NATIONAL QUALIFICATIONS

Mathematics
Higher Prelim Examination 2010/2011
Paper 1
Assessing Units 1 & 2

Time allowed - 1 hour 30 minutes

Read carefully

Calculators may NOT be used in this paper.

Section A - Questions 1 - 20 (40 marks)

Instructions for the completion of **Section A** are given on the next page.

For this section of the examination you should use an **HB pencil**.

Section B (30 marks)

1. Full credit will be given only where the solution contains appropriate working.
2. Answers obtained by readings from scale drawings will not receive any credit.

Read carefully

1 Check that the answer sheet provided is for **Mathematics Higher Prelim 2010/2011 (Section A)**.
2 For this section of the examination you must use an **HB pencil** and, where necessary, an eraser.
3 Make sure you write your **name**, **class** and **teacher** on the answer sheet provided.
4 The answer to each question is **either** A, B, C or D. Decide what your answer is, then, using your pencil, put a horizontal line in the space below your chosen letter (see the sample question below).
5 There is **only one correct** answer to each question.
6 Rough working should **not** be done on your answer sheet.
7 Make sure at the end of the exam that you hand in your answer sheet for Section A with the rest of your written answers.

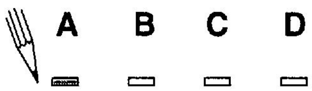
Sample Question

A line has equation $y = 4x - 1$.

If the point $(k, 7)$ lies on this line, the value of k is

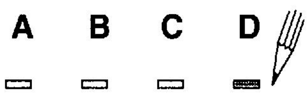
A 2
B 27
C 1·5
D -2

The correct answer is **A** $\rightarrow$ 2. The answer **A** should then be clearly marked in pencil with a horizontal line (see below).



Changing an answer

If you decide to change an answer, carefully erase your first answer and using your pencil, fill in the answer you want. The answer below has been changed to **D**.



FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Trigonometric formulae:

$$
\begin{aligned}
\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\sin 2A &= 2 \sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}
$$

SECTION A
ALL questions should be attempted

1. A sequence is defined by the recurrence relation

$$U_{n+1} = 3U_n - 6 \text{ with } U_0 = 2 \cdot 4 .$$

The value of U_2 is

A $-2 \cdot 4$

B -3

C 0

D 2

2. Two functions are defined on suitable domains as

$$f(x) = 2x - 4 \text{ and } g(x) = x^2 + 1 .$$

$f(g(1))$ is

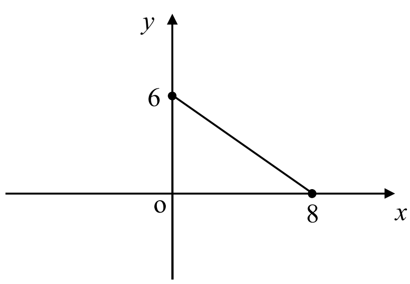
A 3

B -3

C 5

D 0

3.



The line above has as its equation

A $y = -\frac{4}{3}x + 6$

B $y = -\frac{3}{4}x + 6$

C $y = \frac{3}{4}x + 6$

D $y = -\frac{3}{4}x + 8$

4. Given that C is a constant of integration, $\int x^{\frac{1}{4}}\,dx$ equals

A $\frac{5}{4}x^{\frac{5}{4}} + C$

B $\frac{1}{4}x^{-\frac{3}{4}} + C$

C $\frac{4}{5}x^{\frac{5}{4}} + C$

D $\frac{1}{4}x^{\frac{5}{4}} + C$

5. $x^2 - 4x + 2$ can be expressed in the form $(x-a)^2 + b$.

What is the value of b?

