*Prelim Examination 2003 / 2004*
*(Assessing Units 1 & 2)*

# MATHEMATICS

## Higher Grade - Paper I (Non~calculator)

**Time allowed - 1 hour 10 minutes**

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Read Carefully

1. **Calculators may not be used in this paper.**
2. Full credit will be given only where the solution contains appropriate working.
3. Answers obtained by readings from scale drawings will not receive any credit.
4. **This examination paper contains questions graded at all levels.**

**FORMULAE LIST**

**Circle:**

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre $(a, b)$ and radius $r$.

**Trigonometric formulae:**

$$
\begin{aligned}
\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\sin 2A &= 2 \sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}
$$

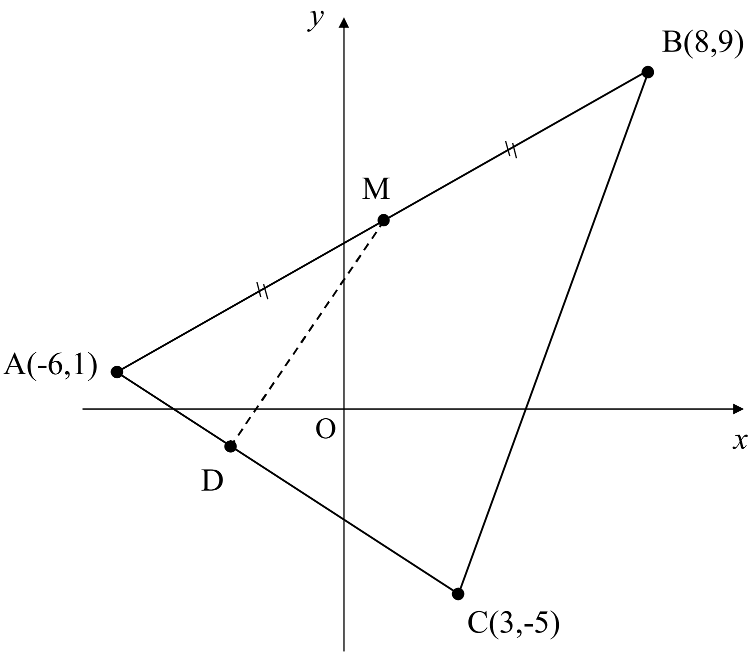
**All questions should be attempted**

**1.** A function is defined on a suitable domain as $f(x) = \frac{1}{x}(x-4)^2$ .

Given that $f'(a) = -3$, find the two possible values of $a$. **5**

**2.** Triangle ABC has vertice A(-6,1), B(8,9) and C(3,-5) as shown.

M is the mid-point of side AB and D is a point on side AC.




(a) Write down the coordinates of M. **1**

(b) Find the equation of MD given that MD is **perpendicular** to side AC. **3**

(c) Hence establish the coordinates of D. **4**

**3.** A recurrence relation is given as $u_{n+1} = 0 \cdot 6u_n + 40$.

(a) Given that $u_1 = 70$, find the intial value, $u_0$, of this sequence. **2**

(b) Hence find the difference between the initial value and the **limit** of this sequence. **3**

**4.** An equation is given as $x^3 - x^2 - ax + b = 0$, where $a$ and $b$ are constants.

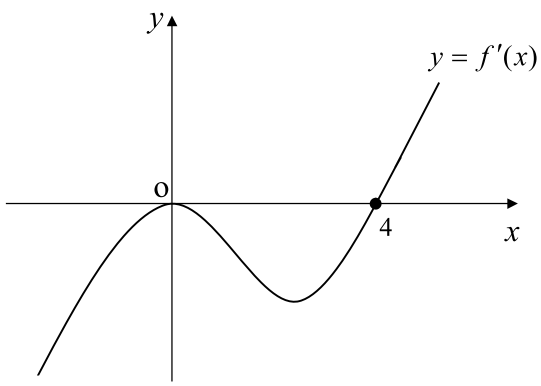
(a) It is known that $x = 2$ and $x = -4$ are two roots of this equation.

Use the above roots to establish the values of the constants $a$ and $b$. **5**

(b) Hence find the third root of this equation. **2**

**5.** A function $f(x)$ has $f'(x)$ as its derivative.

Part of the graph of $y = f'(x)$ is shown below.




Sketch a possible graph for the original function, $y = f(x)$. **3**

**6.** Two functions are defined on suitable domains as

$$f(x) = \frac{x - 2p}{3} \quad \text{and} \quad g(x) = x^2 + p \quad \text{, where } p \text{ is a constant.}$$

(*a*) Show clearly that the composite function $g(f(x))$ can be expressed in the form

$$g(f(x)) = \tfrac{1}{9}\left(x^2 - 4px + 4p^2 + 9p\right)$$ **3**

(*b*) The equation $g(f(x)) = 4p^2$ has **no real** roots.

