NATIONAL
QUALIFICATIONS

Mathematics
Higher Prelim Examination 2009/2010
Paper 1
Assessing Units 1, 2 and 3

Time allowed - 1 hour 30 minutes

Read carefully

Calculators may NOT be used in this paper.

Section A - Questions 1 - 20 (40 marks)

Instructions for the completion of **Section A** are given on the next page.

For this section of the examination you should use an **HB pencil**.

Section B (30 marks)

1. Full credit will be given only where the solution contains appropriate working.
2. Answers obtained by readings from scale drawings will not receive any credit.

Read carefully

1 Check that the answer sheet provided is for **Mathematics Higher Prelim 2009/2010 (Section A)**.
2 For this section of the examination you must use an **HB pencil** and, where necessary, an eraser.
3 Make sure you write your **name**, **class** and **teacher** on the answer sheet provided.
4 The answer to each question is **either** A, B, C or D. Decide what your answer is, then, using your pencil, put a horizontal line in the space below your chosen letter (see the sample question below).
5 There is **only one correct** answer to each question.
6 Rough working should **not** be done on your answer sheet.
7 Make sure at the end of the exam that you hand in your answer sheet for Section A with the rest of your written answers.

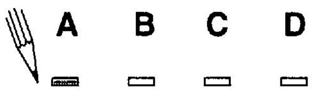
Sample Question

A line has equation $y = 4x - 1$.

If the point $(k,7)$ lies on this line, the value of k is

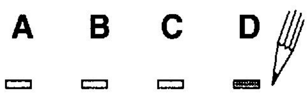
A 2
B 27
C $1{\cdot}5$
D -2

The correct answer is **A** $\rightarrow$ 2. The answer **A** should then be clearly marked in pencil with a horizontal line (see below).



Changing an answer

If you decide to change an answer, carefully erase your first answer and using your pencil, fill in the answer you want. The answer below has been changed to **D**.



FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Trigonometric formulae:

$$\begin{aligned}
\sin(A \pm B) &= \sin A\cos B \pm \cos A\sin B \\
\cos(A \pm B) &= \cos A\cos B \mp \sin A\sin B \\
\sin 2A &= 2\sin A\cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

Scalar Product: $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}|\cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$.

or

$\boldsymbol{a}.\boldsymbol{b} = \boldsymbol{a}_1\boldsymbol{b}_1 + \boldsymbol{a}_2\boldsymbol{b}_2 + \boldsymbol{a}_3\boldsymbol{b}_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

SECTION A
ALL questions should be attempted

1. A straight line with gradient 2 passes through the points (–2,3) and (–6, *k*).
The value of *k* is

A 11

B -5

C 1

D -13

2. Which of the following gives **equal roots** for the equation $x^2 + kx + 8 = 0$

A $k = 8\sqrt{2}$

B $k = 32$

C $k = 6$

D $k = 4\sqrt{2}$

3. The **rate of change** of the function $f(x) = \dfrac{4}{(x-1)^2}$ when $x = 3$ is

A -4

B 1

C -2

D -1

4. Given that $\cos A = \frac{\sqrt{2}}{5}$, the exact value of $\cos 2A$ is

A $\frac{2\sqrt{2}}{5}$

B $-\frac{17}{25}$

C $-\frac{21}{25}$

D cannot be found

5. $\displaystyle\int_0^{2a} 4x\, dx$ is

A 4

B $16a^2$

C $8a^2$

D $4a^2$

6. The graph of $y = \frac{1}{2}\log_{10} x$ crosses the x-axis at the point

A $(2, 0)$

B $(1, 0)$

C $(10, 0)$

D $(20, 0)$

7. If $f(x) = 3x$ and $g(x) = x^2 - 6x$ then $g(f(1))$ is equal to

A -15

B -9

C 0

D 3

8. **Three** of the following are equal in value. Which is the exception?

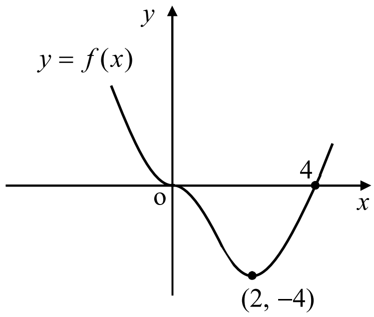
A $\cos\frac{\pi}{3}$

B $\sin\frac{5\pi}{6}$

C $\cos\frac{4\pi}{3}$

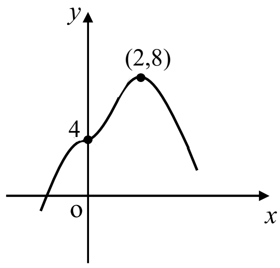
D $-\sin\frac{11\pi}{6}$

9. Part of the graph of the function $y = f(x)$ is shown opposite.

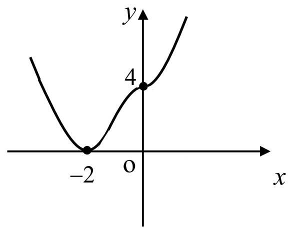


Which of the following graphs represents the related function $y = f(-x) + 4$?

