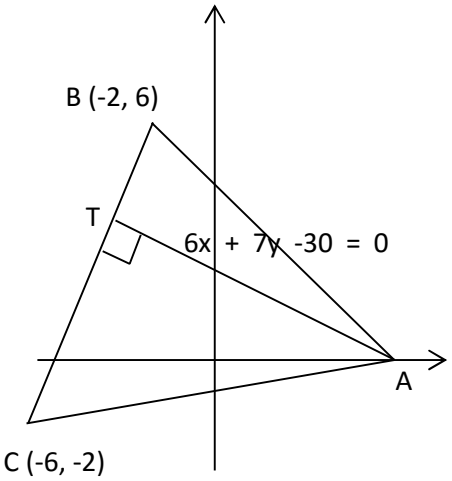
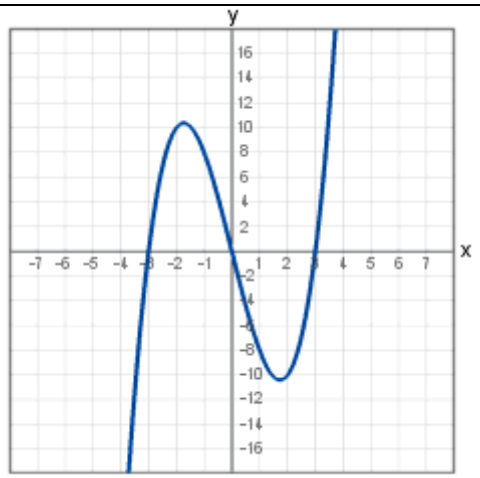
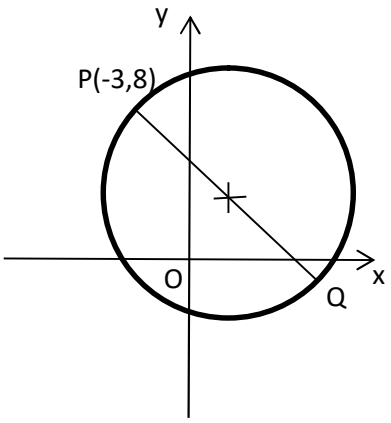
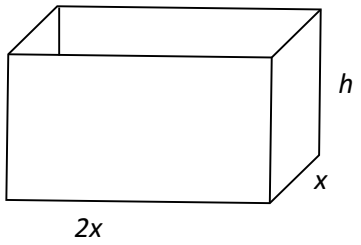
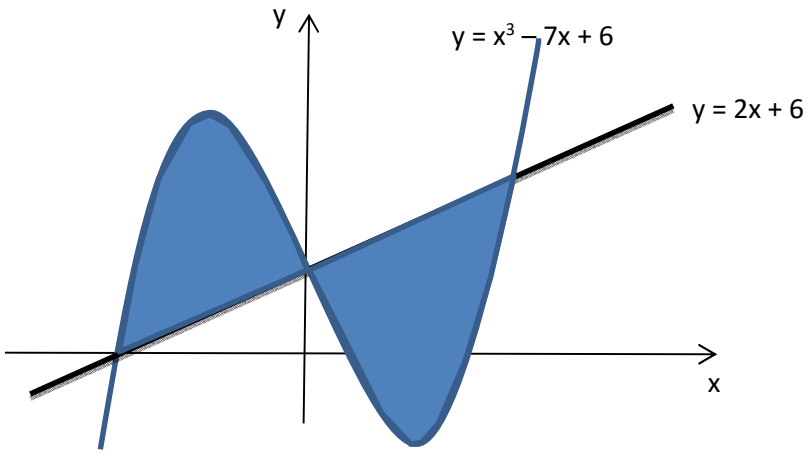
SECTION B
ALL questions should be attempted.