A 6

B -4

C -2

D 2

6. Given that $\sin a = \frac{1}{\sqrt{2}}$, where $0 < a < \frac{\pi}{2}$, the exact value of $2\sin 2a$ is

A 2

B $\frac{4}{\sqrt{2}}$

C 1

D cannot be found

7. The function $f(x) = px^2 - 20x$ is such that $f(-2) = 0$.

The value of p is

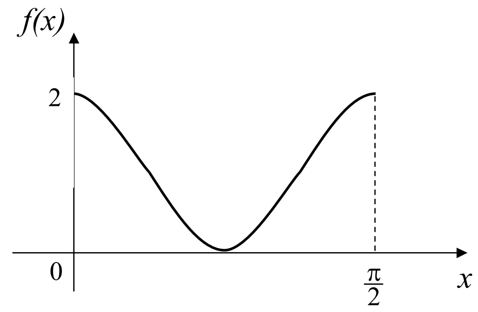
A 10

B 0

C -20

D -10

8. The diagram opposite shows part of the graph of a trigonometrical function.



The most likely function could be $f(x) = ...$

A $2\cos 4x$

B $\cos 4x$

C $\cos 2x + 1$

D $\cos 4x + 1$

9. If $x - 2$ is a factor of the polynomial $x^3 - 2x^2 + kx - 10$, then k equals

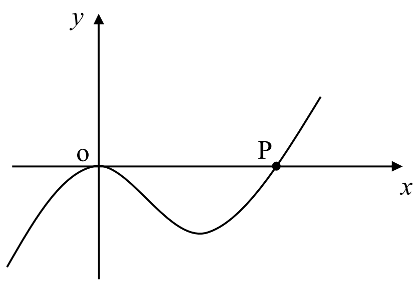
A 5

B -13

C 3

D -5

10. The sketch below shows part of the curve $y = x^3 - 5x^2$.



The gradient of the tangent to the curve at the point P(5, 0) is

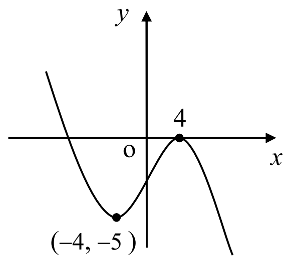
A -125

B 0

C 25

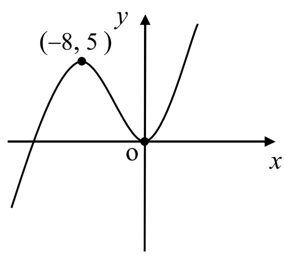
D 50

11. Part of the graph of the function $y = f(x)$ is shown below.

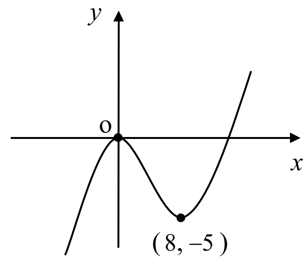


Which of the following graphs represents the related function $y = -f(x+4)$?

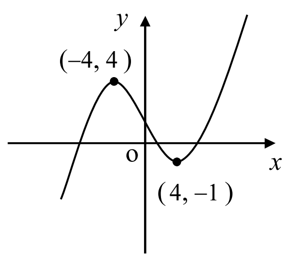
A



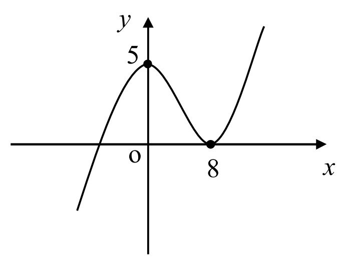
B



C



D



12. The equation of the line passing through $(2, -1)$ and parallel to the line with equation $2x + 3y + 5 = 0$ is

A $y + 1 = \frac{3}{2}(x - 2)$

B $y + 1 = -\frac{2}{3}(x - 2)$

C $y - 2 = -\frac{2}{3}(x + 1)$

D $y + 1 = -2(x - 2)$

13. Which of the following expressions is/are equal to $(9a)^{-\frac{3}{2}}$?

(1) $\dfrac{9}{a^{\frac{3}{2}}}$

(2) $\dfrac{1}{27\sqrt{a^3}}$

(3) $\dfrac{1}{9a^{\frac{3}{2}}}$

A all three expressions

B only expression (1)

C only expression (2)

D expressions (2) and (3)

14. The equation of a circle, centre $(-2, 6)$, with the x-axis as a tangent is

A $(x-6)^2+(y+2)^2=36$

B $(x+2)^2+(y-6)^2=4$

C $(x-2)^2+(y+6)^2=36$

D $(x+2)^2+(y-6)^2=36$

15. The equation $4x^2+k=0$ has real roots. The range of values of k is

A $k \leq 0$

B $k > 0$

C $-2 < k < 2$

D $k < 0$

16. A function is defined as $f(x) = x^3 + 1$.

Which of the following statements **is true** about this function?

A it is never increasing

B it is never stationary

C it is never decreasing

D it has two stationary points

17. If $x = \sqrt{3} - 1$, then x^2 equals

A 2

B 4

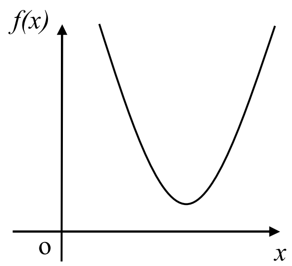
C $10 - 2\sqrt{3}$

D $4 - 2\sqrt{3}$

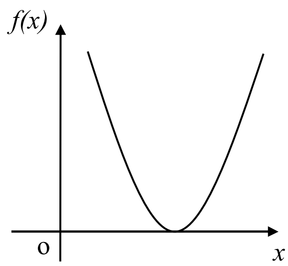
18. A quadratic function, *f*, where $f(x) = ax^2 + bx + c$, is such that $a < 0$ and $b^2 - 4ac < 0$.

