

MADRAS COLLEGE



Mathematics
Higher Prelim Examination 2011/2012
Paper 1
Assessing Units 1 & 2

**NATIONAL
QUALIFICATIONS**

Time allowed - 1 hour 30 minutes

Read carefully

Calculators may NOT be used in this paper.

Section A - Questions 1 - 20 (40 marks)

Instructions for the completion of **Section A** are given on the next page.

For this section of the examination you should use an **HB pencil**.

Section B (30 marks)

1. Full credit will be given only where the solution contains appropriate working.
2. Answers obtained by readings from scale drawings will not receive any credit.

Read carefully

- 1 Check that the answer sheet provided is for **Mathematics Higher Prelim 2011/2012 (Section A)**.
- 2 For this section of the examination you must use an **HB pencil** and, where necessary, an eraser.
- 3 Make sure you write your **name, class and teacher** on the answer sheet provided.
- 4 The answer to each question is **either** A, B, C or D. Decide what your answer is, then, using your pencil, put a horizontal line in the space below your chosen letter (see the sample question below).
- 5 There is **only one correct** answer to each question.
- 6 Rough working should **not** be done on your answer sheet.
- 7 Make sure at the end of the exam that you hand in your answer sheet for Section A with the rest of your written answers.

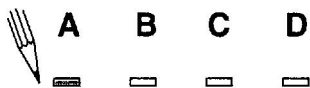
Sample Question

A line has equation $y = 4x - 1$.

If the point $(k, 7)$ lies on this line, the value of k is

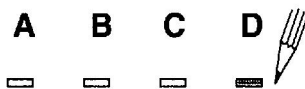
- A 2
- B 27
- C 1.5
- D -2

The correct answer is **A** \rightarrow 2. The answer **A** should then be clearly marked in pencil with a horizontal line (see below).



Changing an answer

If you decide to change an answer, carefully erase your first answer and using your pencil, fill in the answer you want. The answer below has been changed to **D**.



FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r .

Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

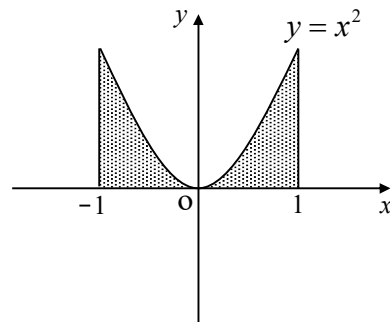
$$= 1 - 2 \sin^2 A$$

SECTION A
ALL questions should be attempted

1. The gradient of any line perpendicular to the line with equation $3x + 2y = 5$ is
- A -3
B $\frac{2}{3}$
C $-\frac{3}{2}$
D $\frac{1}{3}$
2. The rate of change of the function $y = x^3$ when $x = -1$ is
- A -1
B 0
C 1
D 3
3. A sequence is defined by the recurrence relation $U_{n+1} = 0.5U_n + 12$ with $U_0 = 16$.
 $U_1 - U_2$ equals
- A 42
B -2
C 4
D 2

4. The shaded area in the diagram equals

- A $\frac{1}{3}$ square units
B 4 square units
C $\frac{2}{3}$ square units
D 1 square unit



5. Two functions, defined on suitable domains, are given as $f(x) = \frac{1}{x} - 4$ and $g(x) = -8x$.

The value of $g(f(0.5))$ is

- A $-4\frac{1}{4}$
- B -8
- C 16
- D -16

6. In each of the following equations x and y are variables.

For which of the equations is $x = 0$, $y = 0$ the only possible solution?

- A $xy = 0$
- B $x - y = 0$
- C $x^2 + y^2 = 0$
- D $x^3 - y^3 = 0$

7. The maximum value of $4 \sin x \cos x$ is

- A 4
- B 1
- C 0
- D 2

8. The remainder on dividing the polynomial $x^3 - 3x + 6$ by $x - 2$ is

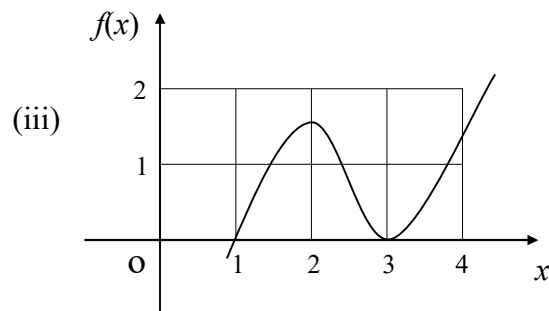
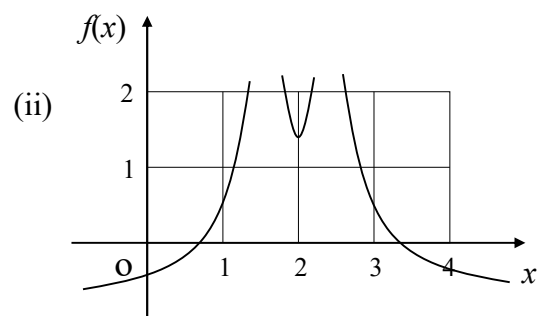
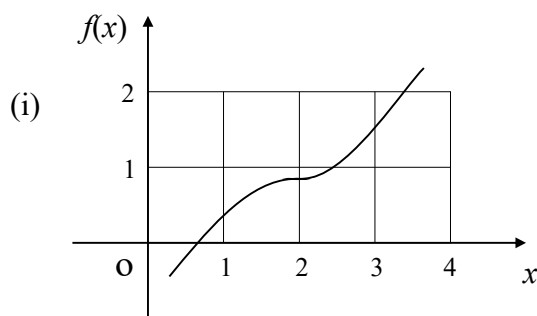
- A 4
- B 8
- C 16
- D 6