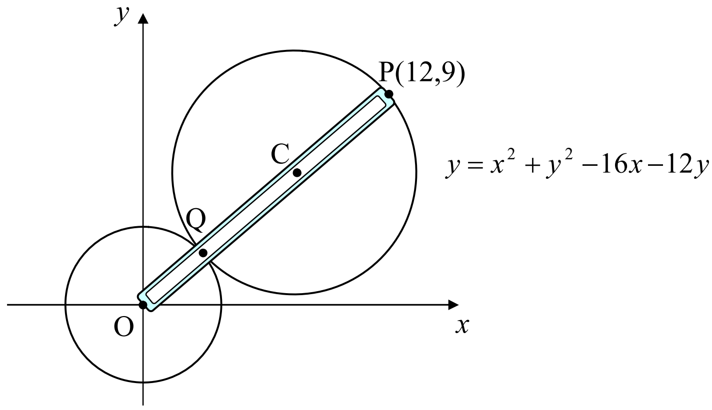
Use this information to find the range of values for $p$ which would allow this. **5**

**7.** The diagram below shows two circles locked together by a connecting rod OP.

The larger circle, centre C, has as its equation $y = x^2 + y^2 - 16x - 12y$ and has PQ as a diameter.

P is the point (12,9) as shown.

The smaller circle is centred on the origin and has OQ as a radius.




(a) Write down the coordinates of C, the centre of the larger circle. **1**

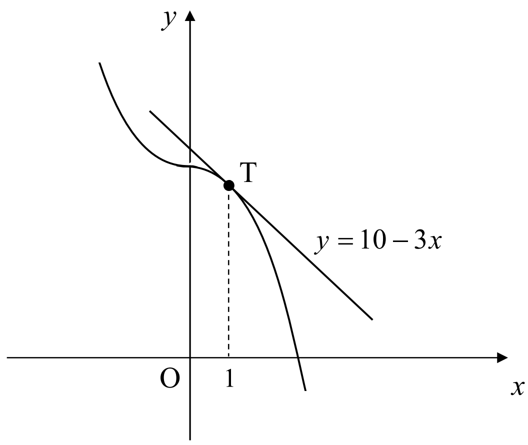
(b) Hence find the coordinates of Q. **1**

(c) Establish the equation of the smaller circle **2**

**8.** Part of the graph of a curve is shown opposite.

The diagram is not drawn to scale.

At the point where $x = 1$ the tangent to the curve has equation $y = 10 - 3x$.

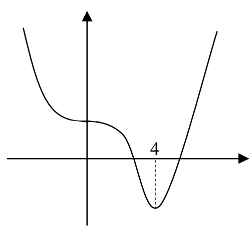



(a) Write down the coordinates of the point of tangency, T. **1**

(b) Given that the curve has as its derivative $\frac{dy}{dx} = -3x^2$, establish the equation of the curve. **4**

**9.** Solve the equation $3\cos 2\theta + 1 = 5\sin\theta$, for $0 < \theta < \pi$. **5**

[ END OF QUESTION PAPER ]