Which of the following is a possible sketch of the graph of *f*?

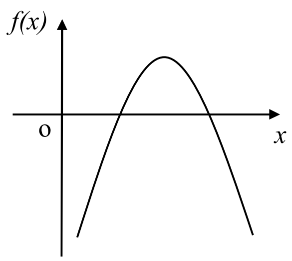
A



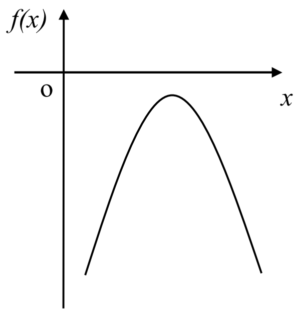
B



C



D



19. If $\sin\theta = \frac{3}{5}$ and $\frac{\pi}{2} < \theta < \pi$, then $\tan\theta$ is equal to

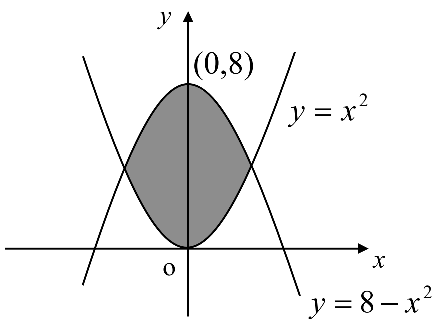
A $-\frac{4}{3}$

B $-\frac{3}{4}$

C $\frac{4}{3}$

D $\frac{3}{4}$

20. The area enclosed between the curves $y = x^2$ and $y = 8 - x^2$ is shown as the shaded area in the diagram below.



Which of the following gives the area of the shaded section?

A $\int_{-\sqrt{8}}^{\sqrt{8}} \left(8 - 2x^2\right) dx$

B $\int_{-2}^{2} \left(8 - 2x^2\right) dx$

C $\int_{-4}^{4} \left(8 - 2x^2\right) dx$

D $\int_{-2}^{2} \left(2x^2 - 8\right) dx$

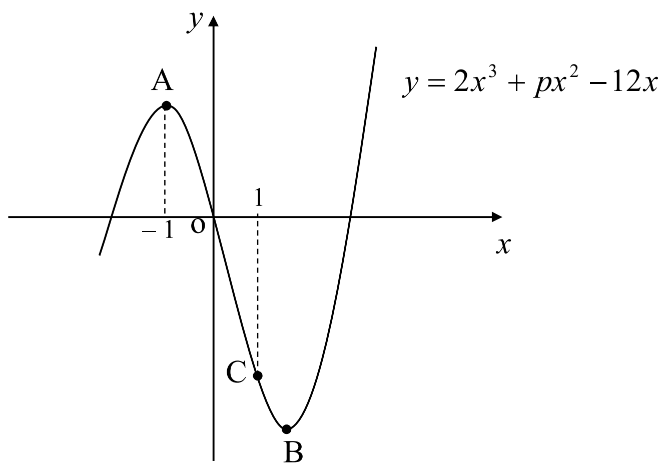
[**END OF SECTION A**]

SECTION B

ALL questions should be attempted

21. The diagram below, which is not drawn to scale, shows part of the curve with equation $y = 2x^3 + px^2 - 12x$, where p is a constant.

A is a stationary point and has -1 as its x-coordinate.



(a) By considering the derivative of y, and using the x-coordinate of point A to help you, show that the p has a value of -3. **4**

(b) Establish the coordinates of B the other stationary point. (all relevant working must be shown) **4**

(c) The point C on the curve has 1 as its x-coordinate.

Find the equation of the tangent to the curve at C. **3**

22. Solve the equation $4\cos 2\theta = 6\cos\theta + 1$, for $0 < \theta < 2\pi$. **6**

23. A sequence of numbers is defined by the recurrence relation $U_{n+1} = aU_n + 20$, where a is a constant.

(a) Given that $U_3 = 36$ and $U_4 = 38$, show **algebraically** , that the value of a is ½ . **2**