9. The function f such that $f(x) = (x-1)(x+5)$ has a stationary value when x equals

- A -5
- B 2
- C -2
- D 1

10. Which of the graphs (i), (ii) or (iii) could be that of a function f such that

$$f'(1) > 0, \quad f'(2) = 0 \quad \text{and} \quad f'(3) > 0?$$



- A (i) only
- B (ii) only
- C (iii) only
- D (i) and (iii) only

11. All the values of x which satisfy $(x-4)(x+3) \geq 0$ are

- A $-4 \leq x \leq 3$
- B $-3 \leq x \leq 4$
- C $x < -3$ or $x > 4$
- D $x \leq -3$ or $x \geq 4$

12. With k being the constant of integration, $\int x^{\frac{1}{2}} dx$ equals

A $\frac{3}{2}x^{\frac{3}{2}} + k$

B $\frac{1}{2x^{\frac{1}{2}}} + k$

C $\frac{1}{2}x^{\frac{3}{2}} + k$

D $\frac{2}{3}x^{\frac{3}{2}} + k$

13. Given that the points $(-2, 1)$, $(0, 7)$ and $(1, k)$ are collinear, then k equals

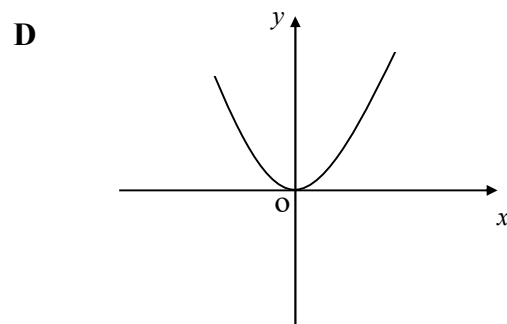
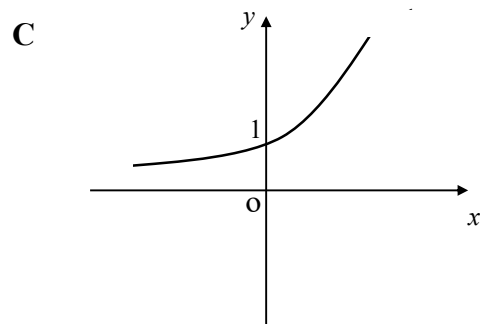
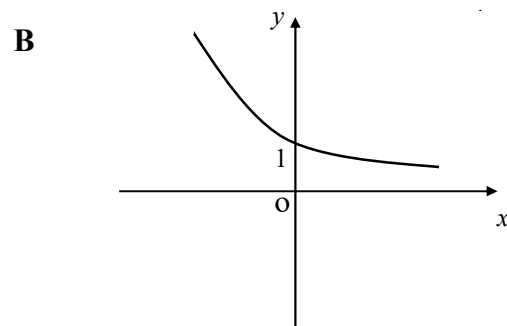
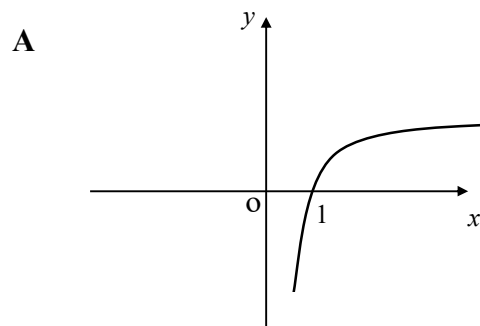
A 13

B 10

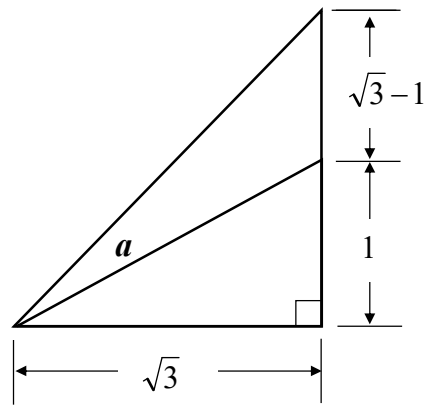
C 0

D -18

14. Which of the following could represent part of the graph of $y = 2^x$?



15. Consider the diagram



Angle a , in radians, is

- A $\frac{\pi}{6}$
B $\frac{\pi}{12}$
C $\frac{\pi}{4}$
D unknown without the use of a calculator
16. Here are 4 terms used to describe the roots of a quadratic equation
(1) real (2) unequal (3) equal (4) non-real
Which of them describe(s) the roots of $2x^2 - 3x + 1 = 0$?

