MADRAS COLLEGE

H1
08/09

HIGHER MATHS
January 2009

Paper 1 Time allowed - 90min

INSTRUCTIONS TO CANDIDATES

READ CAREFULLY

Calculators may <u>NOT</u> be used in this paper.

SECTION A - Questions 1 - 20 (40 marks) Use the Answer Sheet

Instructions for the completion of **Section A** are given on the next page.

For this section of the examination you should use an **HB pencil**.

SECTION B (30 marks) Use the blank folded paper

Write the following at the top of the first page of the blank folded paper:
Your name (top left)
Your maths set (top centre)
Your teacher's name (top right)
and just below this information copy the following grid

21	22	23	24	Total

For this section of the examination:

1. Full credit will be given only where the solution contains appropriate working.
2. Answers obtained by readings from scale drawings will not receive any credit.

Read carefully

1 Check that the answer sheet provided is for **Mathematics Higher Prelim 2008/2009 (Section A)**.
2 For this section of the examination you must use an **HB pencil** and, where necessary, an eraser.
3 Make sure you write your **name**, **class** and **teacher** on the answer sheet provided.
4 The answer to each question is **either** A, B, C or D. Decide what your answer is, then, using your pencil, put a horizontal line in the space below your chosen letter (see the sample question below).
5 There is **only one correct** answer to each question.
6 Rough working should **not** be done on your answer sheet.
7 Make sure at the end of the exam that you hand in your answer sheet for Section A with the rest of your written answers.

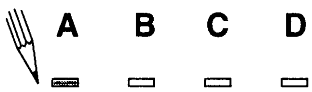
Sample Question

A line has equation $y = 4x - 1$.

If the point $(k, 7)$ lies on this line, the value of k is

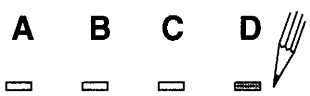
A 2
B 27
C 1·5
D -2

The correct answer is **A** $\rightarrow$ 2. The answer **A** should then be clearly marked in pencil with a horizontal line (see below).



Changing an answer

If you decide to change an answer, carefully erase your first answer and using your pencil, fill in the answer you want. The answer below has been changed to **D**.



FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Trigonometric formulae:

$$\begin{aligned} \sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\ \cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\ \sin 2A &= 2\sin A \cos A \\ \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A \end{aligned}$$

SECTION A

ALL questions should be attempted

1. If $f(x) = 2x^{\frac{3}{2}}$ then $f'(4)$ equals

A 16

B 4

C $25\frac{3}{5}$

D 6

2. Triangle ABC has vertices A$(-3, -3)$, B$(12, -1)$ and C(6, 11).

The gradient of the **altitude** through B is

A $-\frac{9}{14}$

B $\frac{14}{9}$

C $-\frac{3}{8}$

D $\frac{8}{3}$

3. The remainder when $x^3 - 11x + 10$ is divided by $(x+3)$ is

A 52

B 16

C 4

D -24

4. The point P$(8, y)$ lies on the circle with equation $x^2 + y^2 - 12x + 4y + 20 = 0$.

The value(s) of y is/are

A 2 only

B -6 only

C -6 and 2

D 6 and -2

5. A sequence is defined by the recurrence relation $U_{n+1} = aU_n - 5$ with $U_0 = 10$.

An expression in terms of a for U_2 is

A $10a - 5$

B $10a^2 - 5$

C $10a^2 - 5a - 5$

D $10a^2$

6. $\int_0^1 4x(x^2 - 2)\, dx$ is

A -3

B -4

C 0

D 12

7. The equation $2x^2 + 8 = kx$ has **no real roots**. k must take the values

A ± 8

B $-8 < k < 8$

C $k < -8$ or $k > 8$

D undefined

8. For which value(s) of x is the function $f(x) = \dfrac{3}{(x+3)(x-2)}$ undefined?

A 3

B 3 and -2

C -3 and 2

D -6

9. The line $ax - 2y + 5 = 0$ is parallel to the line with equation $3x + y - 4 = 0$.

The value of a is

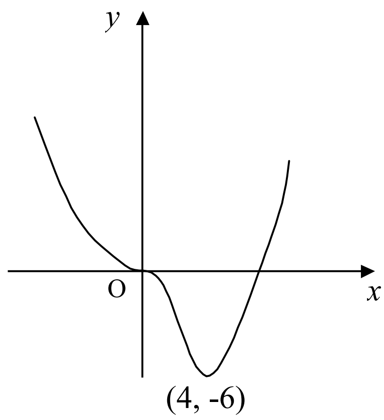
A -3

B -6

C $\frac{2}{3}$

D $-\frac{3}{2}$

10.