(b) Hence find the limit of this sequence. **2**

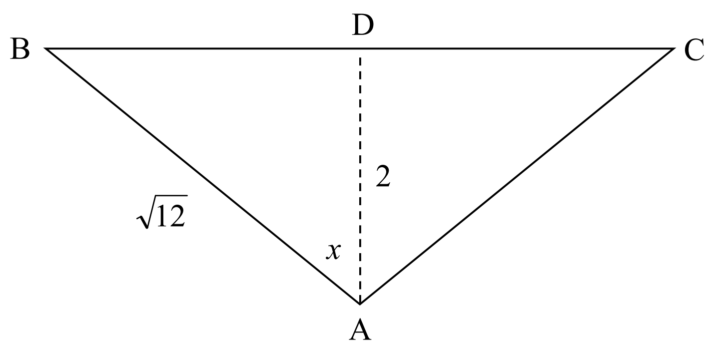
(c) The sequence is such that $L = \boldsymbol{k}U_0$, where L is the limit of the sequence, U_0 is the the initial value and $\boldsymbol{k}$ is a number.

Find the value of $\boldsymbol{k}$. **3**

24. In the diagram below, which is not drawn to scale, triangle ABC is isosceles with AB = AC.

D is the mid-point of BC. $\text{AB} = \sqrt{12}$ units and $\text{AD} = 2$ units as shown.

Angle $\text{BAD} = x$.



(a) Show clearly that $\sin x = \dfrac{\sqrt{2}}{\sqrt{3}}$. **3**

(b) Hence show that $\sin \text{BAC} = \dfrac{2\sqrt{2}}{3}$. **3**

[**END OF SECTION B**]

[**END OF QUESTION PAPER**]

Mathematics
Higher Prelim Examination 2010/2011
Paper 2
Assessing Units 1 & 2

NATIONAL
QUALIFICATIONS

Time allowed - 1 hour 10 minutes

Read carefully

1. **Calculators may be used in this paper.**
2. Full credit will be given only where the solution contains appropriate working.
3. Answers obtained from readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Trigonometric formulae:

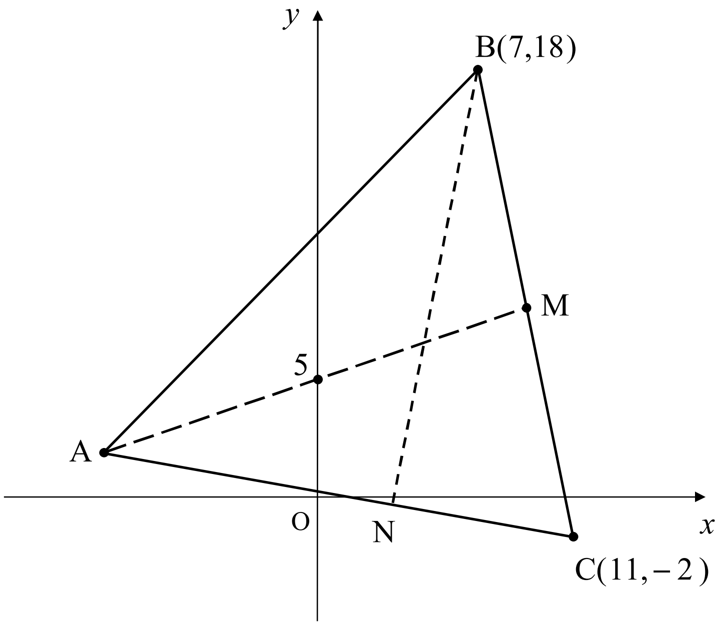
$$\begin{aligned}
\sin(A \pm B) &= \sin A\cos B \pm \cos A\sin B \\
\cos(A \pm B) &= \cos A\cos B \mp \sin A\sin B \\
\sin 2A &= 2\sin A\cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

ALL questions should be attempted

1. In the diagram below triangle ABC has two of its vertices as B(7,18) and C(11, – 2).

M is the mid-point of BC. The line AM crosses the y-axis at (0, 5).

BN is an altitude of the triangle.



(a) Show that the equation of the median AM is 3y = x + 15 **4**

(b) Given that the equation of side AB is $y = x + 11$, establish the coordinates of vertex A. **3**

(c) Hence find the equation of the altitude BN. **3**

2. (a) If $x - 1$ is a factor of $3x^3 + kx^2 + 4x - 13$, show that the value of k is 6. **3**

(b) Hence find the x-coordinate of the single stationary point on the curve with equation $y = 3x^3 + kx^2 + 4x - 13$ when k takes this value. **4**

3. Two functions, defined on suitable domains, are given as

$$f(x) = 3px + \tfrac{1}{2}p \quad \text{and} \quad g(x) = 3px^2 - 2x, \text{ where } p \text{ is a constant.}$$