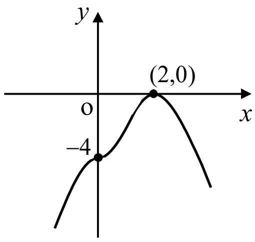
A



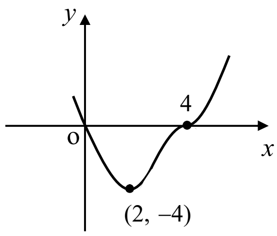
B



C



D



10. If $\boldsymbol{a}$ and $\boldsymbol{b}$ are two **unit vectors** which are perpendicular to each other then $\boldsymbol{a} \cdot (2\boldsymbol{a} + 3\boldsymbol{b})$ is equal to

A 2

B 5

C 0

D 3

11. A recurrence relationship is defined as $U_{n+1} = 0 \cdot 6U_n + b$.

If the limit of the recurrence relationship is 20, the value of b is

A 12

B 50

C $33\frac{1}{3}$

D 8

12. The minimum turning value of the function $f(x) = x^2 - 4x + 8$ is

A 4

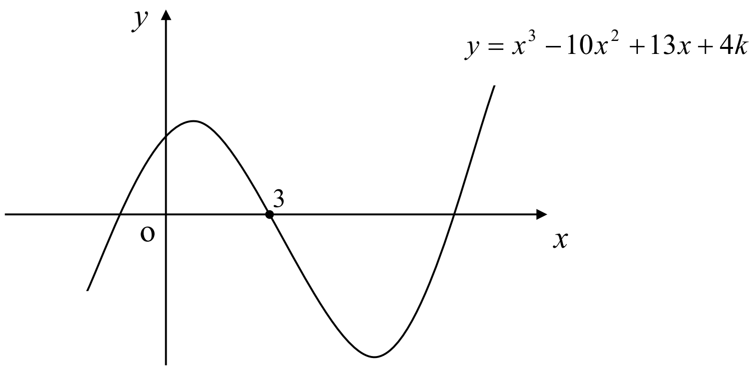
B 2

C 8

D -4

13. The diagram below shows part of the curve with equation $y = x^3 - 10x^2 + 13x + 4k$.

The curve crosses the x-axis at (3,0).



The value of k is

A 12

B 6

C 0

D -6

14. If $a^{\frac{1}{2}} = \sqrt{8}\, b^{\frac{3}{2}}$ then b equals

A $\dfrac{a^{\frac{1}{3}}}{\sqrt{8}}$

B $\dfrac{a^{\frac{1}{3}}}{2}$

C $\dfrac{a^{\frac{3}{4}}}{\sqrt{8}}$

D $\dfrac{a^{\frac{1}{3}}}{8}$

15. The equation of the circle, centre $(-5,6)$ and having the x-axis as a tangent is

A $(x+5)^2+(y-6)^2=25$

B $(x-5)^2+(y+6)^2=25$

C $(x+5)^2+(y-6)^2=36$

D $(x-5)^2+(y+6)^2=36$

16. $\sin 3x^\circ \sin 2x^\circ - \cos 3x^\circ \cos 2x^\circ$ equals

A $\sin x^\circ$

B $-\cos x^\circ$

C $\cos 5x^\circ$

D $-\cos 5x^\circ$

17. The maximum value of $\sqrt{5}\cos x^\circ + 2\sin x^\circ$ is

A 3

B 9

C 7

D $\sqrt{5}+2$

18. P, Q and R are points such that $\overrightarrow{PQ}=\begin{pmatrix}2\\0\\1\end{pmatrix}$, $\overrightarrow{PR}=\begin{pmatrix}1\\1\\2\end{pmatrix}$ and R is $(0,2,1)$.

The coordinates of Q are

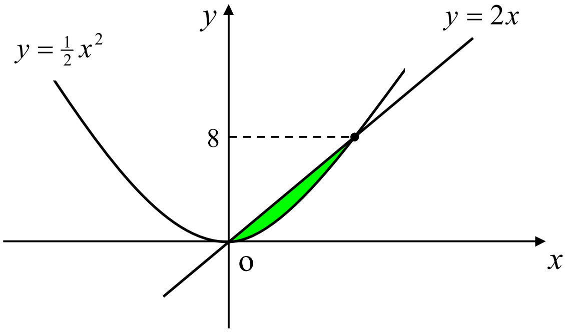
A $(-3,1,-2)$

B $(-1,-1,0)$

C $(1,-3,-2)$

D $(1,1,0)$

19. The diagram, **which is not drawn to scale**, shows parts of the graphs of the curve $y = \frac{1}{2}x^2$ and the line $y = 2x$.



