NATIONAL QUALIFICATIONS

Mathematics

Higher Mini-Prelim Examination 2010/2011

Assessing Unit 3 + revision from Units 1 & 2

Time allowed - 1 hour 10 minutes

Read carefully

1. **Calculators may be used in this paper.**
2. Full credit will be given only where the solution contains appropriate working.
3. Answers obtained from readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Trigonometric formulae:

$$\begin{aligned}
\sin(A \pm B) &= \sin A\cos B \pm \cos A\sin B \\
\cos(A \pm B) &= \cos A\cos B \mp \sin A\sin B \\
\sin 2A &= 2\sin A\cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

Scalar Product:

$\boldsymbol{a} \cdot \boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}|\cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$.

or

$\boldsymbol{a} \cdot \boldsymbol{b} = \boldsymbol{a}_1\boldsymbol{b}_1 + \boldsymbol{a}_2\boldsymbol{b}_2 + \boldsymbol{a}_3\boldsymbol{b}_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$ $\cos ax$	$a\cos ax$ $-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$ $\cos ax$	$-\frac{1}{a}\cos ax + C$ $\frac{1}{a}\sin ax + C$

SECTION A

In this section the correct answer to each question is given by one of the alternatives **A**, **B**, **C** *or* **D**.
Indicate the correct answer by writing **A**, **B**, **C** *or* **D** *opposite the number of the question on your answer paper.*
Rough working may be done on the paper provided. 2 marks will be given for each correct answer.

1. If $f(x) = (2x-1)^4$ then $f'(1)$ equals

A 4

B 1

C 2

D 8

2. The **maximum** value of the function $g(x) = 3\sin x + 2\cos x$ is

A $\sqrt{13}$

B 5

C 0

D 2

3. The radius of the circle with equation $x^2 + y^2 + 4x - 2y = 4$ is

A 2

B 3

C 1

D $\sqrt{24}$

4. When $5\sin x^\circ + 3\cos x^\circ$ is written in the form $k\sin(x-\alpha)^\circ$ where $0 \le \alpha \le 360$, $\tan\alpha$ is equal to

A $\dfrac{5}{3}$

B $-\dfrac{5}{3}$

C $-\dfrac{3}{5}$

D $\dfrac{3}{5}$

5. The value of $\log_{\sqrt{2}} 4$ is

A 2

B $4\sqrt{2}$

C $\frac{1}{4}$

D 4

6. Given that the vectors $\begin{pmatrix} 1 \\ 4 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} p \\ -2 \\ 3 \end{pmatrix}$ are perpendicular, the value of p is

A 0

B 8

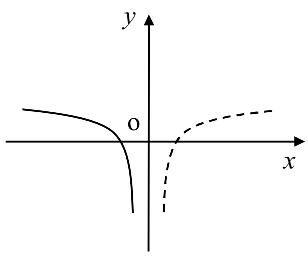
C 4

D -6

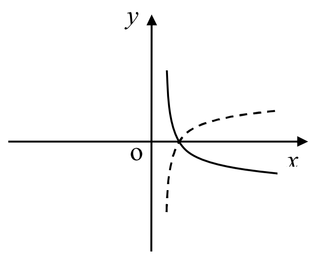
7. Part of the graph of $y = \log_{10} x$ is shown in each diagram below as a broken line.

Which diagram also shows, as a full line, part of the graph of $y = \log_{10} \frac{1}{x}$?

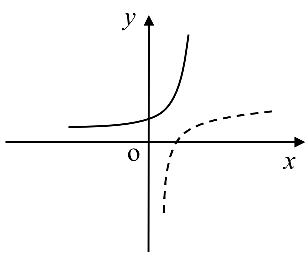
A



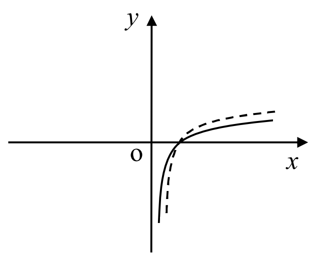
B



C



D



8. $\boldsymbol{a} = \begin{pmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ g \end{pmatrix}$ is a **unit** vector. Which of the following could be the value of g?