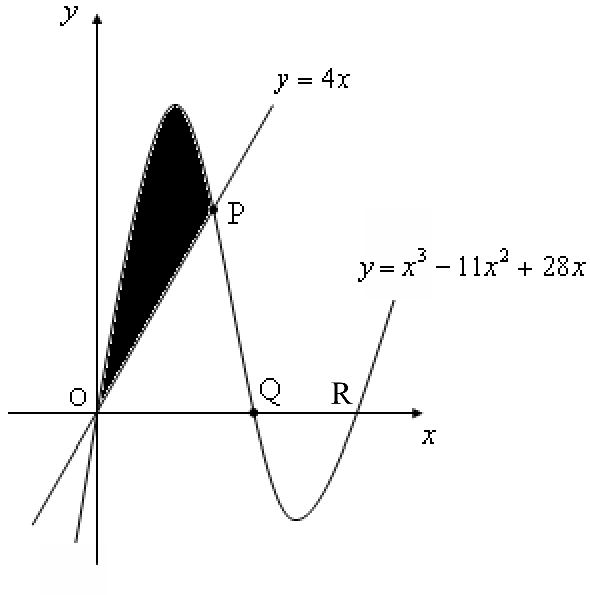
(a) Show clearly that the composite function $f(g(x))$ can be written in the form $f(g(x)) = ax^2 + bx + c$, and write down the values of a, b and c in terms of p. **4**

(b) Hence find the value of p, where $p > 0$, such that the equation $f(g(x)) = 0$ has **equal roots**. **3**

4. The diagram below, **which is not drawn to scale**, shows part of the curve with equation $y = x^3 - 11x^2 + 28x$ and the line $y = 4x$.

The line and the curve intersect at the origin and the point P.

The curve also crosses the x-axis at Q and R.



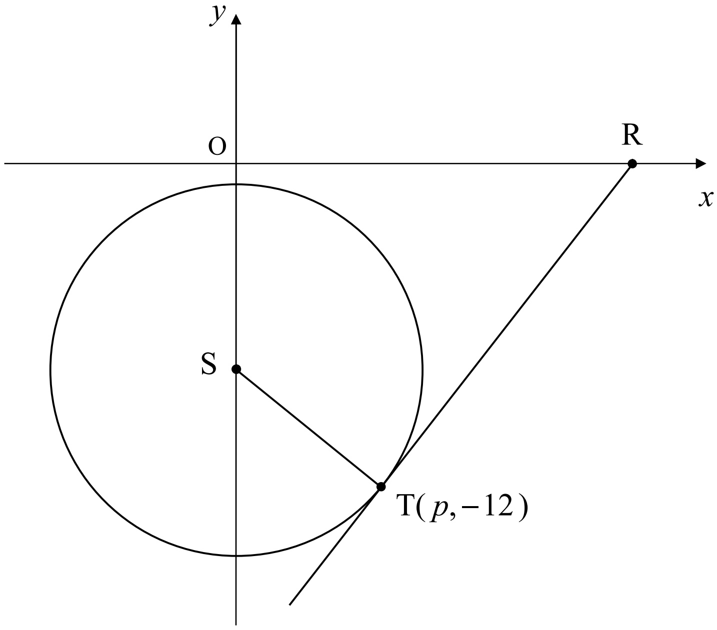
(a) Find the coordinates of P. **2**

(b) Find the coordinates of Q and R **3**

(c) Calculate the shaded area in the diagram. **5**

5. The circle, centre S, has as its equation $x^2 + y^2 + 16y + 12 = 0$.

T$(p, -12)$ is a point of tangency.



(a) Write down the coordinates of S, the centre of the circle. **1**

(b) Find the value of p, the x-coordinate of T. **2**

(c) Find the equation of the tangent through T and hence state the coordinates of R. **4**

(d) Establish the equation of the circle which passes through the points S, T and R. **3**

6. The cost of laying one mile of service piping to a wind farm is estimated by means of the formula

$$C = \frac{1800}{a} + 450a,$$

where C is the cost in tens of pounds and a is the cross-sectional area of the tube in square inches.

What cross-sectional area is the most economical to use? **5**

7. A formula is given as $E = \sin^2\theta - \frac{1}{2}\sin\theta - 1$ for $0 \le \theta \le \dfrac{\pi}{2}$.

(a) Express E in the form $E = (\sin\theta + p)^2 + q$ and write down the values of p and q . **2**

(b) Hence, or otherwise, state the minimum value of E and the corresponding replacement for θ . Give your answer correct to 2 decimal places. **3**

8. A function, defined on a suitable domain, has as its derivative $f'(x) = 3x^2 - \dfrac{10}{x^2}$.

(a) Given that $f(2) = 3$, find $f(x)$. **5**

(b) Hence find $f(1)$. **1**

[**END OF QUESTION PAPER**]

Mathematics

Higher Prelim Examination 2010/2011

NATIONAL QUALIFICATIONS

Paper 1 - Section A - Answer Sheet

NAME :	CLASS :

TEACHER :

You should use an HB pencil.
Erase all incorrect answers thoroughly.