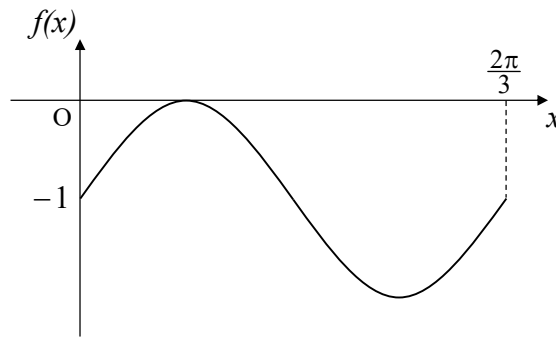
- A (4) only
B (3) only
C (1) and (3)
D (1) and (2)

17. The circle with equation $x^2 + y^2 = 25$ is moved 6 units to the left parallel to the x -axis and 4 units down parallel to the y -axis.

The equation of the circle in this new position is

- A $(x - 6)^2 + (y + 4)^2 = 25$
B $(x + 6)^2 + (y + 4)^2 = 25$
C $(x + 6)^2 + (y - 4)^2 = 25$
D $(x - 6)^2 + (y - 4)^2 = 25$

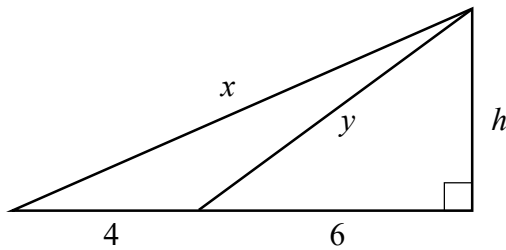
18. The diagram below shows part of the graph of a trigonometrical function.



The most likely function could be $f(x) = \dots$

- A $-\sin x$
- B $-\cos 3x$
- C $\sin 3x - 1$
- D $1 - \sin 3x$

19.



From the above diagram, the value of $x^2 - y^2$ is

- A 64
- B 16
- C 8
- D 4

20. Solve $2\sin x = \sqrt{3}$ for x , where $0 \leq x \leq 2\pi$

- A $\pi/3$ and $5\pi/3$
- B $\pi/3$ and $2\pi/3$
- C $\pi/6$ and $5\pi/6$
- D $\pi/6$ and $11\pi/6$

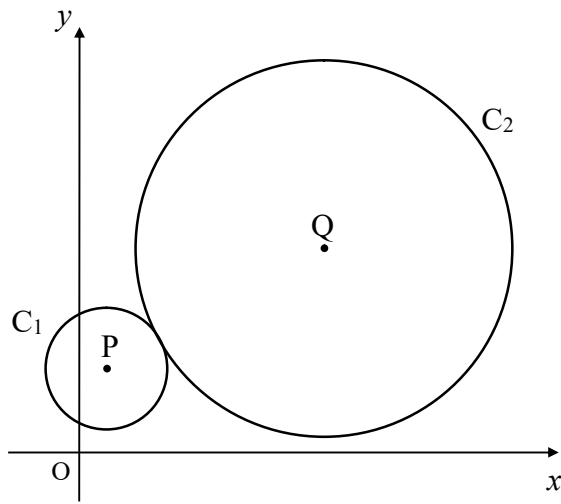
[END OF SECTION A]

SECTION B

ALL questions should be attempted

21. The circle C_1 has $P(1, 3)$ as its centre and a radius of $\sqrt{5}$ units.

The circle C_2 has as its equation $x^2 + y^2 - 18x - 14y + 85 = 0$.

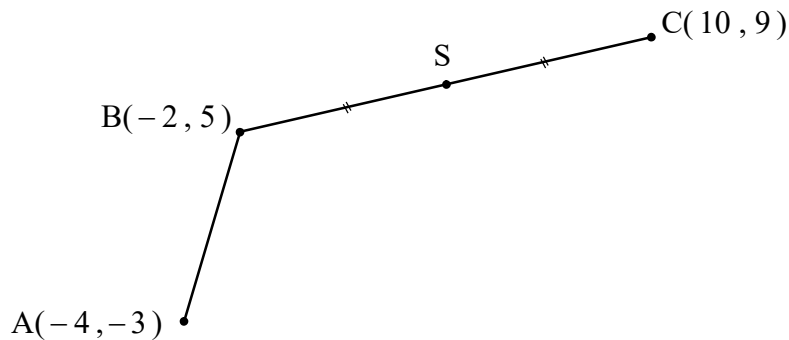


(a) Find the coordinates of Q , the centre of C_2 , and the radius of this larger circle. **3**