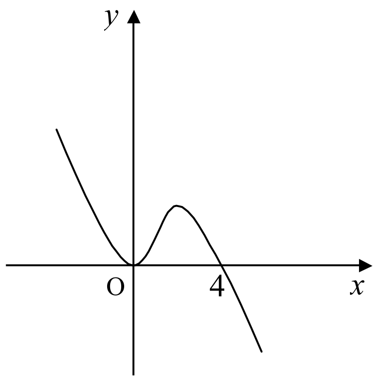


The diagram shows part of the graph of $y = f(x)$.

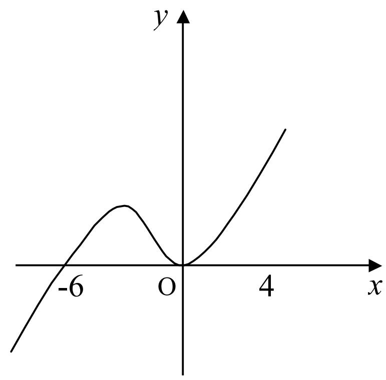
It has stationary points at (0, 0) and $(4, -6)$.

Which of the following could be part of the graph of the derived function $y = f'(x)$?

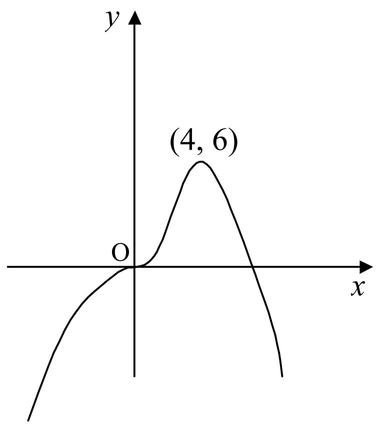
A



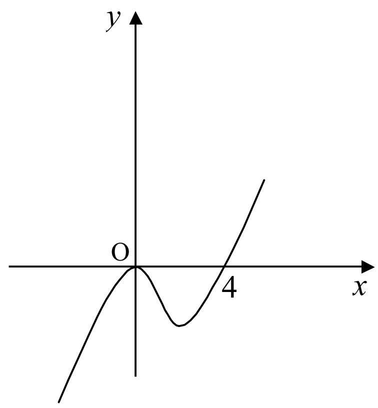
B



C



D



11. The two sequences defined by the recurrence relations $U_{n+1} = 0 \cdot 5U_n + 20$ and $V_{n+1} = 0 \cdot 2V_n + k$ have the same limit. The value of k is

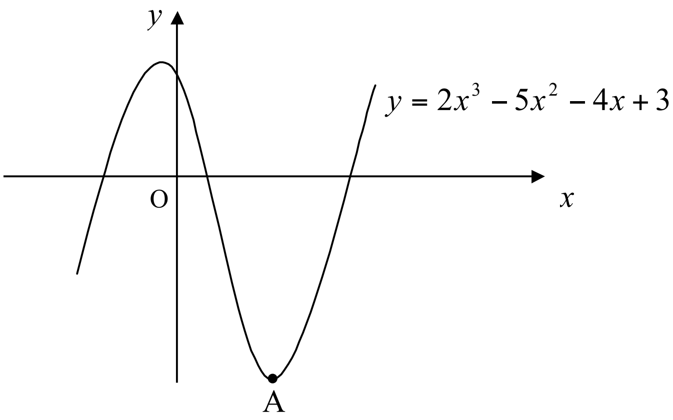
A 8

B 20

C 40

D 32

12. The diagram shows part of the curve with equation $y = 2x^3 - 5x^2 - 4x + 3$.



The x- coordinate of the point A is

A $\frac{1}{3}$

B 2

C 3

D -9

13. The function f is defined as $f(x) = \dfrac{x-6}{x}$, $x \neq 0$. The value of $f(f(3))$ equals

A 7

B -7

C -5

D -1

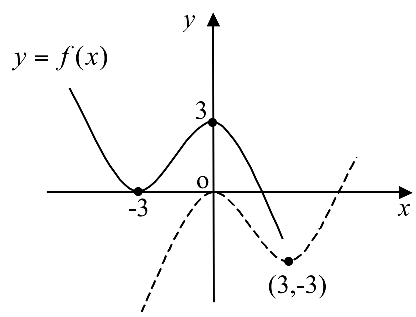
14. The diagram shows the graph of $y = f(x)$ as a full line and the graph of a related function as a broken line. The equation of the related function is