Indicate your choice of answer with a single mark as in this example →

A	B	C	D
▭	▬	▭	▭

	A	B	C	D
1	▭	▭	▭	▭
2	▭	▭	▭	▭
3	▭	▭	▭	▭
4	▭	▭	▭	▭
5	▭	▭	▭	▭
6	▭	▭	▭	▭
7	▭	▭	▭	▭
8	▭	▭	▭	▭
9	▭	▭	▭	▭
10	▭	▭	▭	▭
11	▭	▭	▭	▭
12	▭	▭	▭	▭
13	▭	▭	▭	▭
14	▭	▭	▭	▭
15	▭	▭	▭	▭
16	▭	▭	▭	▭
17	▭	▭	▭	▭
18	▭	▭	▭	▭
19	▭	▭	▭	▭
20	▭	▭	▭	▭

Section A	/ **40**	
Section B	/ **30**	
Total (P1)	/ **70**	
Total (P2)	/ **60**	
Overall Total	/ **130**	**%**

Please make sure you have filled in all your details above before handing in this answer sheet.

Higher Grade - Paper 1 2010/2011 ANSWERS - Section A

1	A
2	D
3	B
4	C
5	C
6	A
7	D
8	D
9	A
10	C
11	A
12	B
13	C
14	D
15	A
16	C
17	D
18	D
19	B
20	B

	A	B	C	D
1	■	□	□	□
2	□	□	□	■
3	□	■	□	□
4	□	□	■	□
5	□	□	■	□
6	■	□	□	□
7	□	□	□	■
8	□	□	□	■
9	■	□	□	□
10	□	□	■	□
11	■	□	□	□
12	□	■	□	□
13	□	□	■	□
14	□	□	□	■
15	■	□	□	□
16	□	□	■	□
17	□	□	□	■
18	□	□	□	■
19	□	■	□	□
20	□	■	□	□

	Give 1 mark for each •	Illustration(s) for awarding each mark
21(a)	**ans: $p = -3$ (4 marks)**	
	•[1] finds $\frac{dy}{dx}$	•[1] $\frac{dy}{dx} = 6x^2 + 2px - 12$
	•[2] knows to sub $x = -1$	•[2] $\frac{dy}{dx} = 6(-1)^2 + 2p(-1) - 12$
	•[3] equates $\frac{dy}{dx}$ to 0	•[3] $6 - 2p - 12 = 0$
	•[4] solves for p	•[4] $p = -3$
(b)	**ans: B(2, –20) (4 marks)**	
	•[1] equates $\frac{dy}{dx}$ to 0	•[1] $\frac{dy}{dx} = 6x^2 - 6x - 12 = 0$
	•[2] factorises and solves for x	•[2] $6(x-2)(x+1) = 0;\ x = 2, -1$
	•[3] subs approp. value to find y-coordinate	•[3] $y = 2(2)^3 - 3(2)^2 - 12(2) = -20$
	•[4] states coordinates of B	•[4] B(2, –20)
(c)	**ans: $y = -12x - 1$ (3 marks)**	
	•[1] subs into equation to find y-coord. of C	•[1] $y = 2(1)^3 - 3(1)^2 - 12(1) = -13$ C$(1, -13)$
	•[2] subs into derivative to find gradient	•[2] $6(1)^2 - 6(1) - 12 = -12$
	•[3] subs into straight line equation	•[3] $y + 13 = -12(x - 1)$
22	**ans: $\theta = \frac{2\pi}{3}$; $\theta = \frac{4\pi}{3}$ (6 marks)**	
	•[1] subs for cos 2θ	•[1] $4(2\cos^2\theta - 1)$.......
	•[2] multiplies and brings terms to LHS	•[2] $8\cos^2\theta - 6\cos\theta - 5 = 0$
	•[3] factorises	•[3] $(4\cos\theta - 5)(2\cos\theta + 1)$
	•[4] solves for cos θ	•[4] $\cos\theta = -\frac{1}{2}$
	•[5] finds one solution	•[5] $\theta = \frac{2\pi}{3}$
	•[6] finds further solution	•[6] $\theta = \frac{4\pi}{3}$