			Marks
21	Triangle ABC has vertex A on the x-axis, as shown in the diagram. B and C are the points (-2, 6) and (-6, –2) respectively. The equation of AB is $6x + 7y - 30 = 0$. (a) State the coordinates of A. (b) Find the equation of the altitude of the triangle from A. (c) The altitude from A meets the line BC at T. Find the coordinates of T.		1 3 4
22	The diagram shows a sketch of the function $y = f(x)$, with turning points at **(-2,10) and (2,-10)** (a) Copy the diagram and on it sketch the graph of $y = f(2x)$. (b) On a separate diagram sketch the graph of $y = 1 - f(2x)$.		2 3
23	Find the range of values of k such that the equation $kx^2 - 2x + 3 = 0$ has no real roots.		4
24	(a) Using the fact that $\frac{5\pi}{12} = \frac{\pi}{4} + \frac{\pi}{6}$ find the exact value of $\cos\left(\frac{5\pi}{12}\right)$ (b) Show that $\cos(A + B) + \cos(A - B) = 2\cos A \cos B$. (c) (i) Express $\frac{\pi}{12}$ in terms of $\frac{\pi}{4}$ and $\frac{\pi}{6}$ (ii) Hence or otherwise find the exact value of $\cos\left(\frac{5\pi}{12}\right) + \cos\left(\frac{\pi}{12}\right)$		3 2 4
	END OF QUESTION PAPER		66

PAPER 2

1	Find the coordinates of the turning points of the curve with equation $y = x^3 - 6x^2 - 15x + 2$ and determine their nature.	8
2	Functions f and g are given by $f(x) = 4x + 3$ and $g(x) = x^2 + 1$ *(a)* (i) Find $p(x)$ where $p(x) = f(g(x))$ (ii) Find $q(x)$ where $q(x) = g(f(x))$ (b) Solve $p'(x) = q'(x)$	3 3 3
3	(a) Show that $x = -1$ is a root of $x^3 + 4x^2 - 7x - 10 = 0$. (b) Hence factorise $x^3 + 4x^2 - 7x - 10$ fully.	4
4.	(a) Show that the point P(-3, 8) lies on circle C_1 with equation $(x - 2)^2 + (y - 3)^2 = 50$ (b) PQ is a diameter of this circle as shown in the diagram. Find the equation of the tangent at Q. (c)Two circles C_2 and C_3 touch C_1 at Q. The radius of each of these circles is twice the radius of circle C_1. Find the equations of circles C_2 and C_3. 	1 5 4
5.	(a) The terms of a sequence satisfy $u_{n+1} = ku_n + 6$. Find the value of k which produces a limit of 4. (b) A sequence satisfies the recurrence relation $u_{n+1} = mu_n + 6$, $u_0 = 4$. (i) Express u_1 and u_2 in terms of m. (ii) Given that $u_2 = 10$, find the value of m which produces a sequence with no limit.	2 5

6.	Solve the equation $2\cos 2x^\circ + 3\sin x^\circ = \sin^2 x^\circ$, in the interval $0 \le x \le 360^\circ$ correct to 1 decimal place.	5
7.	An open box measures internally x units by $2x$ units by h units and has an inner surface area of 18 units²  (a) show that the Volume, V units , of the cuboid is given by $V = \frac{2}{3}x(9 - x^2)$ (b) Find the exact value of x for which this volume is a maximum	3 5
8.	The diagram shows a curve with equation $y = x^3 - 7x + 6$ and the line with equation $y = 2x + 6$ (a) Find the points of intersection of curve and the line.  (b) Calculate the total shaded area.	3 9
	END OF QUESTION PAPER	60