The shaded area above can be represented by

A $\displaystyle\int_0^8 \left(2x - \tfrac{1}{2}x^2\right) dx$

B $\displaystyle\int_0^4 \left(\tfrac{1}{2}x^2 - 2x\right) dx$

C $\displaystyle\int_0^4 \left(2x - \tfrac{1}{2}x^2\right) dx$

D $\displaystyle\int_0^8 \left(\tfrac{1}{2}x^2 + 2x\right) dx$

20. Given that $\sin x = a^2 - 1$ and $\cos x = a - 1$ then $\tan x$ equals

A a

B $a + 1$

C $a^3 - a^2 - a + 1$

D $\dfrac{1}{a+1}$

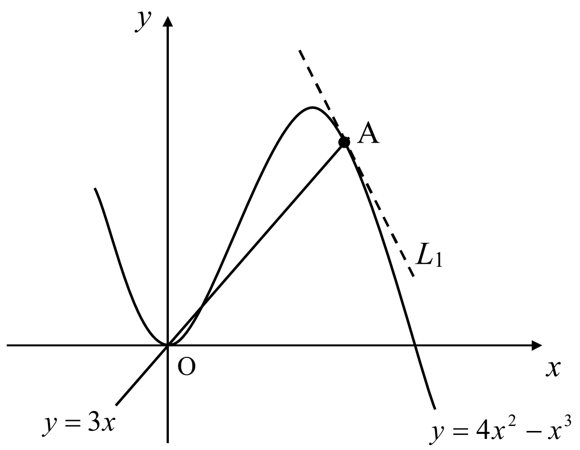
[END OF SECTION A]

SECTION B

ALL questions should be attempted

21. The diagram below, **which is not drawn to scale**, shows part of the graph of the curve $y = 4x^2 - x^3$ and the straight line $y = 3x$.

The line intersects the curve at three points.



(a) Find the coordinates of the point A. **4**

(b) The line L_1 is the tangent to the curve at A.

Establish the equation of the line L_1. **4**

22. Angle x is acute and is such that $\cos x = \frac{\sqrt{2}}{10}$.

(a) Show clearly that the exact value of $\sin x$ is $\frac{7\sqrt{2}}{10}$. **3**

(b) Hence find the value of the constant of integration, C, if

$$\int \left(2\cos 2x\right) dx = 0 \cdot 5$$

and $\sin x$ and $\cos x$ take the above values. **4**

23. (a) Express $\log_2 a - \log_2 b = 3$ in the form $a = kb$ and state the value of k. **3**

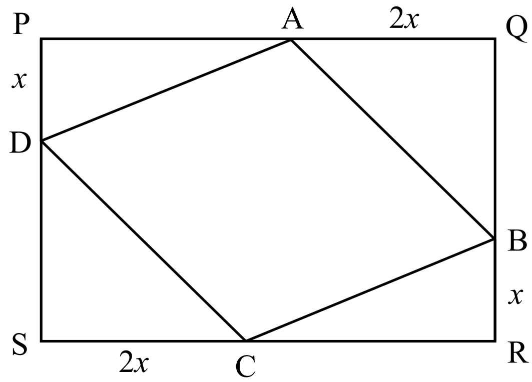
(b) When a and b take the same values as in part (a) a second equation can be written as

$$\log_2 a + \log_2 b = 1.$$

Find the values of a and b where $a > 0$ and $b > 0$. **3**

24. PQRS is a rectangle **measuring 6 units by 4 units**.

Points A, B, C and D are points on the sides of the rectangle such that AQ = SC = $2x$ and PD = BR = x as shown.



