
Mathematics
Higher Mini-Prelim Examination 2009/2010

**NATIONAL
QUALIFICATIONS**

Assessing Unit 3 + revision from Units 1 & 2

Time allowed - 1 hour 10 minutes

Read carefully

1. **Calculators may be used in this paper.**
2. Full credit will be given only where the solution contains appropriate working.
3. Answers obtained from readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

Scalar Product: $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}| |\mathbf{b}| \cos \theta$, where θ is the angle between \mathbf{a} and \mathbf{b} .

or

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 \text{ where } \mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x) dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

SECTION A

In this section the correct answer to each question is given by one of the alternatives **A**, **B**, **C** or **D**. Indicate the correct answer by writing **A**, **B**, **C** or **D** opposite the number of the question on your answer paper.

Rough working may be done on the paper provided. 2 marks will be given for each correct answer.

1. The function $f(x) = 2 \sin x^\circ + \cos x^\circ$ has a **minimum** value of

- A -2
- B 0
- C -3
- D $-\sqrt{5}$

2. Which of the following is a correct assumption from the statement $\log_b a = c$?

- A $a^c = b$
- B $c^a = b$
- C $b^c = a$
- D $c^b = a$

3. What is the value of $\int_0^\pi \sin x \, dx$?

- A -2
- B $+1$
- C $+2$
- D 0

4. P and Q have position vectors $\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$ respectively.

The length of PQ is

- A 1
- B $\sqrt{17}$
- C $\sqrt{21}$
- D $\sqrt{13}$

5. Given that $\cos P = \frac{1}{\sqrt{6}}$, where $0 < p < \frac{\pi}{2}$, the value of $\cos 2P$ is

A $\frac{1}{\sqrt{3}}$

B $\frac{2}{\sqrt{6}}$

C $\frac{\sqrt{5}}{\sqrt{6}}$

D $-\frac{2}{3}$

6. An equation is such that $\log x + \log(x+1) = \log 6$, where $x > 0$.

The value of x is

A 2

B 1

C 3

D 6

7. The gradient of the tangent to the curve $y = \sin x^\circ$ at the point where $x = 60^\circ$ is

A $\frac{\sqrt{3}}{2}$

B $\frac{1}{2}$

C $-\frac{1}{2}$

D 0

8. Vectors \mathbf{a} and \mathbf{b} are such that $|\mathbf{a}| = |\mathbf{b}| = 2$ with P being the angle between the vectors.

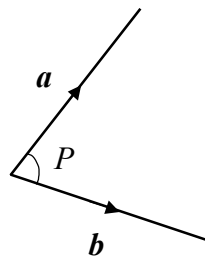
If $\mathbf{a} \cdot \mathbf{b} = 0.8$, the value of $\cos P$ is

A $3 \cdot 2$

B $0 \cdot 4$

C $0 \cdot 2$

D $0 \cdot 05$



[END OF SECTION A]

SECTION B

ALL questions should be attempted

9. A function is defined on a suitable domain as $f(x) = \frac{-16}{(2x-1)^2}$.

(a) Show clearly that the derivative of this function can be written in the form

$$f'(x) = \frac{k}{(2x-1)^n}$$

and write down the values of k and n .

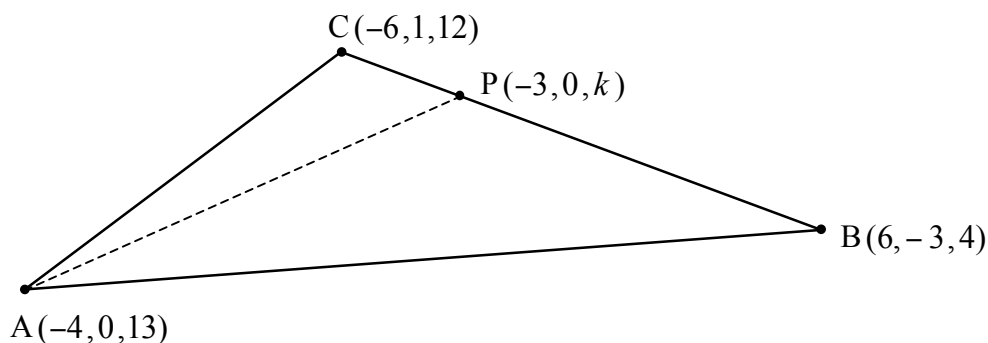
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(b) Hence find x when $f'(x) = 1$ and $x > 0$.

3

10. In the diagram below A, B and C have coordinates $(-4, 0, 13)$, $(6, -3, 4)$ and $(-6, 1, 12)$ respectively.

P lies on BC and has coordinates $(-3, 0, k)$



(a) Find the value of k .

3

(b) Hence calculate the size of angle APB.

5

11. A formulae for mass decay is given as $M_t = M_0 e^{-0.02t}$, where t is time elapsed in hours, M_0 is the initial mass in grams and M_t is the mass remaining after t hours.

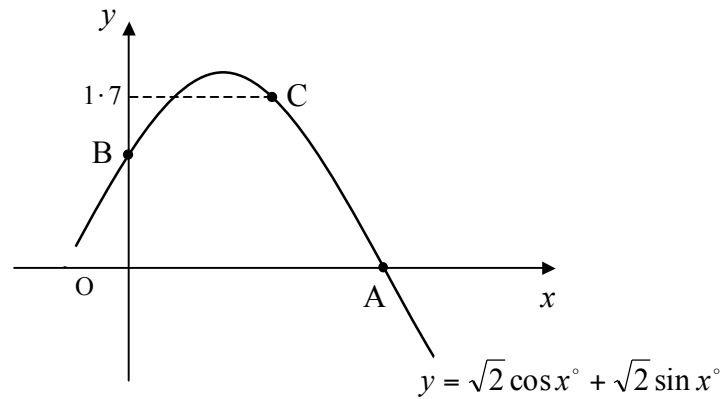
How long will it take for an initial mass of 40 grams to decay down to 28 grams?

Give your answer correct to the nearest minute.

5

12. If $\frac{dy}{dx} = \sqrt{4x+1}$, find an expression for y in terms of x given that $y = 9.5$ when $x = 2$. 5

13. Part of the graph of $y = \sqrt{2} \cos x^\circ + \sqrt{2} \sin x^\circ$ is shown below.



- (a) Express $y = \sqrt{2} \cos x^\circ + \sqrt{2} \sin x^\circ$ in the form $y = k \cos(x - a)^\circ$, where $k > 0$. 3
- (b) Hence state the coordinates of A and B rounding the coordinates to **3 significant figures** where necessary. 2
- (c) By solving the equation $\sqrt{2} \cos x^\circ + \sqrt{2} \sin x^\circ = 1.7$, find the coordinates of point C. 4

[END OF SECTION B]

[END OF QUESTION PAPER]