A $y = -f(x) - 3$

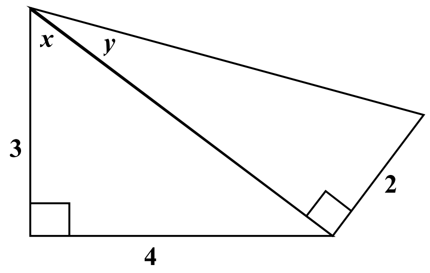
B $y = f(x-6) - 3$

C $y = f(-x) - 3$

D $y = f'(x)$



15. The diagram shows two right-angled triangles with lengths as shown.



The exact value of $\cos(x + y)$ is

A $\dfrac{8}{\sqrt{29}}$

B $\dfrac{7}{5\sqrt{29}}$

C $\dfrac{23}{5\sqrt{29}}$

D $\dfrac{3}{\sqrt{29}}$

16. A circle has centre A(1, 3) and radius $\sqrt{5}$. Another circle has centre B(9, 7) and radius $3\sqrt{5}$.

Which of the following is true for these two circles?

A they intersect at two points

B they touch externally

C they touch internally

D they do not intersect or touch

17. The maximum value of $\dfrac{12}{x^2 - 4x + 10}$ is

A 2

B -2

C 6

D -6

18. A ball is thrown upwards reaching a height of 'h' metres after 't' seconds where $h(t) = 2 + 12t - 3t^2$. The time taken, in seconds, to reach its maximum height is

A 2

B 3

C 4

D 5

19. The exact value of $\sin\frac{2\pi}{3} - \cos\frac{7\pi}{6}$ is

A 0

B 1

C $\sqrt{3}$

D $\frac{\sqrt{3}}{4}$

20. (x, y) is a solution for the system of equations

$$x^2 + 7y^2 = 16$$

$$x - 3y = 0.$$

Possible values for $x + y$ are

(1) 0 (2) 4 (3) -4

A (1) only

B (2) only

C (2) and (3) only

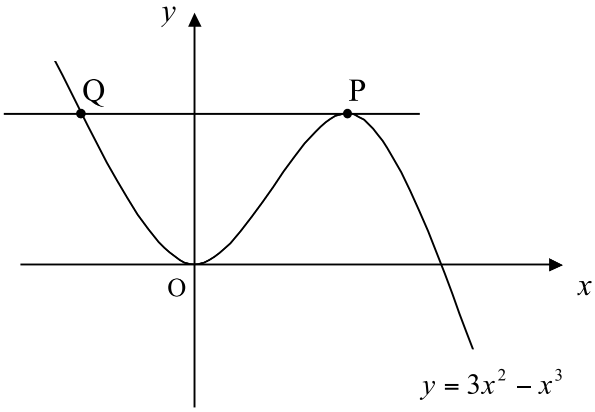
D (1), (2) and (3)

[END OF SECTION A]

SECTION B

ALL questions should be attempted

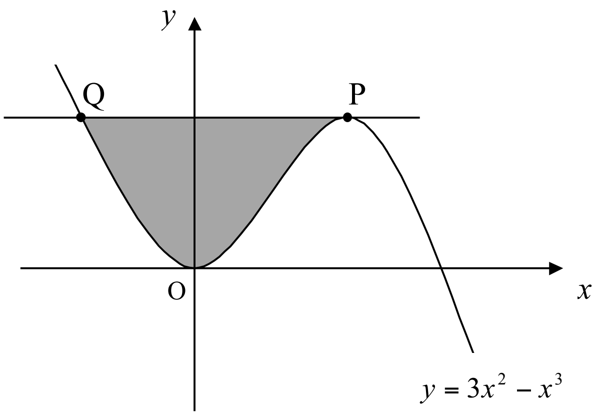
21. Part of the graph of the curve with equation $y = 3x^2 - x^3$ is shown below.
The diagram is not drawn to scale.