(b) Show clearly that C_1 touches C_2 at a single point. **3**

22. Given that $\int_0^a (4 - 3x)^2 dx = 8$, find the value of a . **5**

23. A, B and C have coordinates $(-4, -3)$, $(-2, 5)$ and $(10, 9)$ respectively as shown.
S is the mid-point of BC.



- (a) Find the equation of the line through S parallel to AB. 4
- (b) Find the coordinates of the point D where ABSD is a parallelogram. 2
24. Three functions, defined on suitable domains, are given as
- $$f(x) = \sin x, \quad g(x) = x^2 \quad \text{and} \quad h(x) = 1 - 2x.$$
- (a) Show clearly that the function k , where $k(x) = h(g(f(x)))$, can be written in its simplest form as $k(x) = \cos 2x$. 3
- (b) Hence find the value of $k\left(\frac{5\pi}{12}\right)$. 3
25. The line $x = 3y + 10$ is a tangent to the circle with equation $x^2 + y^2 - 4x - 8y - 20 = 0$ at the point P.
- A second line with equation $y = kx - 4$ also passes through P.
- Find the value of k , the gradient of this second line. 7

[END OF SECTION B]

[END OF QUESTION PAPER]

MADRAS COLLEGE



Mathematics
Higher Prelim Examination 2011/2012
Paper 2
Assessing Units 1 & 2

**NATIONAL
QUALIFICATIONS**

Time allowed - 1 hour 10 minutes

Read carefully

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FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

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Trigonometric formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

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$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

ALL questions should be attempted

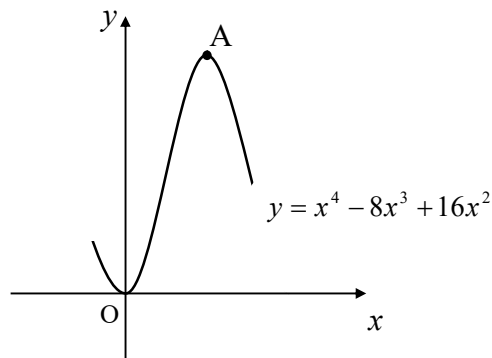
1. (a) A function f , defined on a suitable domain, is given as $f(x) = (x-1)^2$.

A second function h is such that $h(x) = x^2(f(x-3))$.

Show clearly that h can be written in the form $h(x) = x^4 - 8x^3 + 16x^2$.

3

(b) Part of the graph of $y = h(x)$ is shown below.



Find the coordinates of point A.

5

2. Two unique sequences are defined by the following recurrence relations

$$U_{n+1} = pU_n + 6 \quad \text{and} \quad U_{n+1} = p^2U_n + 9, \quad \text{where } p \text{ is a constant.}$$

(a) If both sequences have the same limit, find the value of p .

4

(b) For both sequences $U_0 = 100$, find the difference between their first terms.