HIGHER MATHS JANUARY PRELIM MARKING SCHEME

Answers Paper 1 Section B		
21 2009 P1 Q21 amended **8 C**	●[1] interpret x-intercept ●[2] find gradient of BC ●[3] know to use $m_1 \times m_2 = -1$ ●[4] state equation of the altitude AT ●[5] state equation of line BC ●[6] prepare to solve simultaneous equat ●[7] solve for x ●[8] solve for y	●[1] A = (5,0) ●[2] $m_{BC} = 2$ ●[3] $m_{AT} = -½$ ●[4] $y - 0 = -½(x - 5)$, $2y = -x + 5$ ●[5] $y - 6 = 2(x + 2)$, $y = 2x + 10$ ●[6] $2(2x + 10) = -x + 5$ etc ●[7] $x = -3$ ●[8] $y = 4$
22 2009 P1 Q22 amended **5 B**	●[1] scaling parallel to the x-axis ●[2] annotate graph ●[3] correct order – reflection, translation ●[4] start to annotate ●[5] complete annotation	3 points (-1,10), the origin, (1,10) ●[1] sketch and mark on point ●[2] other two points correct 3 points (-1,-9), (0,1), (1,11) ●[3] reflect in x-axis, vertical translation ●[4] sketch and mark one point ●[5] two final points marked
23 2007 P1 Q4 Amended **4 C**	●[1] know to use discriminant < 0 for no real roots ●[2] interpret the values of a, b and c ●[3] substitute ●[4] solve an inequation	●[1] $b^2 - 4ac < 0$ ●[2] $a = k$, $b = -2$, $c = 3$ ●[3] $4 - 12k < 0$ ●[4] $k > ⅓$
24 2009 P1 Q24 Amended **6 C** **3 B**	●[1] expand compound angle ●[2] substitute exact values ●[3] process to a single fraction ●[4] start proof ●[5] complete proof ●[6] identify steps ●[7] identify A and B ●[8] Substitute in 2cosAcosB	●[1] $\cos\frac{\pi}{4}\cos\frac{\pi}{6} - \sin\frac{\pi}{4}\sin\frac{\pi}{6}$ ●[2] $\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$ ●[3] $\frac{\sqrt{3}-1}{2\sqrt{2}}$ ●[4] $\cos A\cos B - \sin A\sin B + ...$ ●[5] $.....\cos A\cos B - \sin A\sin B$ ●[6] $\left(\frac{\pi}{12}\right) = \left(\frac{\pi}{4} - \frac{\pi}{6}\right)$ ●[7] $A = \frac{\pi}{4}, \quad B = \frac{\pi}{6}$ ●[8] $2 \times \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}$

	●[9] process	●[9] $\frac{\sqrt{3}}{\sqrt{2}}$
Answers - Paper 2		
1 2009 Q1 amended **8 C**	●[1] know to differentiate ●[2] differentiate ●[3] set derivative to zero ●[4] factorise ●[5] solve for x ●[6] evaluate y coordinates ●[7] justification (nature table) ●[8] interpret results	●[1] $\frac{dy}{dx} = \ldots$ (1 term correct) ●[2] $3x^2 - 12x - 15$ ●[3] $\frac{dy}{dx} = 0$, STATED EXPLICITLY ●[4] $3(x+1)(x-5) = 0$ ●[5] x = -1, ●[6] x = 5 ●[6] y = 10, y = -98 ●[7] $< \quad -1 \quad > \quad < \quad 5 \quad >$ +tve 0 -tve 0 +tve ●[8] max at (-1,10) min at (5,-98)
2 2009 Q2 amended **6 C**	●[1] substitute for g(x) in f(x) ●[2] complete ●[3] sub and complete for q(x) ●[4] simplify ●[5] differentiate ●[6] solve	●[1] $f(x^2+1)$ ●[2] $4(x^2+1)+3$ ●[3] $(4x+3)^2+1$ ●[4] $4x^2+7$ ●[5] $8x$ ●[5] $16x^2+24x+10$ $32x+24$ ●[6] x = -1
3 2009 Q3 amended **5 C**	●[1] know to use synthetic division with x = -1, or use f(-1) = 0 ●[2] no remainder ●[3] complete quadratic factor ●[4] factorise fully	●[1] -1 | 1 4 -7 -10 1 3 -10 0 ●[2] no remainder, so x=-1 is a root 0 can be highlighted or in bold ●[3] $(x+1)(\mathbf{x^2+3x-10})$ ●[4] (x +1)(x + 5)(x - 2) stated explicitly
4. 2009 Q4 amended **6 C** 4 A	●[1] substitute ●[2] find centre ●[3] use midpoint for Q ●[4] find gradient of radius	●[1] $(-3-2)^2 + (8-3)^2 = 50$ ●[2] centre = (2,3) ●[3] Q= (7,-2) ●[4] $m_{rad} = -1$