A $\frac{1}{2}$

B 1

C -1

D $\frac{1}{\sqrt{2}}$

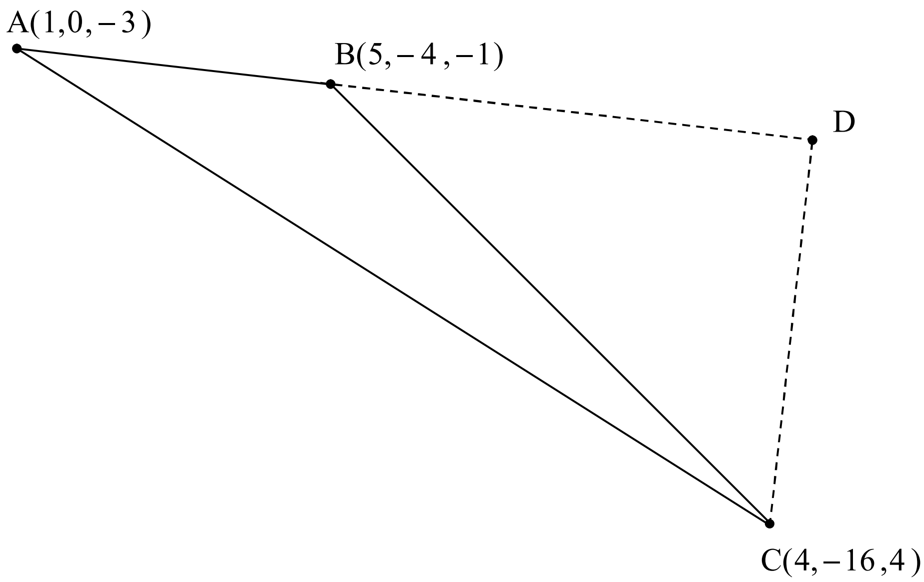
[END OF SECTION A]

SECTION B

ALL questions should be attempted

9. Triangle ABC has vertices A$(1,0,-3)$, B$(5,-4,-1)$ and C$(4,-16,4)$ respectively.

A, B and D are collinear such that $\frac{\text{AB}}{\text{BD}} = \frac{2}{3}$.



(a) Show that the coordinates of D are (11, –10, 2). **2**

(b) Hence show clearly that angle ADC is a right angle. **4**

(c) Prove that angle ABC is obtuse. **3**

10. A function is defined as $f(x) = 6\cos^2 \tfrac{1}{2}x^\circ + \sqrt{3}\sin x^\circ$.

(a) By using the fact that $\cos^2 x^\circ = \tfrac{1}{2}(\cos 2x^\circ + 1)$ show clearly that this function can be expressed in the form

$$f(x) = 3\cos x^\circ + \sqrt{3}\sin x^\circ + 3.$$ **3**

(b) Express $3\cos x^\circ + \sqrt{3}\sin x^\circ + 3$ in the form $k\cos(x-\alpha)^\circ + 3$ where $0 < \alpha < 360$ and $k > 0$. **3**

(c) Hence solve the equation $f(x) = 0$ for $200 < x < 360$. **4**

11. A function *f* is given by $f(x) = \left(x^2 + 3\right)^{\frac{1}{2}}$.

(a) Find $f'(x)$ **3**

(b) Find algebraically the values of *x* for which $f'(x) = \dfrac{1}{2}$. **3**

12. Given that $(x+1)$ and $(x-3)$ are both factors of $2x^3 - 5x^2 + ax + b$, find *a* and *b*. **4**

13. Solve $\log_3(x^2 - 4) - \log_3(x-2) = 3$ **5**

[**END OF SECTION B**]

[**END OF QUESTION PAPER**]

Higher Mini Prelim - Unit 3 — 2010/2011 (Answers + Marking Scheme)

Section A - Answers

1	D	2	A	3	B	4	C
5	D	6	B	7	B	8	D

2 marks each (16 marks)