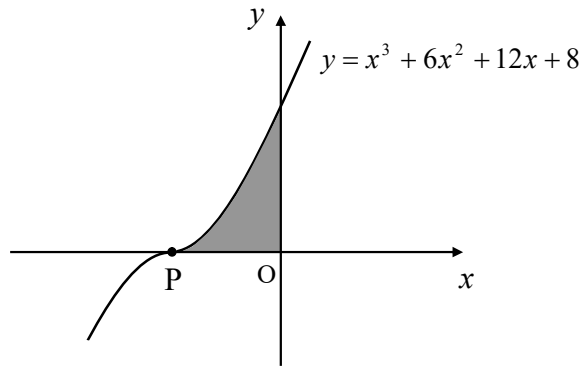
3

3. Solve algebraically the equation

$$5 \sin 2x^\circ + 4 \sin x^\circ = 0 \quad \text{for } 0 \leq x < 360.$$

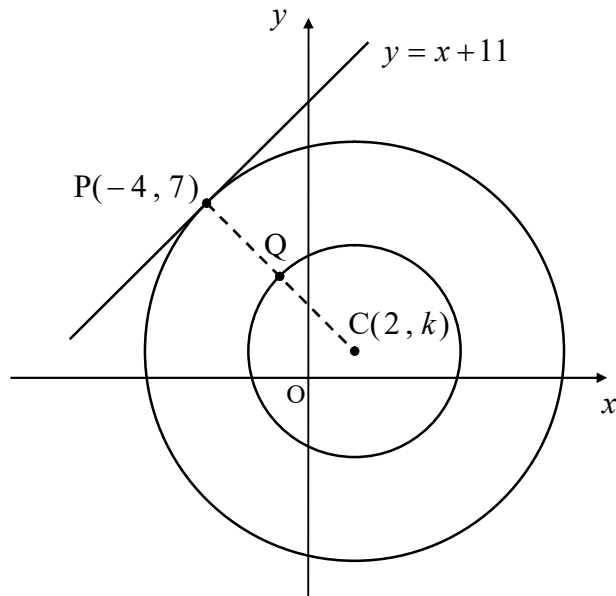
5

4. Part of the graph of $y = x^3 + 6x^2 + 12x + 8$ is shown in the diagram.



- (a) Find the coordinates of P. 3
- (b) Hence calculate the shaded area. 4

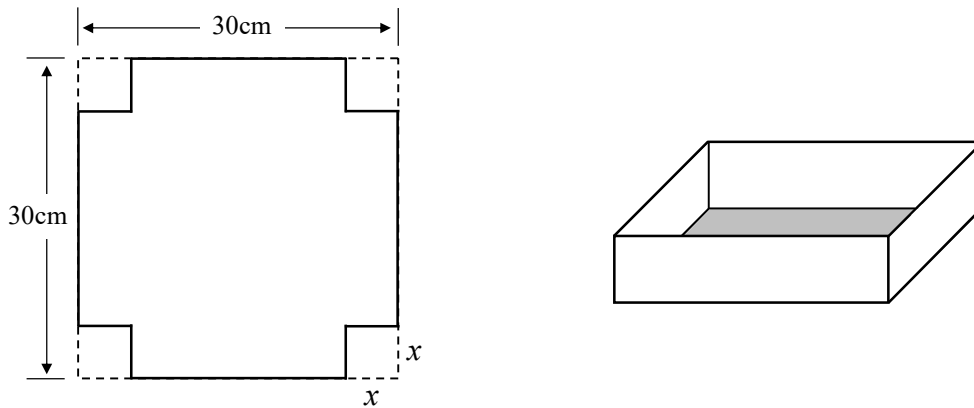
5. The diagram shows two concentric circles with centre $C(2, k)$.
 The larger of the two circles has the line with equation $y = x + 11$ as a tangent.
 The point $P(-4, 7)$ is the point of tangency between this line and the circle.



- (a) By considering gradients, find the value of k , the y -coordinate of the point C . 3
- (b) Hence find the equation of the smaller circle given that Q is the mid-point of PC . 3

6. From a square sheet of metal of side 30 centimetres, equal squares of side x centimetres are removed from each corner.

The sides are then folded up and sealed to form an open cuboid.



- (a) Show that the volume of this resulting cuboid is given by

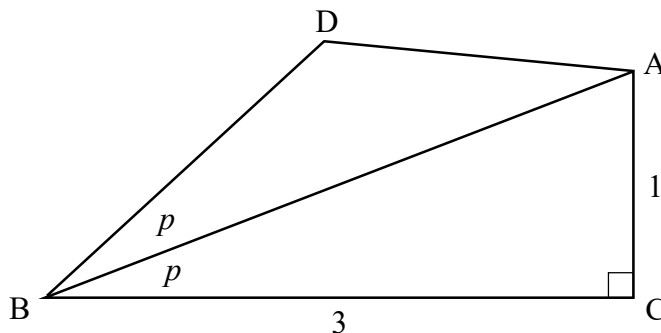
$$V(x) = 4x^3 - 120x^2 + 900x. \quad 3$$

- (b) If the cuboid is to have **maximum** possible volume, what size of square should be removed from each corner? 5

- (c) How many litres of water would this particular cuboid hold? 1

7. Consider the diagram below. Angle $ABC = \text{angle } DBA = p$.

Triangle ACB is right-angled with BC equal to 3 and CA equal to 1 unit.



Show clearly that the **exact value** of $\cos DBC$ is $\frac{4}{5}$. 5

8. A designer is testing two model racing cars along a straight track.

Each car completes a single run and the following information is recorded.

	Speed	Distance
Car A	$k - x$	3
Car B	k	$4x$



(a) Given that both cars completed the run in **exactly the same time**, show clearly that the following equation can be constructed.

$$4x^2 - 4kx + 3k = 0 \quad 3$$

(b) Find the value of the constant k if the equation $4x^2 - 4kx + 3k = 0$ has **equal roots** and $k > 0$. 3

(c) Hence find x when k takes this value. 2

9. The tangent to the curve $y = x + \frac{p}{\sqrt{x}}$, at the point where $x = 4$, is parallel to the line with equation $x + y = 10$.

Find the value of p . 5

[END OF QUESTION PAPER]

Paper 1 - Section A - Answer Sheet

NAME :

CLASS :

TEACHER :

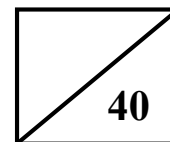
You should use an HB pencil.
 Erase all incorrect answers thoroughly.