| | Give 1 mark for each • | Illustration(s) for awarding each mark |
|---|---|---|
| **1.** | **ans:** $a = \pm 2$ **5 marks**<br>•1 for expanding brackets<br>•2 for diff. first two terms (constant goes)<br>•3 for diff. 3rd term<br>•4 equating to -3<br>•5 solving to answer | •1 $f(a) = a - 8 + 16a^{-1}$ (or equiv.)<br>•2 $f'(a) = 1$ ..........<br>•3 $f'(a) =$ ............ $-16a^{-2}$ (or equiv.)<br>•4 $1 - \frac{16}{a^2} = -3$<br>•5 $a^2 - 16 = -3a^2 \Rightarrow a^2 = 4 \therefore a = \pm 2$ |
| **2.** | **(a) ans:** M(1,5) **1 mark**<br>•1 answer<br><br>**(b) ans:** $2y = 3x + 7$ **3 marks**<br>•1 for gradient of AC<br>•2 for perpendicular gradient<br>•3 for equation (any form)<br><br>**(c) ans:** D(-3,-1) **4 marks**<br>•1 for strategy (system of equ's)<br>•2 equation of AC<br>•3 finding first coordinate<br>•4 finding second coordinate | (a) •1 M(1,5)<br><br>(b) •1 $m_{AC} = \frac{-5-1}{3+6} = -\frac{2}{3}$<br>•2 $m_{MD} = \frac{3}{2}$<br>•3 $y - 5 = \frac{3}{2}(x-1)$<br><br>(c) •1 using a system<br>•2 $y - 1 = -\frac{2}{3}(x+6)$<br>•3 $y = -1$<br>•4 $2(-1) = 3x + 7 \therefore x = -3$ |
| **3.** | **(a) ans:** $U_0 = 50$ **2 marks**<br>•1 for setting up recurrence<br>•2 for calculating answer<br>**(b) ans:** diff. = 50 **3 marks**<br>•1 for knowing how to find a limit<br>•2 for calculating limit<br>•3 answer | (a) •1 $70 = 0\cdot6U_0 + 40$<br>•2 $U_0 = 30 \div 0\cdot6 = 50$<br><br>(b) •1 $L = \frac{b}{1-a}$ (or equivalent)<br>•2 $L = \frac{40}{1-0\cdot6} = 100$<br>•3 diff. = 100 - 50 = 50 |
| **4.** | **(a) ans:** $a = 14$ , $b = 24$ **5 marks**<br>•1 knowing to use synthetic division<br>•2 first equation<br>•3 second equation<br>•4 finding first constant<br>•5 finding 2nd constant<br><br>**(b) ans:** $x = 3$ **2 marks**<br>•1 substit. a & b in coefficient (or equiv.)<br>•2 solving to zero for 3rd root | (a) •1 2 \| 1 -1 -a b<br>•2 $b + 4 - 2a = 0$ (or equiv.)<br>•3 $b + 4a - 80 = 0$<br>•4 $b = 24$<br>•5 $24 - 2a = -4 \therefore a = 14$<br><br>(b) •1 $x^2 + x - 12 = 0$ (or equiv.)<br>•2 $(x+4)(x-3) = 0$ , $\therefore x = 3$ |
| **5.** | **ans:** see sketch opposite **3 marks**<br>•1 downward point of inflexion on $y$-axis (anywhere on y-axis including the origin)<br>•2 for minimum T.P.<br>•3 for 4 marked on $x$-axis @ min. T.P. | possible example …..   |

| | Give 1 mark for each • | Illustration(s) for awarding each mark |
|---|---|---|
| **6.** | **(a) ans:** proof **3 marks**<br>•1 set up composite function<br>•2 squaring out bracket (+ the 9)<br>•3 desired form<br>**(b) ans:** $0 < p < \frac{1}{4}$ **5 marks**<br>•1 for equating and simplifying<br>•2 for discriminant statement<br>•3 for $a$, $b$ and $c$<br>•4 substituting and solving (probably solving for roots)<br>•5 final statement | (a) •1 $g(f(x)) = \left(\frac{x-2p}{3}\right)^2 + p$<br>•2 $g(f(x)) = \frac{x^2-4px+4p^2}{9} + p$<br>•3 $g(f(x)) = \frac{1}{9}\left(x^2-4px+4p^2+9p\right)$<br>(b) •1 $\frac{1}{9}\left(x^2-4px+4p^2+9p\right) = 4p^2$<br>$x^2-4px+9p-32p^2 = 0$<br>•2 $b^2-4ac<0$ (stated or implied)<br>•3 $a=1,\ b=-4p,\ c=9p-32p^2$<br>•4 $36p(4p-1)=0\ \therefore\ p=0\ or\ p=\frac{1}{4}$<br>•5 answer between the roots since min. |
| **7.** | **(a) ans:** C(8,6) **1 mark**<br>•1 answer<br>**(b) ans:** Q(4,3) **1 mark**<br>•1 for answer<br>**(c) ans:** $x^2+y^2=25$ **2 marks**<br>•1 finding $r^2$ or $r$ by pyth.<br>•2 equation (centred on O) | (a) •1 C(8,6)<br>(b) •1 Q(4,3) …. various methods<br>(c) •1 $r^2 = 4^2+3^2 = 25$<br>•2 centred on the origin<br>$\therefore\ x^2+y^2=25$ |
| **8.** | **(a) ans:** T(1,7) **1 mark**<br>•1 for sub. to answer<br>**(b) ans:** $y = 8-x^3$ (or equiv.) **4 marks**<br>•1 for knowing to integrate<br>•2 for integrating correctly<br>•3 for subst. (1,7) to find C<br>•4 for answer | (a) •1 $y = 10-3(1)\ \therefore\ y=7$<br>(b) •1 $y = \int \frac{dy}{dx}.dx$ (stated or implied)<br>•2 $y = \frac{-3x^3}{3} + C$<br>•3 $7 = -(1)^3 + C$<br>•4 C = 8 leading to answer |
| **9.** | **ans:** $\theta = \frac{\pi}{6}\ or\ \frac{5\pi}{6}$ **5 marks**<br>•1 for double angle substitution<br>•2 solving to zero<br>•3 factorising and solving<br>•4 discarding one root (no solution)<br>•5 finding correct answers (no marks off if 30º and 150º) | •1 $3(1-2\sin^2\theta)+1 = 5\sin\theta$<br>•2 $6\sin^2\theta + 5\sin\theta - 4 = 0$<br>•3 $(3\sin\theta+4)(2\sin\theta-1)=0$<br>$\therefore\ \sin\theta = \frac{-4}{3}\ or\ \sin\theta = \frac{1}{2}$<br>•4 $\cancel{\sin\theta = \frac{-4}{3}}$ discarded no solution<br>•5 from $\sin\theta = \frac{1}{2}$<br>$\theta = \frac{\pi}{6}\ or\ \frac{5\pi}{6}$ |

