NATIONAL QUALIFICATIONS

Mathematics
Higher Mini-Prelim Examination 2008/2009

Assessing Unit 3 + revision from Units 1 & 2

Time allowed - 1 hour 10 minutes

Read carefully

1. **Calculators may be used in this paper.**
2. Full credit will be given only where the solution contains appropriate working.
3. Answers obtained from readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Trigonometric formulae:

$$\begin{aligned}
\sin(A \pm B) &= \sin A\cos B \pm \cos A\sin B \\
\cos(A \pm B) &= \cos A\cos B \mp \sin A\sin B \\
\sin 2A &= 2\sin A\cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

Scalar Product: $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}|\cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$.

or

$$\boldsymbol{a}.\boldsymbol{b} = \boldsymbol{a}_1\boldsymbol{b}_1 + \boldsymbol{a}_2\boldsymbol{b}_2 + \boldsymbol{a}_3\boldsymbol{b}_3 \quad \text{where } \boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \text{ and } \boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

SECTION A

In this section the correct answer to each question is given by one of the alternatives **A**, **B**, **C** *or* **D**. *Indicate the correct answer by writing* **A**, **B**, **C** *or* **D** *opposite the number of the question on your answer paper.*
Rough working may be done on the paper provided. 2 marks will be given for each correct answer.

1. A is the point $(-4, 6, 5)$ and B is the point $(-1, 3, 2)$. The components of $\overrightarrow{AB}$ are

A $\begin{pmatrix} -3 \\ 3 \\ 3 \end{pmatrix}$ **B** $\begin{pmatrix} -5 \\ 9 \\ 7 \end{pmatrix}$

C $\begin{pmatrix} 3 \\ -3 \\ -3 \end{pmatrix}$ **D** $\begin{pmatrix} 5 \\ -9 \\ -7 \end{pmatrix}$

2. The gradient of the tangent to the curve $y = 3\sin 2x$ at the point where $x = \frac{\pi}{6}$ is

A $3\sqrt{3}$

B 3

C -3

D $-3\sqrt{3}$

3. The circle $x^2 + y^2 + 11x + 7y + 10 = 0$ cuts the x-axis at the points P and Q.

The length of PQ is

A 3

B 7

C 9

D 11

4. Given that C is a constant of integration, then $\int (4x+3)^{-\frac{1}{2}}\, dx$ equals

A $(4x+3)^{\frac{1}{2}} + C$

B $\frac{1}{2}(4x+3)^{\frac{1}{2}} + C$

C $\frac{1}{4}(4x+3)^{\frac{1}{2}} + C$

D $-2(4x+3)^{-\frac{3}{2}} + C$

5. The derivative of $(3-4x)^3$ with respect to x is

A $-\dfrac{(3-4x)^4}{16}$

B $\dfrac{(3-4x)^4}{4}$

C $-(3-4x)^4$

D $-12(3-4x)^2$

6. Vector $\boldsymbol{a}$ has components $\boldsymbol{a} = \begin{pmatrix} 3 \\ -2 \\ k \end{pmatrix}$.

If $|\boldsymbol{a}| = 4$, then the value of k is

A 3

B -1

C -13

D $\sqrt{3}$

7. Solve $\log_3 3x + \log_3 x = 3$, for x where $x > 0$.

A 1

B $\frac{27}{4}$

C 3

D $\frac{3}{4}$

8. The maximum value of $3\sin x - 4\cos x + 5$ is

A 10

B 0

C 4

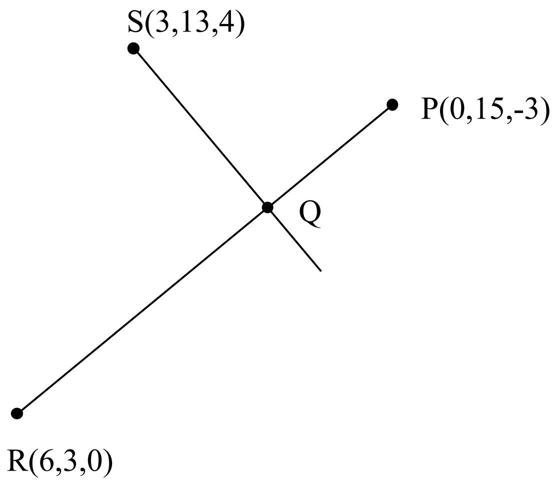
D -5

[END OF SECTION A]

SECTION B

ALL questions should be attempted

9. Consider the diagram below.



(a) Given that Q divides PR in the ratio $1:2$, find the coordinates of Q. **3**

(b) Hence prove that angle SQR is a right angle. **4**

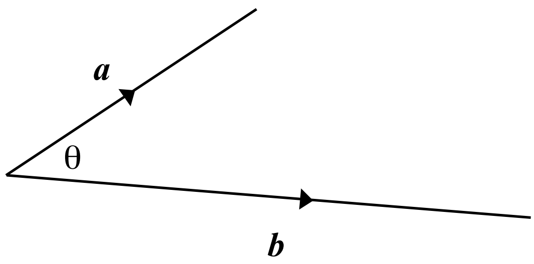
10. Evaluate $\int_0^1 \frac{6}{(3-2x)^2}\, dx$. **5**

11. Solve the equation $\sin x^\circ + 3\cos x^\circ = 2$ for $0 < x \leq 360$. **6**

12. Find the coordinates of the point on the curve $y = x^3 - x^2 - 4x + 2$ where the gradient of the tangent is 1 and $x < 0$. **4**

13. The diagram shows two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ where $\boldsymbol{a} = \begin{pmatrix} 4 \\ 0 \\ 2 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 6 \\ -3 \\ 0 \end{pmatrix}$.

The angle between the vectors is θ.



(a) Show clearly that $\cos\theta = \frac{4}{5}$. **3**

(b) Hence, or otherwise, find the exact value of $\cos 2\theta$. **2**

14. The mass of radium-226 remaining after a decay period of t years can be calculated using the formula

$M_t = M_0 e^{kt}$, where M_0 is the initial mass, M_t is the mass remaining after t years and k is a constant.

(a) Find the value of the constant k, given that a sample of radium-226 takes 500 years to **decay to 80%** of its initial mass.
Give your answer correct to 2 significant figures. **5**

(b) Hence calculate the approximate percentage mass remaining, of a sample of radium-226, after a period of 5 thousand years.
Give your answer correct to the nearest percent. **2**

[**END OF SECTION B**]

[**END OF QUESTION PAPER**]