	●[5] find gradient of tangent ●[6] state equation of tangent ●[7] state radius ●[8] know how to find centre ●[9] equation of one circle ●[10] equation of the other circle	●[5] $m_{tan} = 1$ ●[6] $y - (-2) = 1(x - 7)$, $y = x - 9$ ●[7] radius = $2 \times \sqrt{50} = 10\sqrt{2}$ ●[8] centre (17,-12) ●[9] $(x + 3)^2 + (y - 8)^2 = 200$ ●[10] $(x - 17)^2 + (y + 12)^2 = 200$
5. 2005 P1 Q6 amended **2 C** **5 B**	●[1] know to find limit ●[2] process ●[3] interpret recurrence relation ●[4] interpret recurrence relation ●[5] arrange in standard form = 0 ●[6] factorise ●[7] use limit conclusion for no limit $m \le -1,\ m \ge 1$ m	●[1] $4 = \frac{6}{1-k}$ ●[2] k = -½ ●[3] $U_1 = 4m + 6$ ●[4] $U_2 = m(4m+6)+6 = 4m^2 + 6m + 6$ ●[5] $4m^2 + 6m + 6 = 10$, $4m^2 + 6m - 4 = 0$ ●[6] $2(2m - 1)(m + 2)$ ●[7] m = -2
6. 2008 P2 Q5 amended **5 B**	●[1] use double angle formula ●[2] obtain standard form (....=0) ●[3] factorise ●[4] process factors ●[5] complete solution	●[1] $\cos 2x = 1 - 2\sin^2 x$ ●[2] $5\sin^2 x - 3\sin x - 2 = 0$ ●[3] $(5\sin x + 2)(\sin x - 1) = 0$ ●[4] $\sin x = -\frac{2}{5},\ x = 203.6^\circ, 336.4^\circ$ ●[5] $\sin x = 1,\ x = 90^\circ$
7. 2004 P2 Q9 amended **5 C** **3 A**	●[1] use area facts ●[2] use volume facts ●[3] complete proof ●[4] arrange in standard form ●[5] differentiate ●[6] set derivative to zero ●[7] process ●[8] justification – nature table	●[1] $A = 4xh + 2xh + 2x^2 = 18$ ●[2] $V = 2x \times x \times h$ ●[3] $V = 2x^2 \times \left(\frac{9 - x^2}{3x}\right) =$ and complete ●[4] $V = 6x - \frac{2}{3}x^3$ ●[5] $\frac{dV}{dx} = 6 - 2x^2$ ●[6] $\frac{dV}{dx} = 0$ STATED EXPLICITLY before factorisation ●[7] $x = \sqrt{3}$ ●[8] nature table maximum: $< \sqrt{3}$ +tve; $\sqrt{3}$ 0; $> \sqrt{3}$ -tve

8. 2011 P2 Q4 amended **12 B**	●[1] Know to equate line and curve	●[1] $x^3-7x+6 = 2x+6$
	●[2] factorises equation	●[2] $x(x-3)(x+3) = 0$ Have accepted $x(x^2-9)=0$
	●[3] solves and finds coordinates	●[3] (0,6), (-3, 0, and (3,12)
	●[1] Know to integrate	●[1] $\int \ldots$ or attempt integration
	●[2] know to deal with areas on each side of the y-axis ●[3] interpret limits on one or both areas	●[2] ●[3] $\int_0^3$ | $\int_{-3}^0$
	●[4] use "upper – lower" on one or both integral	●[4] $(2x+6)-(x^3-7x+6)$ | $(x^3-7x+6)-(2x+6)$ $-x^3+9x$ | x^3-9x
	●[5] integrate	●[5] $-\frac{x^4}{4}+\frac{9x^2}{2}$ | $\frac{x^4}{4}-\frac{9x^2}{2}$
	●[6] substitute in limits	●[6] $\left(-\frac{3^4}{4}+\frac{9\times3^2}{2}\right)-0$ | $0-\left(\frac{3^4}{4}-\frac{9\times3^2}{2}\right)$
	●[7] & ●[8] evaluate the area of one or both sides ●[9] state total area	●[7] $\frac{81}{4}$ | ●[8] $\frac{81}{4}$ Allow -81/4 → area = 81/4units² ●[9] $\frac{81}{2}$
	For a statement that the graph is symmetrical so total area = 2 x area 1 Have given ●[2] ●[9] but not ●[8]	For so total area = 2 x area 1 with no mention of symmetry Have given ●[9] but not ●[2] or ●[8]

TOTAL 60