MADRAS COLLEGE

Mathematics
Higher Prelim Examination 2013/2014
March Prelim Exam

NATIONAL
QUALIFICATIONS

Time allowed - 1 hour 30 minutes

Read carefully

Calculators may be used in this paper.

Section A - Questions 1 - 9 (18 marks)

1.Please construct an answer grid at the top of your paper for the multiple choice answers

Section B Questions 10 – 15 (36 marks)

1. Full credit will be given only where the solution contains appropriate working.
2. Answers obtained by readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Trigonometric formulae:

$$\begin{aligned}
\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\sin 2A &= 2 \sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

Scalar Product: $\boldsymbol{a} \cdot \boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}|\cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$.

or

$\boldsymbol{a} \cdot \boldsymbol{b} = \boldsymbol{a}_1\boldsymbol{b}_1 + \boldsymbol{a}_2\boldsymbol{b}_2 + \boldsymbol{a}_3\boldsymbol{b}_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\, dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

1. What is the value of $(\boldsymbol{i}+2\boldsymbol{j}).(\boldsymbol{j}+2\boldsymbol{k})$?

A 9
B 2
C 1
D 0

2. What is the angle between the vectors $\begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$?

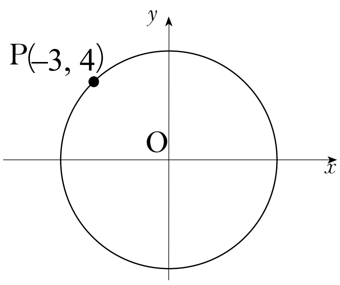
A $\frac{\pi}{6}$

B $\frac{\pi}{4}$

C $\frac{\pi}{3}$

D $\frac{\pi}{2}$

3. The point P(−3, 4) lies on the circle $x^2 + y^2 = 25$ as shown in the diagram.



What is the gradient of the tangent at P ?

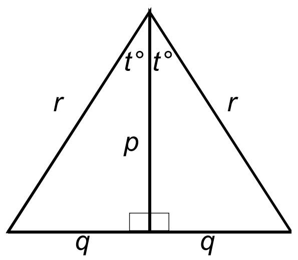
A $-\frac{4}{3}$

B $-\frac{1}{5}$

C $\frac{3}{4}$

D $\frac{5}{3}$

4. The diagram shows an isosceles triangle with lengths as shown.



Express sin $2t°$ in terms of p, q and r.

A $\sin 2t^\circ = \dfrac{2q^2}{r^2}$

B $\sin 2t^\circ = \dfrac{2q}{r}$

C $\sin 2t^\circ = \dfrac{2p}{r}$

D $\sin 2t^\circ = \dfrac{2pq}{r^2}$

5. Given that $\log_{10}(y) = 2\log_{10}(x) + \log_{10}(3)$, express y in terms of x.

A $y = 2x + 3$

B $y = 6x$

C $y = 3x^2$

D $y = 3 \times 2^x$

6. Find $\displaystyle\int_0^{\frac{\pi}{4}} \cos 2x \; dx.$

A $-2\sqrt{2}$

B $\dfrac{1}{2}$

C 0

D $\sqrt{2}$

7. The line joining the points $(-2, -3)$ and $(6, k)$ has gradient $\frac{2}{3}$.

What is the value of k ?

A $\frac{14}{3}$

B $\frac{7}{3}$

C $-\frac{1}{3}$

D -9

8. Given that $\boldsymbol{u} = \begin{pmatrix} 3 \\ -4 \\ 1 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} -2 \\ -1 \\ 1 \end{pmatrix}$, what is the magnitude of $(\boldsymbol{u} - \boldsymbol{v})$?

A 1

B $\sqrt{20}$

C $\sqrt{32}$

D $\sqrt{34}$

9. The maximum value of $1 - \cos\left(x - \frac{\pi}{6}\right)$, $0 \leq x < 2\pi$ occurs when $x = t$.

What is the value of t ?

A 0

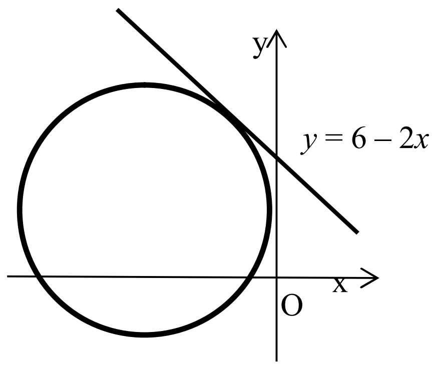
B $\frac{\pi}{6}$

C $\frac{2\pi}{3}$

D $\frac{7\pi}{6}$

SECTION B

ALL questions should be attempted.