**Total 50 marks**

*Prelim Examination 2003 / 2004*
*(Assessing Units 1 & 2)*

# MATHEMATICS
## Higher Grade - Paper II

**Time allowed - 1 hour 30 minutes**

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Read Carefully

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**FORMULAE LIST**

**Circle:**

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

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**Trigonometric formulae:**

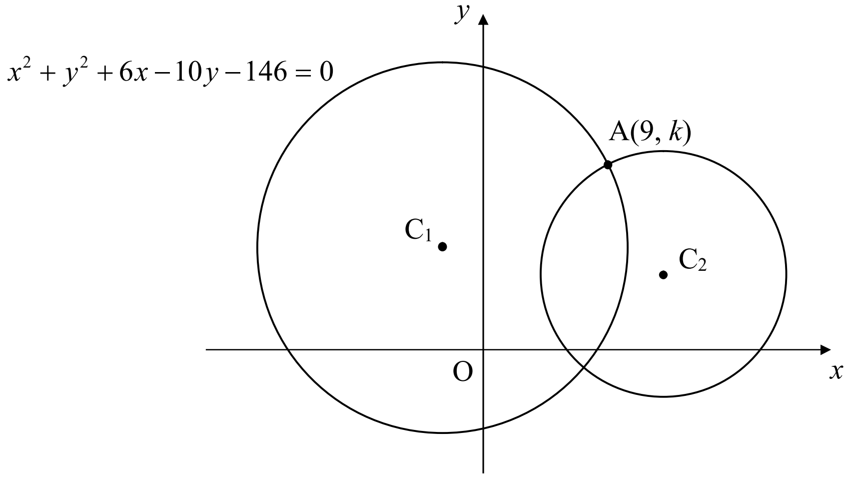
$$\begin{aligned}
\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\sin 2A &= 2 \sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

**All questions should be attempted**

**1.** Two intersecting circles are shown in the diagram below.

The circle, centre $C_1$, has $x^2 + y^2 + 6x - 10y - 146 = 0$ as its equation.

The point A(9, $k$) lies on the circumference of both circles.



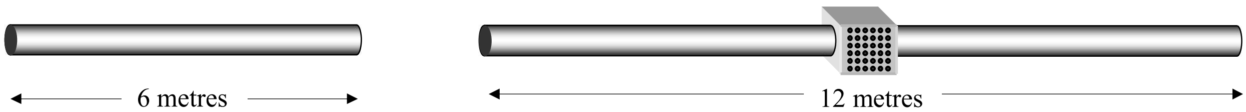

(a) Establish the value of $k$. **2**

(b) The second circle has the point $C_2(p, 3)$ as its centre.

Given that angle $C_1AC_2$ is a right-angle, find the value of $p$. **5**

(c) Hence find the equation of a third circle which passes through $C_1$, A and $C_2$. **4**

**2.** In a steam turbine the blades are rotated using superheated steam. Superheated steam has many advantages, one being its ability to travel long distances (through tubing) with minimal heat loss. One way of keeping the temperature of the superheated steam as constant as possible is to apply heat, through heated elements, at intervals along the tubing (see diagram).




(a) In a particular turbine superheated steam enters the tubes at a temperature of 1050°F. It is known that the steam loses 2% of its temperature for every metre of tubing travelled.