(a) Show that the area of ABCD is given by the function $A(x) = 4x^2 - 14x + 24$. **4**

(b) Hence find the value of x which minimises the area of ABCD and calculate this minimum area. **5**

[**END OF SECTION B**]

[**END OF QUESTION PAPER**]

Mathematics

Higher Prelim Examination 2009/2010

Paper 2

Assessing Units 1, 2 and 3

NATIONAL QUALIFICATIONS

Time allowed - 1 hour 10 minutes

Read carefully

1. **Calculators may be used in this paper.**
2. Full credit will be given only where the solution contains appropriate working.
3. Answers obtained from readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Trigonometric formulae:

$$
\begin{aligned}
\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\sin 2A &= 2 \sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}
$$

Scalar Product: $\boldsymbol{a} . \boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}| \cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$.

or

$$\boldsymbol{a} . \boldsymbol{b} = \boldsymbol{a}_1\boldsymbol{b}_1 + \boldsymbol{a}_2\boldsymbol{b}_2 + \boldsymbol{a}_3\boldsymbol{b}_3 \quad \text{where } \boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \text{ and } \boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Table of standard derivatives:

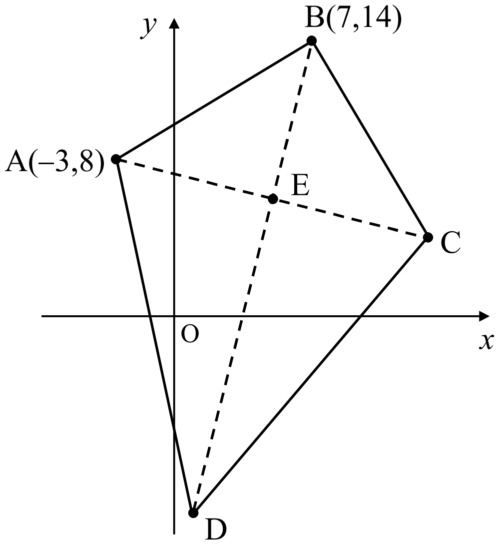
$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

ALL questions should be attempted

1. Kite ABCD has two of its vertices at A(–3,8) and B(7,14) as shown.



(a) Given that the equation of the longer diagonal BD is $y = 4x - 14$, find the equation of the short diagonal AC expressing your answer in the form $ax + by + c = 0$ and write down the values of a, b and c. **4**

(b) Find the coordinates of E, the point of intersection of the two diagonals. **3**

(c) Hence establish the coordinates of C. **2**

2. Two functions are defined on suitable domains as $f(x) = 4x + 1$ and $g(x) = \dfrac{1}{x-1}$.

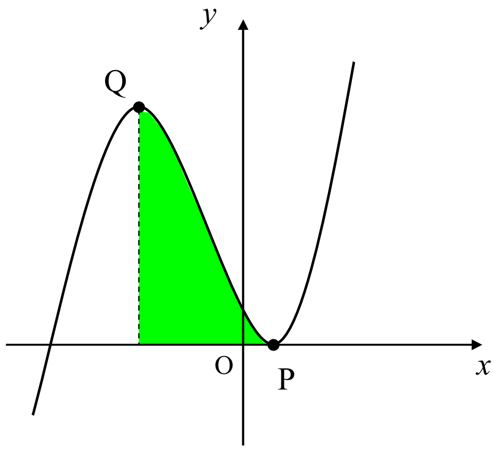
(a) If $h(x) = f(g(x))$, show clearly that $h(x)$ can be written as

$$h(x) = \frac{x+3}{x-1}.$$ **3**

(b) Show that value of $h(\sqrt{5})$ can be expressed in the form $p + \sqrt{5}$ and write down the value of p. **4**

3. A function is defined on the set of real numbers as $f(x) = 2x^3 + 3x^2 - 12x + 7$.

Part of the graph of $y = f(x)$ is shown below.



(a) Find the coordinates of the stationary points P and Q. **5**

(b) Calculate the shaded area in the diagram. **4**

4. Solve algebraically the equation

$$15\sin 2x^\circ = 10\cos x^\circ \qquad \text{for} \qquad 0 \leq x < 360\,.$$ **5**

5. A and B are the points $(-1, 4, -2)$ and $(7, -12, 30)$ respectively.

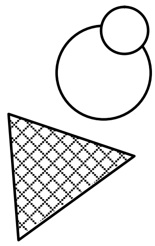
(a) Find the coordinates of the point C given that $\dfrac{AC}{CB} = \dfrac{3}{5}$. **4**

(b) A fourth point D is such that angle ACD $= 90^\circ$.