	Give 1 mark for each •	Illustration(s) for awarding each mark
23(a)	**ans:** $\boldsymbol{a = \frac{1}{2}}$ **(2 marks)**	
	•¹ substitutes values	•¹ $38 = a \times 36 + 20$
	•² solves for a	•² $a = \frac{1}{2}$
(b)	**ans: 40** **(2 marks)**	
	•¹ knows how to find limit	•¹ $L = \frac{20}{\frac{1}{2}}$
	•² answer	•² 40
(b)	**ans:** $\boldsymbol{k = 5}$ **(3 marks)**	
	•¹ knows to find U_0	•¹ evidence of working backwards to U_0
	•² evaluates U_0	•² $U_2 = 32$; $U_1 = 24$; $U_0 = 8$;
	•³ finds k	•³ $k = \frac{40}{8} = 5$
24(a)	**ans: proof** **(3 marks)**	
	•¹ finds length of BD	•¹ $BD = \sqrt{8}$
	•² finds expression for sin x.	•² $\sin x = \frac{\sqrt{8}}{\sqrt{12}}$
	•³ simplifies to answer	•³ $\sin x = \frac{\sqrt{8}}{\sqrt{12}} = \frac{2\sqrt{2}}{2\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}}$
(b)	**ans: proof** **(3 marks)**	
	•¹ realises double angle	•¹ $\sin 2x = 2\sin x \cos x$
	•² finds cos x	•² $\cos x = \frac{2}{\sqrt{12}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$
	•³ substitutes and simplifies to answer	•³ $\sin 2x = 2 \times \frac{\sqrt{2}}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{2\sqrt{2}}{3}$
		Total: 70 marks

Higher Grade Paper 2 2010/2011 Marking Scheme

	Give 1 mark for each •	Illustration(s) for awarding each mark
1(a)	**ans: $3y = x + 15$ (3 marks)**	
	•[1] finds midpoint of BC	•[1] midpoint BC = (9, 8)
	•[2] finds gradient of AM	•[2] $m_{AM} = \frac{1}{3}$
	•[3] subs into equation of straight line	•[3] $y - 8 = \frac{1}{3}(x - 9)$ **or** $y = \frac{1}{3}x + 5$
(b)	**ans: A(– 9, 2) (3 marks)**	
	•[1] knows to use system of equations	•[1] evidence
	•[2] solves for x and y	•[2] $x = -9$; $y = 2$
	•[3] states coordinates of E	•[3] A(– 9, 2)
(c)	**ans: $y = 5x - 17$ (3 marks)**	
	•[1] finds gradient of AC	•[1] $m_{AB} = -\frac{1}{5}$
	•[2] finds gradient of altitude	•[2] $m_{alt} = 5$
	•[3] subs into equation of straight line	•[3] $y - 18 = 5(x - 7)$
2(a)	**ans: $k = 6$ (3 marks)**	
	•[1] knows to use synthetic division	•[1] evidence
	•[2] uses synthetic division correctly	•[2] $\begin{array}{c\|cccc} 1 & 3 & k & 4 & -13 \\ & & 3 & k+3 & k+7 \\ \hline & 3 & k+3 & k+7 & k-6 \end{array}$
	•[3] equates remainder to 0 and solves for k	•[3] $k - 6 = 0$; $k = 6$
(b)	**ans: $x = -\frac{2}{3}$ (4 marks)**	
	•[1] finds derivative	•[1] $\frac{dy}{dx} = 9x^2 + 12x + 4$
	•[2] makes derivative equal to 0 for SP	•[2] $9x^2 + 12x + 4 = 0$ at SP
	•[3] factorises	•[3] $(3x + 2)(3x + 2) = 0$
	•[4] solves for x	•[4] $x = -\frac{2}{3}$