Calculate the expected temperature of the superheated steam as it leaves a plain 6 metre length of tubing. Give your answer correct to the nearest degree. **3**

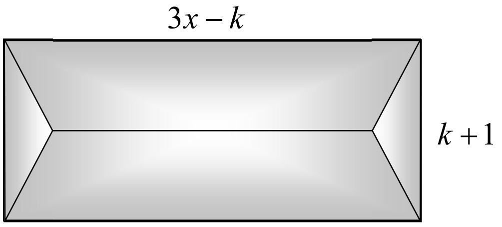
(b) Heating elements are placed every 6 metres but not at the beginning <u>or</u> the end of the tubing. Each of these elements increases the temperature of the steam passing over it by 60°F.

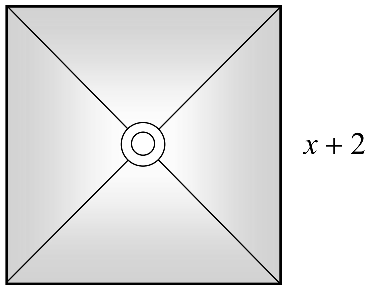
Calculate the temperature of the steam as it leaves a 30 metre section of this tubing. **3**

(c) With this system in place, calculate the approximate temperature of the steam leaving a tube of infinite length. **3**

**3.** A householder is considering two different designs for a conservatory.

One design has a rectangular base measuring $3x - k$ by $k + 1$ metres and the other design is square based with side $x + 2$ metres. Both $x$ and $k$ are constants.



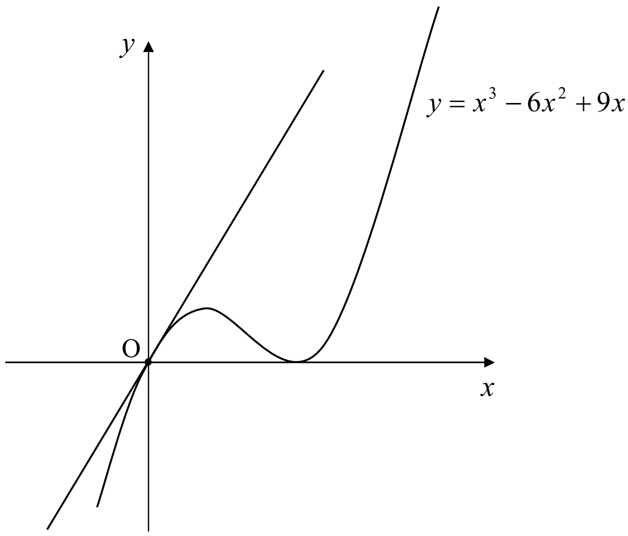




(a) With both designs having the same base **area**, show clearly that the following equation can be formed.

$$x^2 + (1 - 3k)x + (k^2 + k + 4) = 0$$ **3**

(b) Given that the above equation has **equal roots**, find first the value of $k$, and then the base area of each conservatory in square metres. **5**

**4.** The diagram, which is not drawn to scale, shows part of the graph of $y = x^3 - 6x^2 + 9x$. The tangent to the curve at the point where $x = 0$ is also drawn.




(a) Establish the equation of the tangent. **3**

(b) This tangent meets the curve at a second point P.

Find the coordinates of P. **4**

**5.** Two functions are defined on suitable domains as

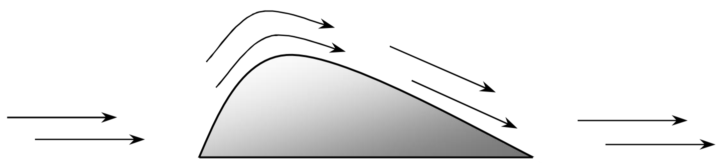
$$f(x) = 2\cos(x)^\circ + 2\sin(x)^\circ \quad \text{and} \quad g(x) = (x)^2.$$

(a) Show that the composite function $g(f(x))$ can be written in the form

$$g(f(x)) = 4(1 + \sin(2x)^\circ). \qquad \textbf{4}$$

(b) Hence solve the equation $g(f(x)) = \cos(x)^\circ + 4$ for $0 \le x < 360$. **4**

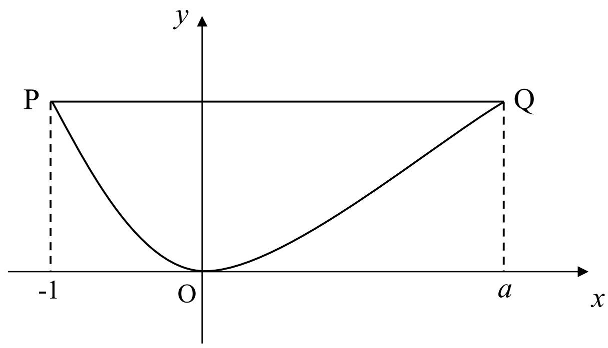
**6.** A designer is working on a new competition helmet for olympic ski racers. He has perfected the design to produce as little drag and as even a wind-flow over the helmet as possible.