Section B - Marking Scheme

	Give 1 mark for each •	Illustration(s) for awarding each mark
9(a)	**ans: (11, – 10, 2)** **(2 marks)**	
	•1 valid method	•1 evidence of using stepping out/section formula
	•2 answer	•2 (11, – 10, 2)
(b)	**ans: proof** **(4 marks)**	
	•1 finds $\overrightarrow{DA}$	•1 $\overrightarrow{DA} = \begin{pmatrix} -10 \\ 10 \\ -5 \end{pmatrix}$
	•2 finds $\overrightarrow{DC}$	•2 $\overrightarrow{DC} = \begin{pmatrix} -7 \\ -6 \\ 2 \end{pmatrix}$
	•3 finds $\overrightarrow{DA}.\overrightarrow{DC}$	•3 $\overrightarrow{DA}.\overrightarrow{DC} = 70 - 60 - 10 = 0$
	•4 conclusion	•4 since $\overrightarrow{DA}.\overrightarrow{DC} = 0$; $\angle$ADC is right angled
(c)	**ans: proof** **(3 marks)**	
	•1 finds $\overrightarrow{BA}$ and $\overrightarrow{BC}$	•1 $\overrightarrow{BA} = \begin{pmatrix} -4 \\ 4 \\ -2 \end{pmatrix}$ $\overrightarrow{BC} = \begin{pmatrix} -1 \\ -12 \\ 5 \end{pmatrix}$
	•2 finds $\overrightarrow{BA}.\overrightarrow{BC}$	•2 $\overrightarrow{BA}.\overrightarrow{BC} = 4 - 48 - 10 = -54$
	•3 conclusion	•3 scalar product < 0 so obtuse angle

	Give 1 mark for each •	Illustration(s) for awarding each mark
10(a)	**ans: proof (3 marks)**	
	•1 applies given info to new function	•1 $\cos^2 \frac{1}{2}x° = \frac{1}{2}(\cos x° + 1)$
	•2 knows to substitute in function	•2 $6[\frac{1}{2}(\cos x° + 1)] + \sqrt{3}\sin x°$
	•3 simplifies to required form	•3 $3(\cos x + 1)] + \sqrt{3}\sin x°; 3\cos x + 3 + \sqrt{3}\sin x°$
(b)	**ans: $\sqrt{12}\cos(x - 30)° + 3$ (3 marks)**	
	•1 finds k	•1 $k = \sqrt{9+3} = \sqrt{12}$
	•2 finds tan α	•2 $\tan\alpha = \frac{\sqrt{3}}{3}$
	•3 finds α	•3 $\alpha = 30°$
(c)	**ans: 240° (4 marks)**	
	•1 equates to 0	•1 $\sqrt{12}\cos(x - 30)° + 3 = 0$
	•2 simplifies	•2 $\cos(x - 30)° = -\frac{3}{\sqrt{12}}$
	•3 finds values	•3 $x = 240°; 360°$
	•4 discards	•4 240°
11(a)	**ans: $\dfrac{x}{(x^2+3)^{1/2}}$ (3 marks)**	
	•1 use of chain rule	•1 $\frac{1}{2}(x^2+3)^{-1/2}$
	•2 differentiates bracket	•2 $2x$
	•3 combines and simplifies	•3 $\dfrac{2x}{2(x^2+3)^{1/2}}$
(b)	**ans: $x = \pm 1$ (3 marks)**	
	•1 equates derivative to $\frac{1}{2}$	•1 $\dfrac{x}{(x^2+3)^{1/2}} = \dfrac{1}{2}$
	•2 rearranges and squares	•2 $2x = (x^2+3)^{1/2}$: $4x^2 = x^2 + 3$
	•3 solves to answer	•3 $3x^2 = 3$: $x^2 = 1$: $x = \pm 1$
12	**ans: $a = -4$; $b = 3$ (4 marks)**	
	•1 uses synthetic division to find one equation	•1 $b - a = 7$
	•2 uses synthetic division to find other eq.	•2 $b + 3a = -9$
	•3 knows to use system of equations	•3 evidence
	•4 solves for a and b	•4 $a = -4$; $b = 3$

	Give 1 mark for each •	Illustration(s) for awarding each mark
13(a)	**ans: $x = 25$ (5 marks)**	
	•[1] applies difference of logs	•[1] $\log_3 \frac{x^2-4}{x-2} = 3$
	•[2] exponentiates	•[2] $\frac{x^2-4}{x-2} = 27$
	•[3] difference of two squares	•[3] $x^2 - 4 = (x+2)(x-2)$
	•[4] simplifies	•[4] $\frac{(x+2)(x-2)}{(x-2)} = x+2$
	•[5] solves to answer	•[5] $x + 2 = 27$
	Sect. B (34 marks)	16 + 34 Total: 50 marks