If D has coordinates $(6, k, 8)$, find the value of k. **4**

6. An ice-cream manufacturer has decided on a new logo for her company.

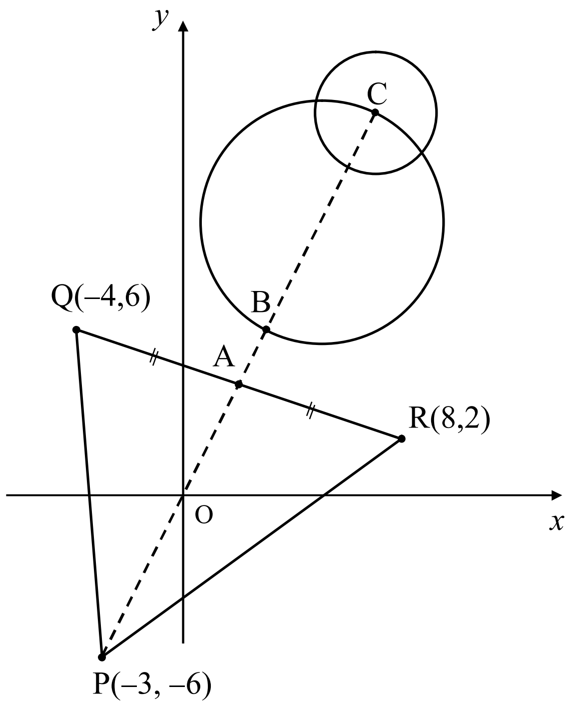
It consists of a triangle and two circles representing a wafer cone and two balls of ice cream.



Placed on a set of rectangular axes the logo is modelled in the diagram below.

The triangle has coordinates P(–3, –6), Q(–4,6) and R(8,2).

A is the midpoint of QR.



(a) Find the equation of PA **3**

(b) When PA is extended it intersects with the larger circle at B and C.

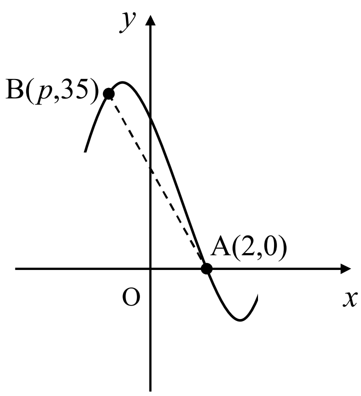
If the larger circle has as its equation $x^2 + y^2 - 10x - 20y + 105 = 0$, find the coordinates of C. **4**

(c) Given that C is the centre of the smaller circle and that its radius is **exactly half** of the larger circle, find the equation of the smaller circle. **3**

7. A curve has as its equation $y = x^3 - kx^2 - 16x + 32$.

Part of the graph of this curve is shown below.

The diagram is not drawn to scale.

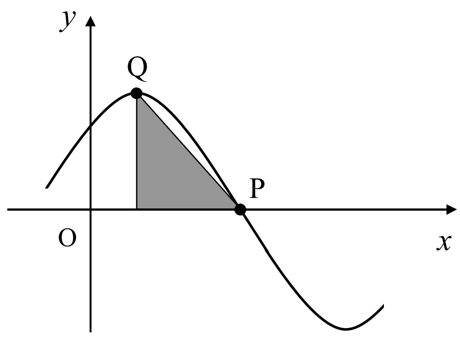


(a) If the curve crosses the x-axis at A(2,0), find k. **3**

(b) The point B(p,35) also lies on this curve, find the value of p. **3**

8. A function is defined on a suitable domain as $f(x) = \sqrt{8}\cos x + \sqrt{8}\sin x$.

Part of the graph of $y = f(x)$ is shown below.



(a) The function can be expressed in the form $f(x) = 4\cos(x - a)$, where $0 < a < \frac{\pi}{2}$.

Find the value of a and hence state the coordinates of the point P. **3**

(b) Show clearly that the exact area of the shaded right-angled triangle is π square units. **3**

[**END OF QUESTION PAPER**]

Mathematics
Higher Prelim Examination 2009/2010

NATIONAL
QUALIFICATIONS

Paper 1 - Section A - Answer Sheet

NAME :	CLASS :

TEACHER :

You should use an HB pencil.
Erase all incorrect answers thoroughly.

Indicate your choice of answer with a single mark as in this example →

A	B	C	D
▭	▬	▭	▭

	A	B	C	D
1	▭	▭	▭	▭
2	▭	▭	▭	▭
3	▭	▭	▭	▭
4	▭	▭	▭	▭
5	▭	▭	▭	▭
6	▭	▭	▭	▭
7	▭	▭	▭	▭
8	▭	▭	▭	▭
9	▭	▭	▭	▭
10	▭	▭	▭	▭
11	▭	▭	▭	▭
12	▭	▭	▭	▭
13	▭	▭	▭	▭
14	▭	▭	▭	▭
15	▭	▭	▭	▭
16	▭	▭	▭	▭
17	▭	▭	▭	▭
18	▭	▭	▭	▭
19	▭	▭	▭	▭
20	▭	▭	▭	▭

Section A	/ 40	
Section B	/ 30	
Total (P1)	/ 70	
Total (P2)	/ 60	
Overall Total	/ 130	%

Please make sure you have filled in all your details above before handing in this answer sheet.