Below is part of his computer aided design showing a flat cross-section of the helmet relative to a set of rectangular axis. The helmet has been rotated through 180º.

The curve PQ has as its equation $y = 3x^2 - x^3$. The line PQ is horizontal.

The $x$-coordinates of P and Q are -1 and $a$ respectively.



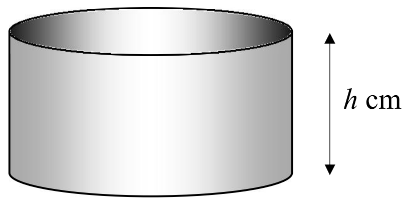

(a) **Show clearly** that the equation of the line PQ is $y = 4$. **1**

(b) Hence determine the value of $a$. **3**

(c) Calculate the **area** enclosed between the line PQ and the curve with equation $y = 3x^2 - x^3$.
Give your answer in square units. **4**

**7.** An cylindrical container, open at the top, has a volume of $64\pi$ cubic centimetres and a height of $h$ centimetres.

(a) Show that $h = \dfrac{64}{r^2}$. **1**




(b) If the radius of the container is $r$ cm, show that the total surface area, $A$, of the container, can be represented by the function

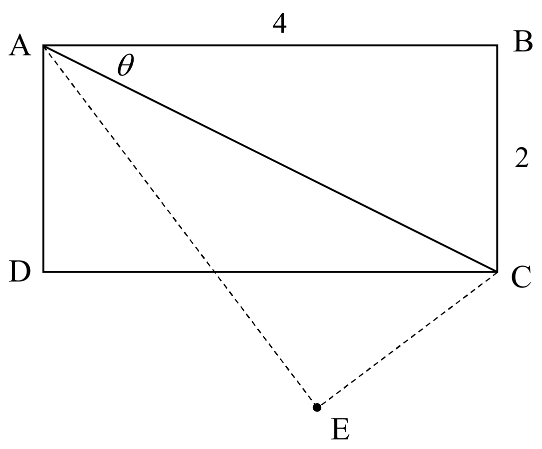
$$A(r) = \frac{128\pi}{r} + \pi r^2.$$ **2**

(c) Hence find the dimensions of the cylinder so that this surface area is a minimum. **5**

**8.** Rectangle ABCD measures 4 units by 2 units as shown. The diagram is not to scale.

Angle BAC = $\theta$ radians.

Point E is the reflected image of B with diagonal AC as the axis of symmetry.




(a) Show clearly that $\cos D\hat{A}E = \sin 2\theta$. **3**

(b) Hence calculate the **exact** value of $\cos D\hat{A}E$. **3**

[ END OF QUESTION PAPER ]