Indicate your choice of answer with a single mark as in this example →

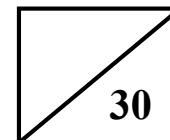
A	B	C	D
<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

	A	B	C	D
1	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
2	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
3	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
4	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
5	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
6	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
7	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
8	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
9	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
10	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
11	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
12	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
13	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
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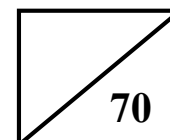
Section A



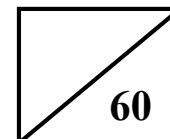
Section B



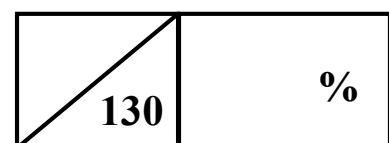
Total (P1)



Total (P2)



Overall Total



Please make sure you have filled in all your details above before handing in this answer sheet.

- 1 B
- 2 D
- 3 B
- 4 C
- 5 C
- 6 C
- 7 D
- 8 B
- 9 C
- 10 A
- 11 D
- 12 D
- 13 B
- 14 C
- 15 B
- 16 D
- 17 B
- 18 C
- 19 A
- 20 B

	A	B	C	D
1	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
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16	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>
17	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
18	<input type="checkbox"/>	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>
19	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
20	<input type="checkbox"/>	<input checked="" type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

	Give 1 mark for each •	Illustration(s) for awarding each mark
21(a)	ans: Q(9, 7); ($\sqrt{45}$) or $3\sqrt{5}$ (3 marks)	<ul style="list-style-type: none"> •¹ Q(9, 7) •² $r^2 = 9^2 + 7^2 - 85$ •³ $r = \sqrt{45}$ or $3\sqrt{5}$
	<ul style="list-style-type: none"> •¹ states centre of C_2 •² knows how to find radius •³ evaluates 	
(b)	ans: proof (3 marks)	<ul style="list-style-type: none"> •¹ $PQ^2 = 8^2 + 4^2$; $PQ = \sqrt{80} = 4\sqrt{5}$ •² $\sqrt{5} + 3\sqrt{5} = 4\sqrt{5}$ •³ distance between centres = sum of radii so circles touch at one point
	<ul style="list-style-type: none"> •¹ finds distance between centres •² finds total of 2 radii •³ conclusion 	
22	ans: $a = 2$ (5 marks)	<ul style="list-style-type: none"> •¹ $\int_0^a 16 - 24x + 9x^2 dx$ •² $[16x - 12x^2 + 3x^3]_0^a$ •³ $16a - 12a^2 + 3a^3 = 8$ •⁴ $(a - 2)(3a^2 - 6a + 4) = 0$ •⁵ $a = 2$
<ul style="list-style-type: none"> •¹ prepares to integrate •² integrates •³ subs and equates to 8 •⁴ factorises (uses synthetic division) •⁵ realises only solution is 2 		
23(a)	ans: $y = 4x - 9$ (4 marks)	<ul style="list-style-type: none"> •¹ S(4, 7) •² $m_{AB} = \frac{5+3}{-2+4} = 4$ •³ $m = 4$ •⁴ $y - 7 = 4(x - 4)$
	<ul style="list-style-type: none"> •¹ find coordinates of S •² finds gradient of AB •³ knows to use parallel gradient •⁴ subs info into equation of straight line 	
(b)	ans: D(2, -1) (2 marks)	<ul style="list-style-type: none"> •¹ evidence of suitable strategy •² D(2, -1)
	<ul style="list-style-type: none"> •¹ evidence of 'stepping out' or other suitable method •² answer 	

	Give 1 mark for each •	Illustration(s) for awarding each mark
24(a)	ans: proof (3 marks) <ul style="list-style-type: none"> ●¹ finds $g(f(x))$ ●² finds $h(g(f(x)))$ ●³ completes proof 	<ul style="list-style-type: none"> ●¹ $g(f(x)) = (\sin x)^2 = \sin^2 x$ ●² $h(g(f(x))) = 1 - 2 \sin^2 x$ ●³ $1 - 2 \sin^2 x = \cos 2x$
(b)	ans: $-\frac{\sqrt{3}}{2}$ (3 marks) <ul style="list-style-type: none"> ●¹ subs value into formula ●² finds equivalent angle ●³ evaluates 	<ul style="list-style-type: none"> ●¹ $\cos 2\left(\frac{5\pi}{12}\right) = \cos \frac{5\pi}{6}$ ●² $-\cos \frac{\pi}{3}$ ●³ $-\frac{\sqrt{3}}{2}$
25	ans: $k = \frac{1}{2}$ (7 marks) <ul style="list-style-type: none"> ●¹ knows to sub line into circle ●² multiplies ●³ simplifies ●⁴ solves for y ●⁵ subs to find x ●⁶ subs point into line ●⁷ solves for k 	<ul style="list-style-type: none"> ●¹ $(3y + 10)^2 + y^2 - 4(3y + 10) - 8y - 20 = 0$ ●² $9y^2 + 60y + 100 + y^2 - 12y - 40 - 8y - 20 = 0$ ●³ $10y^2 + 40y + 40 = 0$ ●⁴ $10(y^2 + 4y + 4) = 0; (y + 2)^2 = 0; y = -2$ ●⁵ $x = 3(-2) + 10 = 4$ ●⁶ $-2 = 4k - 4$ ●⁷ $k = \frac{1}{2}$