Higher Grade - Paper 1 2009/2010 ANSWERS - Section A

1	B
2	D
3	D
4	C
5	C
6	B
7	B
8	C
9	B
10	A
11	D
12	A
13	B
14	B
15	C
16	D
17	A
18	D
19	C
20	B

	A	B	C	D
1	□	■	□	□
2	□	□	□	■
3	□	□	□	■
4	□	□	■	□
5	□	□	■	□
6	□	■	□	□
7	□	■	□	□
8	□	□	■	□
9	□	■	□	□
10	■	□	□	□
11	□	□	□	■
12	■	□	□	□
13	□	■	□	□
14	□	■	□	□
15	□	□	■	□
16	□	□	□	■
17	■	□	□	□
18	□	□	□	■
19	□	□	■	□
20	□	■	□	□

	Give 1 mark for each •	Illustration(s) for awarding each mark
21(a)	**ans: A(3, 9)** **(4 marks)**	
	•¹ equates 2 equations and collects to LHS	•¹ $4x^2 - x^3 = 3x; 4x^2 - x^3 + 3x = 0$
	•² factorises	•² $x(x-3)(x-1) = 0$
	•³ solves for x chooses appropriate value	•³ $x = 0, 1, 3; x = 3$
	•⁴ substitutes and states point A	•⁴ $y = 3 \times 3 = 9$; A(3, 9)
(b)	**ans: $y + 3x = 18$** **(4 marks)**	
	•¹ know to take derivative	•¹ $\frac{dy}{dx} = 8x - 3x^2$
	•² knows to substitute	•² $8(3) - 3(3)^2$
	•³ evaluates to find gradient	•³ $m = -3$
	•⁴ substitutes into $y - b = m(x-a)$	•⁴ $y - 9 = -3(x-3)$
22(a)	**ans: proof** **(3 marks)**	
	•¹ uses right triangle and finds third side	•¹ triangle with sides $\sqrt{2}$, 10 and $\sqrt{98}$
	•² writes ratio of sin	•² $\sin x = \frac{\sqrt{98}}{10}$
	•³ shows simplification to answer	•³ $\sqrt{98} = \sqrt{49} \times \sqrt{2} = 7\sqrt{2}$
	OR	
	•¹ knows to sub in appropriate formula	•¹ $\sin^2 x = 1 - \cos^2 x$
	•² subs in formula and starts to simplify	•² $\sin^2 x = 1 - \left(\frac{\sqrt{2}}{10}\right)^2 = 1 - \frac{2}{10} = \frac{98}{100}$
	•³ shows simplification to answer	•³ $\sin x = \sqrt{\frac{98}{100}} = \frac{\sqrt{98}}{10} = \frac{7\sqrt{2}}{10}$
(b)	**ans: $C = 0 \cdot 22$** **(4 marks)**	
	•¹ integrates correctly	•¹ $\int (2\cos 2x)\, dx = \sin 2x + C$
	•² replacement (equates)	•² $2\sin x \cos x + C$ .....
	•³ substitutes values and equates	•³ $2\left(\frac{7\sqrt{2}}{10}\right)\left(\frac{\sqrt{2}}{10}\right) + C = 0 \cdot 5$
	•⁴ calculation to answer	•⁴ $\frac{28}{100} + C = 0 \cdot 5 \therefore C = 0 \cdot 5 - 0 \cdot 28 = 0 \cdot 22$

	Give 1 mark for each •	Illustration(s) for awarding each mark
23(a)	**ans: $a = 8b \;\therefore\; k = 8$** **(3 marks)**	
	•[1] writes a a single log	•[1] $\log_2\left(\frac{a}{b}\right) = 3$
	•[2] changes to index form	•[2] $\frac{a}{b} = 2^3 = 8$
	•[3] answer	•[3] $a = 8b \;\therefore\; k = 8$
(b)	**ans: $a = 4$, $b = \frac{1}{2}$** **(3 marks)**	
	•[1] single log + index form	•[1] $\log_2 ab = 1 \;\therefore\; ab = 2$
	•[2] first value	•[2] $a = 8(\frac{2}{a}) \Rightarrow a^2 = 16 \;\therefore\; a = 4$ (or equiv.)
	•[3] second value	•[3] $4(a) = 2 \;\therefore\; b = \frac{1}{2}$ *(pupils may solve simult. by elimination … award marks accordingly)*
24(a)	**ans: proof** **(4 marks)**	
	•[1] finds expressions for missing dimensions	•[1] $(6-2x)$ and $(4-x)$
	•[2] finds area of triangles	•[2] $x(6-2x)$ and $2x(4-x)$
	•[3] subtracts from area of rectangle	•[3] $24-(6x-2x^2+8x-2x^2)$
	•[4] simplifies to answer	•[4] $24-6x+2x^2-8x+2x^2$
(b)	**ans: $\frac{7}{4}$; $11\frac{3}{4}$** **(5 marks)**	
	•[1] knows to make derivative equal to 0	•[1] $\frac{dy}{dx} = 0$
	•[2] finds derivative a	•[2] $\frac{dy}{dx} = 8x - 14 = 0$
	•[3] solves for x and justifies	•[3] $x = \frac{7}{4}$; table of values or second derivative
	•[4] subs value to find area	•[4] $a = 4\left(\frac{7}{2}\right)^2 - 14\left(\frac{7}{4}\right) + 24$
	•[5] answer	•[5] $11\frac{3}{4}$
		Total: 70 marks