(a) Establish the coordinates of the stationary point P. **4**

(b) The horizontal line through P meets the curve again at Q.
Find the coordinates of Q. **3**

(c) Hence calculate the shaded area shown in the diagram below.



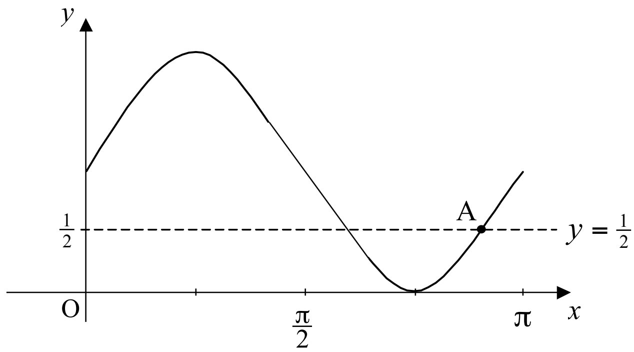
6

22. Two functions, defined on suitable domains, are given as $f(x) = x^2 - 1$ and $g(x) = 2 - x$.

(a) Show that $f(g(a))$ can be expressed in the form $pa^2 + qa + r$ and write down the values of p, q and r. **4**

(b) Hence find a if $f(g(a)) = 8$ and $a > 0$. **2**

23. The diagram below shows part of the graph of $y = \sin 2x + 1$, for $0 \le x \le \pi$, and the line with equation $y = \frac{1}{2}$.



Find the coordinates of the point A. **4**

24. A recurrence relation is defined by the formula $U_{n+1} = 0 \cdot 6U_n + 24$.

(a) Establish the limit of this sequence. **2**

(b) Given now that U_1 is exactly half of this limit, find U_0, the initial value of the sequence. **2**

(c) A second recurrence relation in the form $U_{n+1} = aU_n + b$ has the **same limit** as the sequence above and is such that $b = 90a$.

Find the values of a and b in this second sequence. **3**

[**END OF SECTION B**]

[**END OF QUESTION PAPER**]

Mathematics

Higher Prelim Examination 2008/2009

NATIONAL QUALIFICATIONS

Paper 1 - Section A - Answer Sheet

NAME :	CLASS :

TEACHER :

You should use an HB pencil.
Erase all incorrect answers thoroughly.

Indicate your choice of answer with a single mark as in this example

A	B	C	D
▭	▬	▭	▭

	A	B	C	D
1	▭	▭	▭	▭
2	▭	▭	▭	▭
3	▭	▭	▭	▭
4	▭	▭	▭	▭
5	▭	▭	▭	▭
6	▭	▭	▭	▭
7	▭	▭	▭	▭
8	▭	▭	▭	▭
9	▭	▭	▭	▭
10	▭	▭	▭	▭
11	▭	▭	▭	▭
12	▭	▭	▭	▭
13	▭	▭	▭	▭
14	▭	▭	▭	▭
15	▭	▭	▭	▭
16	▭	▭	▭	▭
17	▭	▭	▭	▭
18	▭	▭	▭	▭
19	▭	▭	▭	▭
20	▭	▭	▭	▭

Section A	/ 40	
Section B	/ 30	
Total (P1)	/ 70	
Total (P2)	/ 60	
Overall Total	/ 130	%

Please make sure you have filled in all your details above before handing in this answer sheet.

MADRAS COLLEGE

H2
08/09

HIGHER MATHS
January 2009

Paper 2 Time allowed - 70min

INSTRUCTIONS TO CANDIDATES

Write the following at the top of the first page of your answer booklet:

Your name (top left)
Your maths set (top centre)
Your teacher's name (top right)

and just below this information copy the following grid

1	2	3	4	5	6	7	8	Total

READ CAREFULLY

1. Full credit will be given only where the solution contains appropriate working.
2. **Calculators may be used in this paper**.
3. Answers obtained by readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{}$[redacted].