| | Give 1 mark for each • | Illustration(s) for awarding each mark |
|---|---|---|
| **1.** | **(a) ans:** $k = 11$ **2 marks**<br>•1 for substituting<br>•2 for solving and choosing answer | (a) •1 $9^2 + y^2 + 6(9) - 10y - 146 = 0$<br>•2 $(y-11)(y+1) = 0$<br>$y = 11$ *or* $y = -1$ $\therefore$ $k = 11$ |
| | **(b) ans:** $p = 13$ **5 marks**<br>•1 for gradient strategy<br>•2 for centre $C_1$<br>•3 for gradient of $C_1A$<br>•4 for gradient of $AC_2$<br>•5 for equating and answer | (b) •1 $m_1 \times m_2 = -1$ (stated or implied)<br>•2 from circle equat…. $C_1(-3,5)$<br>•3 $m_{C_1A} = \frac{11-5}{9+3} = \frac{1}{2}$<br>•4 $\therefore m_{AC_2} = -2$<br>•5 $\frac{3-11}{p-9} = -2$ , $\therefore$ $p = 13$ |
| | **(c) ans:** $(x-5)^2 + (y-4)^2 = 65$ **4 marks**<br>•1 for realising $C_1C_2$ is a diameter<br>•2 for mid-point of $C_1C_2$<br>•3 for the value of $r^2$<br>•4 for answer | (c) •1 strategy from right-angle<br>•2 centre is (5,4)<br>•3 $r^2 = 4^2 + 7^2 = 65$ (or equivalent)<br>•4 $(x-5)^2 + (y-4)^2 = 65$<br>(pupils may use other methods, mark at discretion) |
| **2.** | **(a) ans:** 930 °F **3 marks**<br>•1 for correct a<br>•2 for setting up calculation<br>•3 for answer (ignore rounding) | (a) •1 $a = (0.98)^6$<br>•2 $u_1 = (0.98)^6 \times 1050$<br>•3 $930 \cdot 13$ |
| | **(b) ans:** $\approx$ 751 °F **3 mark**<br>•1 for setting up recurrence (line 1)<br>•2 for working lines down to $u_5$<br>•3 for realising not to add 60 at final ans. | (b) •1 $u_1 = (0 \cdot 98)^6 \times 1050 = 930 + 60 = 990$<br>•2 $u_2 = 937$ , $u_3 = 890$ , $u_4 = 848$<br>•3 $u_5 = (0 \cdot 98)^6 \times 848 = 751$ |
| | **(c) ans:** $\approx$ 466 °F **3 marks**<br>•1 for knowing how to find the limit<br>•2 for calculating the limit<br>•3 for realising to subtract 60 to find ans. | (c) •1 $L = \frac{b}{1-a}$ (or equivalent)<br>•2 $L = \frac{60}{1-(0 \cdot 98)^6} \approx 526$<br>•3 $526 - 60 = 466$ |
| **3.** | **(a) ans:** proof **3 marks**<br>•1 for equating<br>•2 for expansions<br>•3 organising + common factor to ans. | (a) •1 $(x+2)^2 = (3x-k)(k+1)$<br>•2 $x^2 + 4x + 4 = 3kx - k^2 + 3x - k$<br>•3 $x^2 + (1-3k)x + (k^2 + k + 4) = 0$ |
| | **(b) ans:** $k = 3$ , Area = 36 $m^2$ **5 marks**<br>•1 for discriminant statement and $a$, $b$ & $c$<br>•2 for substitution and expansion<br>•3 for solving and choosing correct root<br>•4 for using $k$ in original equ. to find $x$<br>•5 for answer | (b) •1 $b^2 - 4ac = 0$ (stated or implied)<br>$a = 1, b = 1-3k, c = k^2 + k + 4$<br>(only 1 mark for above, mark given for $a$, $b$ & $c$ in PI)<br>•2 $(1-3k)^2 - 4(k^2 + k + 4) = 0$<br>$1 - 6k + 9k^2 - 4k^2 - 4k - 16 = 0$<br>•3 $5(k-3)(k+1) = 0$ , $\therefore$ $k = 3$ *or* $-1$<br>•4 $x^2 - 8x + 16 = 0$ , $\therefore x = 4$<br>•5 $A = (x+2)^2 = (4+2)^2 = 36$ (or equiv.) |

| | Give 1 mark for each • | Illustration(s) for awarding each mark |
|---|---|---|
| **4.** | **(a) ans:** $y = 9x$ **3 marks**<br>•1 for differentiating<br>•2 for substituting for gradient<br>•3 correct equation<br><br>**(b) ans:** P(6,54) **4 marks**<br>•1 strategy of a system<br>•2 combining and equating to zero<br>•3 for $x$- coordinate<br>•4 for $y$-coordinate | (a) •1 $\frac{dy}{dx} = 3x^2 - 12x + 9 = m$<br>•2 @ $x = 0$ , $m = 9$<br>•3 line through origin $\therefore y = 9x$<br><br>(b) •1 attempts to form a system of equat.s<br>•2 $x^3 - 6x^2 + 9x = 9x$<br>$x^3 - 6x^2 = 0$<br>•3 $x^2(x-6) = 0 \therefore \cancel{x=0}$ *or* $x = 6$<br>•4 $y = 9(6) = 54$ |
| **5.** | **(a) ans:** proof **4 marks**<br>•1 for attempting composite<br>•2 for expansion<br>•3 for realising 4<br>•4 for introducing double angle then ans.<br><br>**(b) ans:** $\{7\cdot2^\circ, 90^\circ, 172\cdot8^\circ, 270^\circ\}$ **4 marks**<br>•1 for equating and solving to zero<br>•2 double angle replacement<br>•3 factorising and finding roots<br>•4 answers | (a) •1 $g(2\cos x + 2\sin x) = (2\cos x + 2\sin x)^2$<br>•2 ..... $= 4\cos^2 x + 8\sin x\cos x + 4\sin^2 x$<br>•3 ...... $= 4 + 8\sin x\cos x$<br>•4 ...... $= 4 + 4(2\sin x\cos x)$<br>...... $= 4 + 4\sin 2x \Rightarrow 4(1+\sin 2x)$<br><br>(b) •1 $4 + 4\sin 2x = \cos x + 4$<br>$4\sin 2x - \cos x = 0$<br>•2 $4(2\sin x\cos x) - ........$<br>•3 $\cos x(8\sin x - 1) = 0$<br>$\cos x = 0$ *or* $\sin x = \frac{1}{8}$<br>•4 $90^\circ$ , $270^\circ$ *or* $7\cdot2^\circ$ , $172\cdot8^\circ$ |
| **6.** | **(a) ans:** proof **1 mark**<br>•1 for clear working to answer<br><br>**(b) ans:** $a = 2$ **3 marks**<br>•1 for knowing to solve equ. of curve to 4<br>•2 for arranging to zero and synth. division<br>•3 for finding other root<br><br>**(c) ans:** $6\frac{3}{4}$ units$^2$ **4 marks**<br>•1 for setting up integral<br>•2 for integrating correctly<br>•3 substituting limits of integration<br>•4 calculating answer | (a) •1 $y = 3(-1^2) - (-1^3) = 3 - (-1) = 4$<br>horizontal line $\therefore y = 4$<br><br>(b) •1 $3x^2 - x^3 = 4$<br>•2 $-x^3 + 3x^2 - 4 = 0$<br>$\begin{array}{c\|cccc} & -1 & 3 & 0 & -4 \\ \hline \end{array}$<br>•3 $\begin{array}{c\|cccc} 2 & -1 & 3 & 0 & -4 \\ & & -2 & 2 & 4 \\ \hline & -1 & 1 & 2 & 0 \end{array}$<br>*Some pupils may put -1 through synth. div and use the quotient to find the other root.*<br><br>(c) •1 $A = \int_{-1}^{2} 4 - \left[3x^2 - x^3\right] dx$<br>•2 $A = \left[4x - x^3 + \frac{x^4}{4}\right]_{-1}^{2}$<br>•3 $A = (8 - 8 + 4) - (-4 + 1 + \frac{1}{4})$<br>•4 $A = (4) - (-2\frac{3}{4}) = 6\frac{3}{4}$ |