	Give 1 mark for each •	Illustration(s) for awarding each mark
1(a)	**ans: $a = 1; b = 4, c = -29$ (4 marks)**	
	•[1] finds gradient of BD	•[1] $m_{BD} = 4$ [from equation]
	•[2] finds gradient of AC	•[2] $m_{AC} = -\frac{1}{4}$
	•[3] subs into $y - b = m(x - a)$ and rearranges	•[3] $y - 8 = -\frac{1}{4}(x + 3); x + 4y - 29 = 0$
	•[4] states values of a, b and c	•[4] $a = 1; b = 4, c = -29$
(b)	**ans: E(5, 6) (3 marks)**	
	•[1] knows to use system of equations	•[1] evidence of equating one variable
	•[2] solves for x and y	•[2] $x = 5; y = 6$
	•[3] states coordinates of E	•[3] E(5, 6)
(c)	**ans: C(13, 4) (2 marks)**	
	•[1] appropriate method	•[1] evidence of 'stepping out' or other method
	•[2] answer	•[2] C(13, 4)
2(a)	**ans: proof (3 marks)**	
	•[1] knows to substitute	•[1] evidence of sub. one function in other
	•[2] substitutes correctly	•[2] $f\left(\frac{1}{x-1}\right) = \frac{4}{x-1} + 1$
	•[3] clearly simplifies to answer	•[3] $\frac{4 + x - 1}{x - 1} = \frac{x + 3}{x - 1}$
(b)	**ans: $p = 2$ (4 marks)**	
	•[1] substitute for x	•[1] $\frac{\sqrt{5} + 3}{\sqrt{5} - 1}$
	•[2] knows to multiply by conjugate surd	•[2] $\frac{\sqrt{5} + 3}{\sqrt{5} - 1} \times \frac{\sqrt{5} + 1}{\sqrt{5} + 1}$
	•[3] multiplies and simplifies	•[3] $\frac{5 + 4\sqrt{5} + 3}{4} = \frac{8 + 4\sqrt{5}}{4} = 2 + \sqrt{5}$
	•[4] states value of p	•[4] $p = 2$

	Give 1 mark for each •	Illustration(s) for awarding each mark
3(a)	**ans: P(1, 0); Q(– 2, 27) (5 marks)**	
	•1 knows derivative = 0 at S.P.	•1 $f\ '(x) = 0$ at SP [stated or implied]
	•2 takes derivative and factorises	•2 $6x^2 + 6x - 12 = 0$; $6(x+2)(x-1) = 0$
	•3 solves for x and chooses appropriate value	•3 $x = -2$ or 1
	•4 substitutes to find y – coordinate	•4 $f(-2) = 2(-2)^3 + 3(-2)^2 - 12(-2) + 7 = 27$
	•5 states coordinates of P and Q	•5 P(1, 0); Q(– 2, 27)
(b)	**ans: 40·5 units^2 (4 marks)**	
	•1 sets up integral	•1 $\int_{-2}^{1} 2x^3 + 3x^2 - 12x + 7\ dx$
	•2 integrates expression	•2 $\left[\frac{x^4}{2} + x^3 - 6x^2 + 7x\right]_{-2}^{1}$
	•3 substitutes values	•3 $\left(\frac{(1)^4}{2} + (1)^3 - 6(1)^2 + 7(1)\right) - \left(\frac{(-2)^4}{2} + (-2)^3 - 6(-2)^2 + 7(-2)\right)$
	•4 evaluates	•4 40·5 units^2
4	**ans: 19·5°, 90°, 160·5°, 270° (5 marks)**	
	•1 subs for sin $2x$°	•1 $15(2\sin x°\cos x°)$.......
	•2 multiplies and simplifies	•2 $30\sin x°\cos x° - 10\cos x° = 0$
	•3 factorises	•3 $10\cos x°(3\sin x° - 1) = 0$
	•4 finds two solutions	•4 $\sin x° = \frac{1}{3}$; $x = 19\cdot5°, 160\cdot5°$
	•5 finds further two solutions	•5 $\cos x° = 0$; $x = 90°, 270°$