Total: 70 marks

	Give 1 mark for each •	Illustration(s) for awarding each mark
1(a)	ans: proof (3 marks)	
	<ul style="list-style-type: none"> ●¹ subs one function into the other ●² multiplies inner bracket ●³ multiplies to answer 	<ul style="list-style-type: none"> ●¹ $f(x-3) = (x-3-1)^2 = (x-4)^2$ ●² $h(x) = [x^2 - 8x + 16]x^2$ ●³ $x^4 - 8x^3 + 16$
(b)	ans: A(2, 16) (5 marks)	
	<ul style="list-style-type: none"> ●¹ knows to make $\frac{dy}{dx} = 0$ ●² differentiates ●³ solves for x ●⁴ chooses correct values & subs to find y ●⁵ states coordinates of A 	<ul style="list-style-type: none"> ●¹ $\frac{dy}{dx} = 0$ ●² $\frac{dy}{dx} = 4x^3 - 24x^2 + 32x = 0$ at SP ●³ $4x(x-4)(x-2) = 0; x = 2, 4$ ●⁴ $y = (2)^4 - 8(2)^3 + 16(2)^2 = 16$ ●⁵ A(2, 16)
2(a)	ans: $p = 0.5$ (4 marks)	
	<ul style="list-style-type: none"> ●¹ gives expression for both limits ●² equates limits ●³ starts to solve ●⁴ solves and discards 	<ul style="list-style-type: none"> ●¹ $L = \frac{6}{1-p}; L = \frac{9}{1-p^2}$ ●² $\frac{6}{1-p} = \frac{9}{1-p^2}$ ●³ $6 - 6p^2 = 9 - 9p; 6p^2 - 9p + 3 = 0$ ●⁴ $3(2p-1)(p-1) = 0; p = 0.5$ or $p = 1$
(b)	ans: 22 (3 marks)	
	<ul style="list-style-type: none"> ●¹ finds 1st term for one RR ●² finds 1st term for other RR ●³ calculates difference in terms 	<ul style="list-style-type: none"> ●¹ $U_1 = \frac{1}{2}(100) + 6 = 56$ ●² $U_1 = \left(\frac{1}{2}\right)^2(100) + 6 = 34$ ●³ $56 - 34 = 22$
3	ans: $0^\circ, 113.6^\circ, 246.4^\circ, 180^\circ$ (5 marks)	
	<ul style="list-style-type: none"> ●¹ subs for $\sin 2x^\circ$ and simplifies ●² factorises ●³ solves for $\sin x^\circ$ and $\cos x^\circ$ ●⁴ solutions from $\sin x^\circ$ ●⁵ solutions from $\cos x^\circ$ 	<ul style="list-style-type: none"> ●¹ $5(2 \sin x^\circ \cos x^\circ) + 4 \sin x^\circ = 0$ $10 \sin x^\circ \cos x^\circ + 4 \sin x^\circ = 0$ ●² $2 \sin x^\circ(5 \cos x^\circ + 2) = 0$ ●³ $\sin x^\circ = 0$ or $\cos x^\circ = -\frac{2}{5}$ ●⁴ $x = 0^\circ, 180^\circ$ ●⁵ $x = 113.6^\circ, 246.4^\circ$

	Give 1 mark for each •	Illustration(s) for awarding each mark
4(a)	ans: P(-2, 0) (3 marks) <ul style="list-style-type: none"> ●¹ equates function to 0 ●² solves using suitable strategy ●³ states coordinates of P 	<ul style="list-style-type: none"> ●¹ $x^3 + 6x^2 + 12x + 8 = 0$ at P ●² suitable strategy leading to $x = -2$ ●³ P(-2, 0)
(b)	ans: 4 square units (4 marks) <ul style="list-style-type: none"> ●¹ knows how to find area ●² integrates ●³ subs values ●⁴ evaluates 	<ul style="list-style-type: none"> ●¹ $\int_{-2}^0 x^3 + 6x^2 + 12x + 8 \, dx$ ●² $\left[\frac{x^4}{4} + 2x^3 + 6x^2 + 8x \right]_{-2}^0$ ●³ $0 - \left(\frac{(-2)^4}{4} + 2(-2)^3 + 6(-2)^2 + 8(-2) \right)$ ●⁴ 4 square units
5(a)	ans: $k = 1$ (3 marks) <ul style="list-style-type: none"> ●¹ finds gradient of CP ●² equates m_{CP} to expression for m_{CP} ●³ solves 	<ul style="list-style-type: none"> ●¹ $m_{\text{given line}} = 1; m_{CP} = -1$ ●² $m_{CP} = \frac{k-7}{6} = -1$ ●³ $k-7 = -6; k = 1$
(b)	ans: $(x-2)^2 + (y-1)^2 = 18$ (3 marks) <ul style="list-style-type: none"> ●¹ finds midpoint of CP ●² finds radius (length of CQ) ●³ subs into general equation of circle 	<ul style="list-style-type: none"> ●¹ Q(-1, 4) ●² $r^2 = 3^2 + 3^2 = 18$ ●³ $(x-2)^2 + (y-1)^2 = 18$

