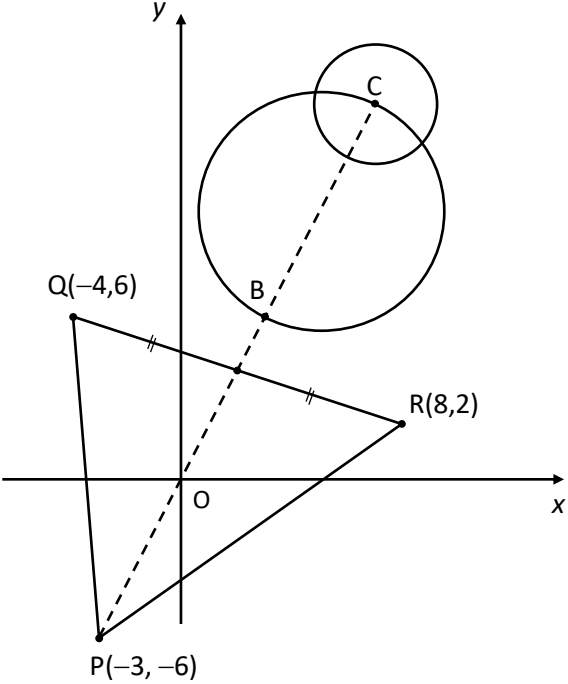
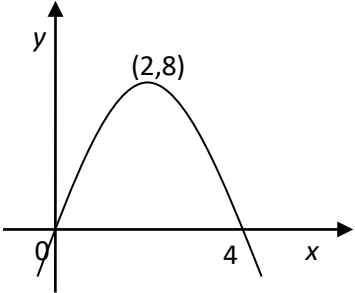
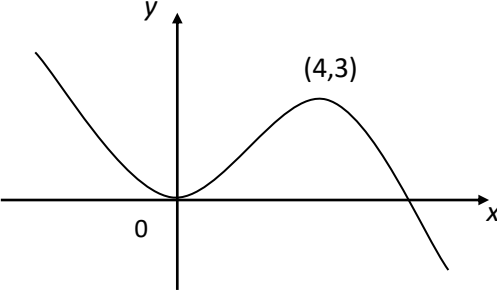
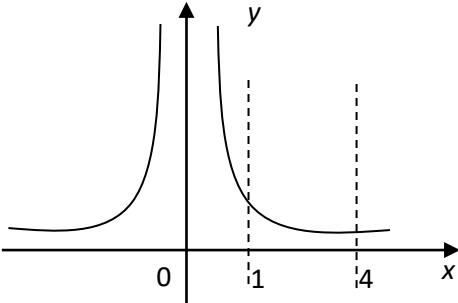


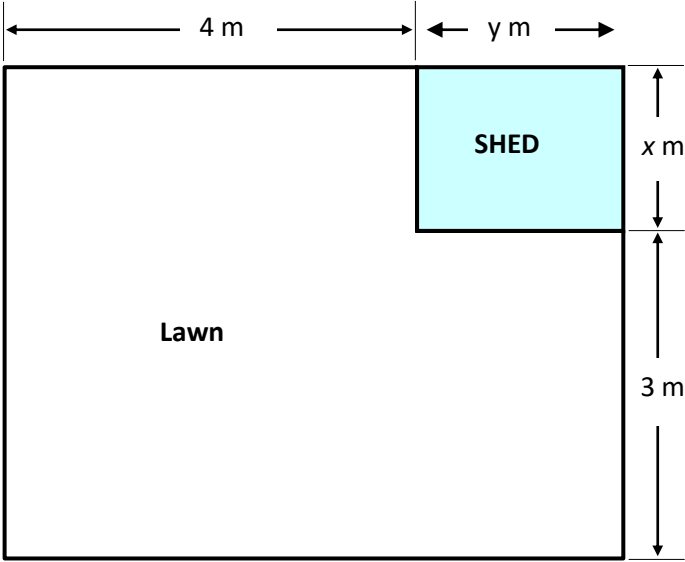
Higher Maths Easter School Prelim – Paper 1		
1	<p>Vectors \mathbf{p}, \mathbf{q}, and \mathbf{r} are defined by</p> $\mathbf{p} = i + j - k, \quad \mathbf{q} = i + 4k, \quad \mathbf{r} = 4i - 3j$ <p>(a) Express $\mathbf{p} - \mathbf{q} + 2\mathbf{r}$ in component form</p> <p>(b) Find the components of the unit vector parallel to \mathbf{r}</p>	2 2
2	Find the equation of the perpendicular bisector of the straight line joining A(2,-1) and B(8,3)	4
3	State the rate of change of the function $f(x) = \sin(x) + 3x$, when $x = \frac{\pi}{3}$	3
4	Find k if $x - 2$ is a factor of $x^3 + kx^2 - 4x - 12$	2
5	<p>State the gradient of the straight line in the diagram below</p> 	2
6	Express $f(x) = 4x^2 + 8x - 5$ in the form $a(x+b)^2 + c$	3
7	<p>Two functions f and g are defined on suitable domains by</p> $f(x) = \frac{1}{x-4} \quad \text{and} \quad g(x) = 2x + 3$ <p>(a) Find the inverse function $g^{-1}(x)$</p> <p>(b) Find an expression for $f(g(x))$. Give your answer in the simplest form</p> <p>(c) State any restrictions on the domain of $f(g(x))$</p>	1 2 1
8	<p>In a right-angled triangle angle x is acute and is such that $\cos x = \frac{\sqrt{2}}{10}$.</p> <p>(a) Show clearly that the exact value of $\sin x$ is $\frac{7\sqrt{2}}{10}$.</p> <p>(b) Hence show that $\sin(x + 45)^\circ = 0.8$.</p>	2 3