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Trigonometric formulae:

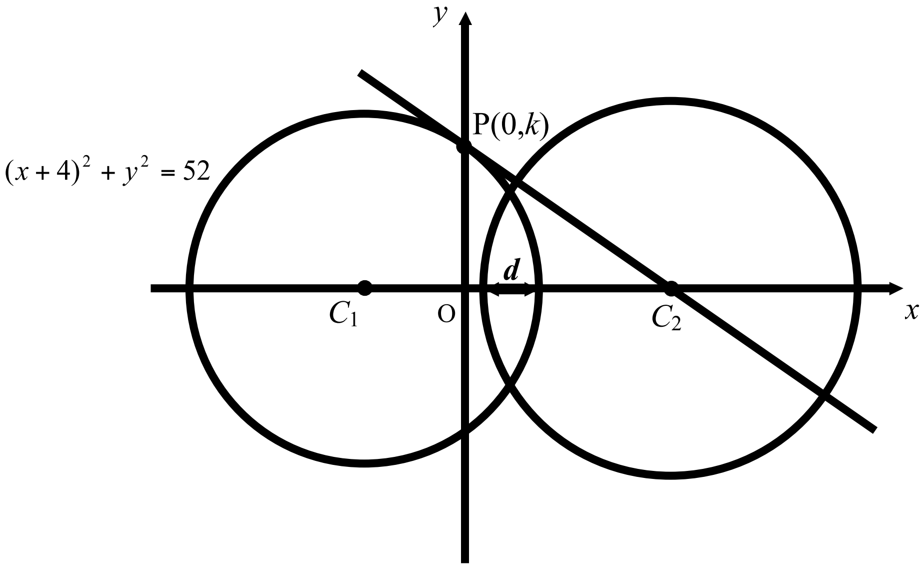
$$\begin{aligned}
\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\sin 2A &= 2 \sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

ALL questions should be attempted

1. Consider the diagram below.

The circle centre C_1 has as its equation $(x+4)^2 + y^2 = 52$.
The point P(0, k) lies on the circumference of this circle and the tangent to this circle through P has been drawn.

A second circle with centre C_2 is also shown.



(a) What is the value of k? **2**

(b) Hence find the equation of the tangent through P. **4**

(c) The tangent through P passes through C_2 the centre of the second circle. State the coordinates of C_2. **1**

(d) Given that the second circle has a radius of 8 units, calculate the distance marked ***d*** on the diagram, giving your answer correct to 1 decimal place. **3**

2. Solve algebraically the equation

$$3\cos 2x^\circ + 4\sin x^\circ - 1 = 0 \qquad \text{for} \qquad 0 \leq x < 360. \qquad \textbf{5}$$

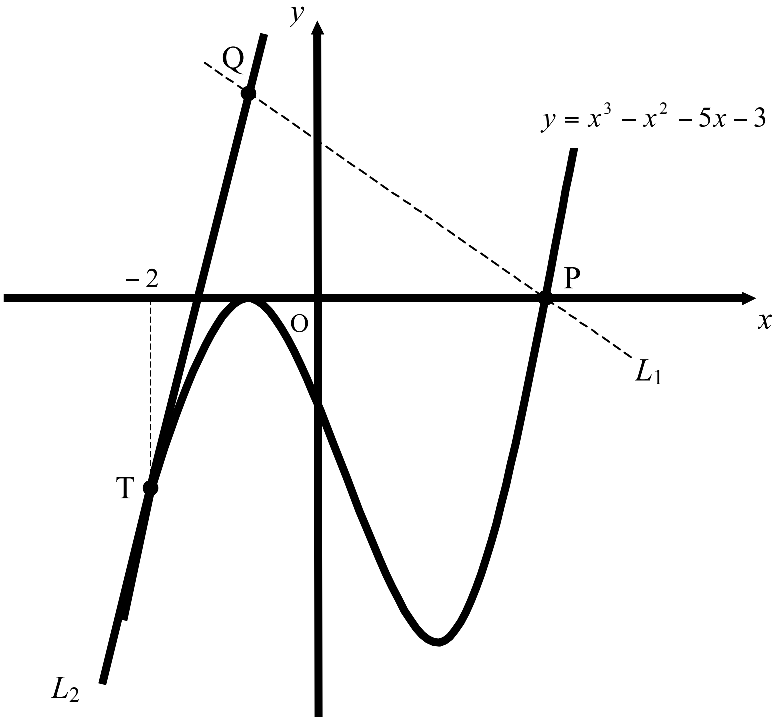
3. A curve has as its derivative [redacted] $= 2x -$ [redacted] .