	Give 1 mark for each •	Illustration(s) for awarding each mark
5(a)	**ans: C(2, – 2, 10)** **(4 marks)**	
	•¹ for strategy	•¹ $5\overrightarrow{AC} = 3\overrightarrow{CB}$ (or otherwise)
	•² for vector algebra and simplifying	•² $5(\boldsymbol{c}-\boldsymbol{a}) = 3(\boldsymbol{b}-\boldsymbol{c}) \Rightarrow 8\boldsymbol{c} = 3\boldsymbol{b}+5\boldsymbol{a}$
	•³ substitution	•³ $8\boldsymbol{c} = 3\begin{pmatrix}7\\-12\\30\end{pmatrix}+5\begin{pmatrix}-1\\4\\-2\end{pmatrix}$
	•⁴ simplifying and answer	•⁴ $8\boldsymbol{c} = \begin{pmatrix}16\\-16\\80\end{pmatrix} \therefore \boldsymbol{c} = \begin{pmatrix}2\\-2\\10\end{pmatrix} \Rightarrow C(2,-2,10)$
(b)	**ans: $k = -4$** **(4 marks)**	
	•¹ strategy	•¹ If angle ACD = 90º then $\overrightarrow{CA} \cdot \overrightarrow{CD} = 0$
	•² for both vectors	•² $\overrightarrow{CA} = \begin{pmatrix}-3\\6\\-12\end{pmatrix}$, $\overrightarrow{CD} = \begin{pmatrix}4\\k+2\\-2\end{pmatrix}$
	•³ scalar product to zero	•³ $-12+(6k+12)+24=0$
	•⁴ answer	•⁴ $6k=-24 \;\therefore\; k=-4$
6(a)	**ans: $y = 2x$** **(3 marks)**	
	•¹ finds midpoint of QR	•¹ midpoint of QR = (2, 4)
	•² finds gradient of PA	•² $m_{PA} = \dfrac{4+6}{2+3} = 2$
	•³ substitutes in $y-b=m(x-a)$	•³ $y-4=2(x-2)$; $y=2x$
(b)	**ans: C(7, 14)** **(4 marks)**	
	•¹ knows to substitute line into circle	•¹ $x^2+(2x)^2-10x-20(2x)+105=0$
	•² multiplies and simplifies	•² $5x^2-50x+105=0$
	•³ factorises and solves	•³ $5(x-3)(x-7)=0$
	•⁴ chooses appropriate value for x and subs	•⁴ $x=3,7;\; x=7,\; y=14$
(c)	**ans: $(x-7)^2+(y-14)^2=5$** **(3 marks)**	
	•¹ finds radius of larger circle	•¹ radius (large) = $\sqrt{25+100-105} = \sqrt{20}$
	•² finds radius of smaller circle	•² radius (small) = $\sqrt{5}$
	•³ subs into $(x-a)^2+(y-b)^2=r^2$	•³ $(x-7)^2+(y-14)^2=5$

	Give 1 mark for each •	Illustration(s) for awarding each mark
7(a)	**ans:** $k = 2$ **(3 marks)**	
	•[1] knows to use synthetic division •[2] makes remainder = 0 •[3] solves for k	•[1] evidence •[2] $8 - 4k = 0$ •[3] $k = 2$
(b)	**ans:** $p = -3$ **(3 marks)**	
	•[1] equates function to 35 •[2] collect terms to LHS and equates to 0 •[3] uses synthetic division to find root	•[1] $p^3 - 2p^2 - 16p + 32 = 35$ •[2] $p^3 - 2p^2 - 16p - 3 = 0$ •[3] $p = -3$
8(a)	**ans:** $a = \frac{\pi}{4}$, $P(\frac{3\pi}{4}, 0)$ **(3 marks)**	
	•[1] for strategy •[2] finding a •[3] coordinates of P	•[1] $\tan a = \frac{\sqrt{8}}{\sqrt{8}}$ •[2] $\tan a = 1 \quad \therefore \; a = \frac{\pi}{4}$ •[3] $\pi - \frac{\pi}{4} = \frac{3\pi}{4} \Rightarrow (\frac{3\pi}{4}, 0)$
(b)	**ans:** **Proof** **(3 marks)**	
	•[1] realising height of triangle •[2] finding horizontal length •[3] calculation to answer	•[1] $h = 4$ •[2] $\frac{3\pi}{4} - \frac{\pi}{4} = \frac{\pi}{2}$ •[3] $A = \frac{1}{2} \times \frac{\pi}{2} \times 4 = \pi$
		Total: 60 marks