	Give 1 mark for each •	Illustration(s) for awarding each mark
3(a)	**ans: $a = 9p^2$; $b = -6p$; $c = \frac{1}{2}p$ (4 marks)**	
	•¹ knows to substitute	•¹ evidence of sub. one function into other
	•² substitutes correctly	•² $3p[x(3px-2)]+\frac{1}{2}p$
	•³ simplifies to correct form	•³ $3p(3px^2-2x)+\frac{1}{2}p$; $9p^2x^2-6px+\frac{1}{2}p$
	•⁴ states values of a, b and c	•⁴ $a = 9p^2$; $b = -6p$; $c = \frac{1}{2}p$
(b)	**ans: $p = 2$ (3 marks)**	
	•¹ knows discriminant = 0	•¹ $b^2-4ac=0$ [stated or implied]
	•² substitutes values and simplifies	•² $(-6p)^2-4\times9p^2\times\frac{1}{2}p=0$; $36p^2-18p^3=0$
	•³ solves and discards	•³ $18p^2(2-p)=0$; $p=2$
4(a)	**ans: P(3, 12); Q(4, 0) (5 marks)**	
	•¹ for P: knows to equates functions	•¹ $x^3-11x^2+28x=4x$
	•² finds x – coord. of P	•² $x=3$
	•³ for Q: equates function to 0	•³ $x^3-11x^2+28x=0$
	•⁴ solves for x	•⁴ $x=4$
	•⁵ states coords. of P and Q	•⁵ P(3, 12); Q(4, 0)
(b)	**ans: $24\frac{1}{12}$ units² (5 marks)**	
	•¹ knows to integrate with limits	•¹ $\int_0^3 \ldots\ldots\ldots$
	•² evidence of top curve – bottom curve	•² $\int_0^3 x^3-11x^2+24x\,dx$
	•³ integrates	•³ $\left[\frac{x^4}{4}-\frac{11x^3}{3}+12x^2\right]_0^3$
	•⁴ subs values	•⁴ $\left[\frac{3^4}{4}-11(3)^3+14(3)^2\right]_0^3$
	•⁵ evaluates integral	•⁵ 29.25 units²

	Give 1 mark for each •	Illustration(s) for awarding each mark
5(a)	**ans: (0, – 8)** **(1 mark)** •[1] states centre of circle	 •[1] (0, – 8)
(b)	**ans: $p = 6$** **(2 marks)** •[1] subs into equation of circle •[2] solves for p	 •[1] $p^2 + 144 - 192 + 12 = 0$ •[2] $p^2 = 36$; $p = 6$
(c)	**ans: $2y = 3x - 42$** **(4 marks)** •[1] finds gradient of ST •[2] finds gradient of tangent •[3] subs into equation of straight line •[4] finds coords of point R	 •[1] $m_{ST} = -2/3$ •[2] $m_{tan} = \frac{3}{2}$ •[3] $y + 12 = \frac{3}{2}(x - 6)$ [or equivalent] •[4] $3x - 42 = 0$; $x = 14$ (14, 0)
(d)	**ans: $(x - 7)^2 + (y + 4)^2 = 65$** **(3 marks)** •[1] finds midpoint of SR (centre of circle) •[2] finds radius •[3] subs into equation of circle	 •[1] centre of circle (7, – 4) •[2] $\sqrt{65}$ •[3] $(x-7)^2 + (y+4)^2 = 42$
6	**ans: $a = 2$** **(5 marks)** •[1] knows to make derivative equal to 0 •[2] prepares to differentiate •[3] finds derivative •[4] attempts to solve for a •[5] solves for a	 •[1] C' = 0 [stated or implied] •[2] $C = = -\frac{16200}{9}a^{-1} + 450a$ •[3] $C' = -\frac{16200}{9a^2} + 450 = 0$ •[4] $-\frac{16200}{9a^2} = -450\,; 4050a^2 = 16200$ •[5] $a^2 = 4$; $a = 2$
7(a)	**ans: $p = -\frac{1}{4}$; $q = -\frac{17}{16}$** **(2 marks)** •[1] completes square •[2] states values of p and q	 •[1] $(\sin\theta - \frac{1}{4})^2 - \frac{17}{16}$ •[2] $p = -\frac{1}{4}$; $q = -\frac{17}{16}$

	Give 1 mark for each •	Illustration(s) for awarding each mark
(b)	**ans: 0·25 radians** **(3 marks)**	
	•[1] states minimum value	•[1] minimum value = $-\dfrac{17}{16}$
	•[2] attempts to find θ	•[2] $\sin\theta = \dfrac{1}{4}$
	•[3] finds value of θ	•[3] θ = 0·25 radians
8(a)	**ans:** $f(x) = x^3 + \dfrac{10}{x} - 10$ **(5 marks)**	
	•[1] knows to integrate	•[1] $f(x) = \int f'(x)\, dx$ [stated or implied]
	•[2] integrates first term correctly	•[2] $= \dfrac{3x^3}{3}$ ....
	•[3] 2nd term integrated correctly + C	•[3] $=$ .... $-\dfrac{10x^{-1}}{-1} + C$
	•[4] equates from additional information	•[4] $3 = 2^3 + \dfrac{10}{2} + C$
	•[5] solves for C and states $f(x) = ...$	•[5] $\therefore\ C = -10$ and $f(x) = x^3 + \dfrac{10}{x} - 10$
(b)	**ans:** $f(1) = 1$ **(1 mark)**	
	•[1] calculates to answer	•[1] $f(1) = 1^3 + \dfrac{10}{1} - 10 = 1$
		Total: 60 marks