	Give 1 mark for each •	Illustration(s) for awarding each mark
6(a)	ans: proof (3 marks) <ul style="list-style-type: none"> ●¹ gives expression for length and breadth ●² subs into formula and starts to simplify ●³ completes simplification to answer 	<ul style="list-style-type: none"> ●¹ $(30 - 2x)$ ●² $x(30 - 2x)^2$ ●³ $x(900 - 120x + 4x^2)$
(b)	ans: $x = 5$ (5 marks) <ul style="list-style-type: none"> ●¹ knows to make derivative = 0 ●² takes derivative ●³ factorises and solves ●⁴ discards ●⁵ justifies answer 	<ul style="list-style-type: none"> ●¹ $V'(x) = 0$ ●² $12x^2 - 240x + 900 = 0$ ●³ $12(x - 5)(x - 15) = 0$ ●⁴ $x = 5$ ●⁵ nature table or 2nd derivative
(c)	ans: 2 litres (1 mark) <ul style="list-style-type: none"> ●¹ calculates volume 	<ul style="list-style-type: none"> ●¹ $20 \times 20 \times 5 = 2000\text{cm}^3 = 2 \text{ litres}$
7	ans: proof (5 marks) <ul style="list-style-type: none"> ●¹ finds length of AB and $\cos p$ ●² realises angle DBC = $2p$ ●³ replaces double angle ●⁴ subs value for $\cos p$ ●⁵ evaluates to answer 	<ul style="list-style-type: none"> ●¹ $AB = \sqrt{10}; \cos p = \frac{3}{\sqrt{10}}$ ●² $\cos DBC = \cos 2p$ ●³ $\cos 2p = 2 \cos^2 p - 1$ [or alternative] ●⁴ $2\left(\frac{3}{\sqrt{10}}\right)^2 - 1$ ●⁵ $2\left(\frac{9}{10}\right) - 1; \frac{18}{10} - 1 = \frac{8}{10} = \frac{4}{5}$

	Give 1 mark for each •	Illustration(s) for awarding each mark
8(a)	ans: proof (3 marks) <ul style="list-style-type: none"> •¹ states expression for both distances •² equates •³ rearranges to answer 	<ul style="list-style-type: none"> •¹ $\frac{3}{k-x}$ and $\frac{4x}{k}$ •² $\frac{3}{k-x} = \frac{4x}{k}$ •³ $3k = 4x(k-x); 3k = 4xk - 4x^2 \dots\dots$
(b)	ans: $k = 3$ (3 marks) <ul style="list-style-type: none"> •¹ knows discriminant = 0 for equal roots •² finds discriminant •³ solves and discards 	<ul style="list-style-type: none"> •¹ $b^2 - 4ac = 0$ for equal roots •² $b^2 - 4ac = (-4k)^2 - 4 \cdot 4 \cdot 3k = 0; 16k^2 - 48k = 0$ •³ $16k(k-3) = 0; k = 3$
(c)	ans: $x = \frac{3}{2}$ (2 marks) <ul style="list-style-type: none"> •¹ subs value for k and rewrites expression •² factorises and solves 	<ul style="list-style-type: none"> •¹ $4x^2 - 12x + 9 = 0$ •² $(2x-3)^2 = 0; x = \frac{3}{2}$
9	ans: $p = 32$ (5 marks) <ul style="list-style-type: none"> •¹ differentiates •² subs value to find gradient •³ finds gradient of given line •⁴ equates gradients •⁵ solves 	<ul style="list-style-type: none"> •¹ $y = x + px^{-\frac{1}{2}}; \frac{dy}{dx} = 1 - \frac{1}{2} px^{-\frac{3}{2}} = 1 - \frac{p}{2x^{\frac{3}{2}}}$ •² $1 - \frac{p}{2(4)^{\frac{3}{2}}} = 1 - \frac{p}{16}$ •³ $x + y = 10; m = -1$ •⁴ $1 - \frac{p}{16} = -1$ •⁵ $-\frac{p}{16} = -2; p = 32$
	Source of questions: Pegasys 2011 -12 Except paper 1 Q 20 – replaced with SQA (Higher) 2011 P1 Q10 but cos changed to sin	Total: 60 marks