9	<p>(a) Find the coordinates of the stationary points for the function $f(x) = x^3 - 12x + 2$ and determine their nature</p> <p>(b) Hence determine the range of values for x for which the function $f(x)$ is strictly increasing</p>	7 2
10	<p>Show that the straight line $y = 10 - 2x$ is a tangent to the circle $x^2 + y^2 + 2x - 4y - 15 = 0$</p>	4
11	<p>The graphs of $y = f(x)$ and $y = g(x)$ intersect at point A on the y-axis as shown in the diagram</p>  <p>If $g(x) = 3x + 4$ and the derived function $f'(x) = 2x - 3$, find $f(x)$</p>	4
12	<p>(a) Find the derivative of the function $f(x) = (8 - x^3)^{1/2}$, $x < 2$</p> <p>(b) Hence integrate $\int \frac{x^2}{(8 - x^3)^{1/2}} dx$</p>	3 2
13	<p>PQR is an equilateral triangle of side 2 units</p> <p>$\vec{PQ} = \mathbf{a}$, $\vec{PR} = \mathbf{b}$, $\vec{QR} = \mathbf{c}$,</p> <p>Evaluate $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c})$ and hence identify two vectors which are perpendicular</p> 	5
14	<p>(a) Given that $3 \log_x y = \log_x y^2 + 2$, find a relationship connecting x and y.</p> <p>(b) Hence find the two values of y when $x = y - 2$.</p>	4 2
END OF PAPER 1		

Higher Maths Easter School Prelim – Paper 2

1	<p>(a) Given three points $R(-1, -8, -2)$, $S(2, -5, 4)$ and $T(3, -4, 6)$</p> <p>(i) Show that R, S and T are collinear</p> <p>(ii) State the ratio in which S divides RT</p> <p>(b) Point F has the coordinates $(5, -1, -1)$. Calculate angle FST</p>	<p>3</p> <p>1</p> <p>5</p>
2	<p>Two sequences are defined by these recurrence relations $U_{n+1} = 3U_n - 0.4$, $U_0 = 1$ and $V_{n+1} = 0.3V_n + 4$, $V_0 = 1$</p> <p>(a) Explain why only one of these sequences approaches a limit as $n \rightarrow \infty$</p> <p>(b) Find algebraically the exact value of the limit</p> <p>(c) For the other sequence, find the smallest value for n for which the nth term exceeds 1000 and the value of that term</p>	<p>1</p> <p>2</p> <p>1</p>
3	<p>In this diagram, the triangle has coordinates $P(-3, -6)$, $Q(-4, 6)$ and $R(8, 2)$</p>  <p>(a) Find the equation of the median from point P</p> <p>(b) When this median is extended, it intersects with the larger circle at B and C. If the larger circle has as its equation $x^2 + y^2 - 10x - 20y + 105 = 0$, find the coordinates of C.</p> <p>(c) Given that C is the centre of the smaller circle and that its radius is exactly half of the larger circle, find the equation of the smaller circle.</p>	<p>3</p> <p>4</p> <p>3</p>

4	<p>The diagram shows a parabola passing through the points $(0,0)$, $(2,8)$ and $(4,0)$.</p> <p>The equation of this parabola is in the form $y = ax(x - b)$.</p> <p>Find the values of a and b.</p>		3
5	<p>(a) Express $2\cos x - 5\sin x$ in the form $k\cos(x + \alpha)^\circ$ stating the values of k and α where $k > 0$ and $0 \leq \alpha \leq 360^\circ$</p> <p>(b) Hence state the range of the function $y = 2\cos x - 5\sin x$</p>	4 2	
6	<p>The diagram shows part of the graph of $y = f(x)$ with turning points at $(0,0)$ and $(4,3)$</p> 	<p>Sketch the graph of</p> <p>(a) $y = f(2x)$</p> <p>(b) $y = 1 - f(2x)$</p>	1 2
7	<p>(a) Express $x^4 - x$ in its fully factorised form</p> <p>(b) Show that $x^4 - x = 0$ has only two real solutions and state the value of these two solutions</p>	5 3	
8	<p>Algebraically solve the equation $\sin 2x = 2\cos^2 x$ for $0 \leq x \leq 2\pi$</p>	5	
9	<p>The diagram shows the graph of the curve $y = \frac{1}{x^2}$ and the straight lines $x = 1$ and $x = 4$</p> 	<p>(a) Find the area enclosed by the curve, the x-axis and the straight lines</p> <p>(b) Find the value of k such that the line $x = k$ divides this area into 2 equal parts</p>	5 4

10	<p>The amount A grams of a radioactive substance at time t minutes is given by $A = A_0 e^{-kt}$, where A_0 is the initial amount of the substance and k is a constant.</p> <p>(a) In 3 minutes, 10 grams of the substance Bismuth are reduced to 9 grams through radioactive decay. Find the value of k</p> <p>(b) The half-life of substance is the length of time in which half of the substance decays. Find the half-life of Bismuth</p>	3 2
11	<p>A plan for a garden is shown below. All dimensions are in metres. The gardener places a rectangular wooden storage shed, with length y metres and width x metres, in one corner.</p>  <p>(a) Given that the area of the shed is 3 square metres, show clearly that the area of the lawn, A square metres, is given in terms of x as</p> $A(x) = 12 + 4x + \frac{9}{x}.$ <p>(b) Hence find the value of x which minimises the area of the lawn, justify your answer.</p>	3 5
	END OF PAPER 2	

FORMULAE LIST

Circle

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Scalar Product: $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta$, where θ is the angle between \mathbf{a} and \mathbf{b} .

Or $\mathbf{a} \cdot \mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + c$
$\cos ax$	$\frac{1}{a} \sin ax + c$