(a) Given that the point (2, 3) lies on this curve, express y in terms of x. **4**

(b) Hence find p if the point $(3,\ p)$ also lies on this curve. **1**

4. The diagram below, **which is not drawn to scale**, shows part of the graph of the curve with equation $y = x^3 - x^2 - 5x - 3$.

Two straight lines are also shown, L_1 and L_2.



(a) Find the coordinates of P. **2**

(b) Line L_1 has a gradient of $-$ [redacted] and passes through the point P.
Find the equation of L_1. **1**

(c) Line L_2 is a tangent to the curve at the point T where $x = -2$.
Find the equation of L_2. **4**

(d) Hence find the coordinates of Q, the point of intersection of the two lines. **3**

5. A company making commercial "glow sticks" have devised a method to test the brightness and consistency of the glow given off.

All glow sticks depend on a chemical process known as chemiluminesence to produce their light. Once a glow stick has been illuminated (by mixing two chemicals together) the brightness of the glow diminishes over a period of time.

When one of their glow sticks is ignited the initial brightness is rated at 200 *gu* (glow units).

(a) During any 1 hour period the glow light is known to lose 8% of its brightness at the beginning of the period.

Calculate the brightness remaining, in *gu*'s, after a period of 4 hours. **2**

(b) At the end of each 4 hour period, the glow light is automatically passed through a tube which has an internal temperature of $-40^{\circ}C$. This lowering of the temperature of the glow light has the effect of allowing it to regain some of its lost brightness. A single pass through this refrigerated tube allows the glow stick to regain 32 glow units.

The 4 hour cycle described above is now left to run uniterrupted for a total of 16 hours.

By considering an appropriate recurrence relation, calculate the brightness remaining, in *gu*'s, after this 16 hour period has been completed.
Your answer must be accompanied with the appropriate working. **3**

(c) If this cycle was left to run over a very long period of time would the brightness of the glow stick ever drop to below half of its initial brightness?
Explain your answer.
Your answer and explanation must be accompanied with the appropriate working. **3**

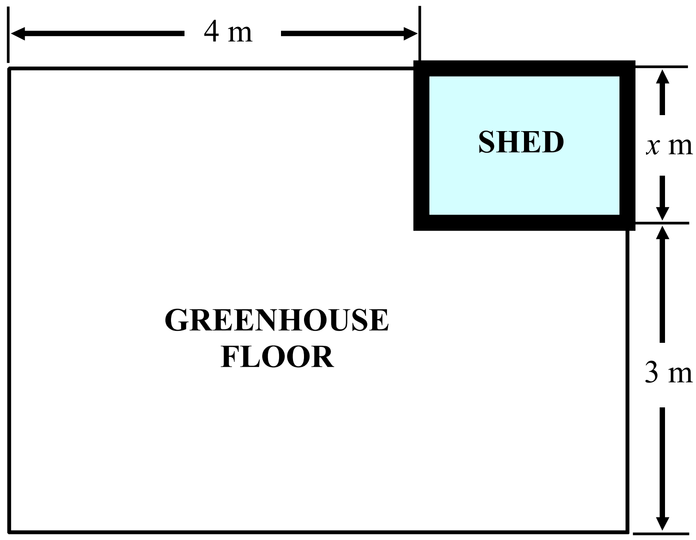
6. (a) If $k =$ [redacted], where k is a real number, show clearly that

$$(k-1)x^2 + 2x + (4k-1) = 0.$$ **3**

(b) Hence find the value of k given that the equation $(k-1)x^2 + 2x + (4k-1) = 0$ has equal roots and $k > 0$. **5**

7. The floor plan of a rectangular greenhouse is shown below. All dimensions are in metres.

The gardener places a rectangular wooden storage shed, of width x metres, in one corner.



(a) Given that the **area of the shed** is 3 square metres, show clearly that the area of greenhouse floor remaining, A square metres, is given in terms of x as

$$A(x) = 12 + 4x + [redacted] .$$ **3**

(b) Hence find the value of x which **minimises** the area of the greenhouse floor remaining, **justifying your answer**. **5**

8. Angle A is acute and such that $\tan A =$ [redacted].

(a) Show clearly that the exact value of $\sin A$ can be written in the form [redacted], and state the value of k. **3**

(b) Hence, or otherwise, show that the value of $\cos 2A$ is exactly [redacted] **3**

[**END OF QUESTION PAPER**]