| | Give 1 mark for each • | Illustration(s) for awarding each mark |
|---|---|---|
| **7.** | **(a) ans:** proof **1 mark**<br>•1 clear working to answer<br><br>**(b) ans:** proof **2 marks**<br>•1 for knowing how to find surface area<br>•2 substitution and manipulation to ans.<br><br>**(c) ans:** $r = 4$ cm , $h = 4$ cm **5 marks**<br><br>•1 knowing to differentiate and solve to zero<br>•2 differentiating correctly<br>•3 for dealing with fraction<br>•4 solving to find $r$<br>•5 finding $h$<br><br>*note: no justification of minimum necessary* | (a) •1 $64\pi = \pi r^2 h$<br>$64 = r^2 h \quad \Rightarrow \quad h = \frac{64}{r^2}$<br><br>(b) •1 $A = 2\pi rh + \pi r^2$<br>•2 $A = 2\pi r\frac{64}{r^2} + \pi r^2$<br>$= \frac{128\pi}{r} + \pi r^2$<br><br>(c) •1 @ min $A'(r) = 0$ (stated or implied)<br>•2 $A'(r) = -128\pi r^{-2} + 2\pi r$<br>•3 $2\pi r - \frac{128\pi}{r^2} = 0 \quad ....... \times r^2$<br>$2\pi r^3 - 128\pi = 0$<br>•4 $2r^3 = 128 \quad \Rightarrow \quad r^3 = 64 \ \therefore \ r = 4$<br>•5 $h = 64 \div 4^2 = 4$ |
| **8.** | **(a) ans:** proof **3 marks**<br><br>•1 for realising $\angle \text{ADE} = 90 - 2\theta$<br>•2 using compound angle replacement<br>•3 exact values to answer<br><br>**(b) ans:** $\frac{4}{5}$ (accept $\frac{16}{20}$ ) **3 marks**<br><br>•1 for using $\sin 2\theta$ and using replacement<br>•2 for calculating hypotenuse<br>•3 substituting exact values then answer | (a) •1 $2\theta + \angle ADE = 90$ (stated or implied)<br>•2 $\cos ADE = \cos(90 - 2\theta)$<br>$= \cos 90 \cos 2\theta + \sin 90 \sin 2\theta$<br>•3 $= (0) \times \cos 2\theta + (1) \times \sin 2\theta$<br>$= \sin 2\theta$<br><br>(b) •1 $\cos ADE = \sin 2\theta = 2\sin\theta\cos\theta$<br>•2 $AC = \sqrt{4^2 + 2^2} = \sqrt{20}$ (or equiv.)<br>•3 $\sin 2\theta = 2 \times \frac{2}{\sqrt{20}} \times \frac{4}{\sqrt{20}} = \frac{16}{20} = \frac{4}{5}$ |

**Total 65 marks**