		Marks
10	D, E and F have coordinates (-1, 3, 4), (3, 1, -2) and (5, 0, -5) respectively. (a) (i) Show that D, E and F are collinear (ii) Find the ratio in which E divides DF (b) G has the coordinates (0, b, -2) Given that DE is perpendicular to GE, find the value of b	4 4
11	Function f and g are defined on suitable domains by $f(x) = \log_2(x^2 - 9x + 14)$ and $g(x) = \log_2(2 - x)$ Solve $f(x) - g(x) = 4$.	5
12	The straight line $y = 6 - 2x$ is a tangent to the circle with equation $x^2 + y^2 + 6x - 4y + k = 0$ Find the value of k.  	6
13	(a) $4\cos x° + \sin x°$ can be expressed in the form $k\cos(x + a)°$, where $k > 0$, and $0 \leq a \leq 360°$. Calculate the values of k and a (b) (i) Hence state the maximum and minimum values of $4\cos(x°) + \sin(x°)$ (ii) Determine the values of x, in the interval $0 \leq x \leq 360°$, at which these maximum and minimum values occur	4 3

14	The size of a human population, H, can be modelled using the equation $H = H_0\, e^{at}$, where H_0 is the population in 2010, t is the time in years since 2010, and a is the annual rate of the increase in the population. a) In 2010 the population of the United Kingdom was approximately 63 million, with an annual rate of increase of 1.4%. Assuming this growth rate remains constant, what would be the population in 2025? b) In 2010 the population of Scotland was approximately 5.2 million, with an annual increase of 0.41% Assuming this growth rate to be constant, how long would it take for Scotland's population to double in size?	2 3
15	A curve has equation y = $\sqrt{(2x-3)}$ Show that the equation of the tangent to the curve at the point (6,3) is $3y = x + 3$	5
	END OF QUESTION PAPER	54

HIGHER MATHS MARCH PRELIM 2014

Multiple Choice

1. **B**	2. **D**	3. **C**	4. **D**	
5. **C**	6. **B**	7. **B**	8. **D**	9. **D**

Section B

10 **8 C**	●[1] use vector approach ●[2] compare two vectors ●[3] complete proof ●[4] state ratio ●[5] use vector approach ●[6] know scalar product = 0 for perpendicular vectors ●[7] start to solve ●[8] complete	●[1] DE $=\begin{pmatrix}4\\-2\\-6\end{pmatrix}$ and EF $=\begin{pmatrix}2\\-1\\-3\end{pmatrix}$ ●[2] DE = 2EF ●[3] DE and DE are parallel and share a common point, hence D,E, and F are collinear ●[4] 2:1 ●[5] GE $=\begin{pmatrix}3\\1-b\\0\end{pmatrix}$ ●[6] DE • GE = 0 ●[7] 4x3 – 2(1 – k) 6x0 = 0 ●[8] b = -5
11 **5 C**	●[1] Use laws of logs ●[2] convert to exponential form ●[3] process conversion ●[4] express in standard form ●[5] find valid solutions	●[1] $\log_2\left(\dfrac{x^2-9x+14}{2-x}\right)=4$ ●[2] $\dfrac{x^2-9x+14}{2-x}=2^4$ ●[3] $x^2-9x+14 = 32-16x$ ●[4] $x^2+7x-18=0$ ●[5] (x -2)(x + 9), x = -9 and x = 2

13 **6 C**	●[1] Substitute ●[2] Expand brackets ●[3] express in standard form ●[4] Know to use discriminant ●[5] interpret a, b & c, substitute ●[6] Solve	●[1] $x^2 + (6 - 2x)^2 + 6x - 4(6 - 2x) + k = 0$. ●[2] $x^2 + 36 - 24x + 4x^2 + 6x - 24 + 8x + k = 0$ ●[3] $5x^2 - 10x + 12 + k = 0$ ●[4] For a tangent $b^2 - 4ac = 0$ ●[5] $b^2 - 4ac = (-10)^2 - 4 \times 5 \times (12 + k)$ ●[6] Solve $-140 = 20k$, $k = -7$
13 **7 C**	●[1] Use addition formulae ●[2] compare coefficients ●[3] process k ●[4] process a ●[5] state maximum and minimum ●[6] find x corresponding to max ●[7] find x corresponding to min For an incorrect answer of ksina = 1 – lost ●[2] can get ●[4] to ●[7] marks if carried through correctly	●[1] $k \cos x \cos a - k \sin x \sin a$ stated explicitly ●[2] $k \cos a = 4$, $k \sin a = -1$ ●[3] $\sqrt{17}$ ●[4] 346° ●[5] $\sqrt{17}$ and $-\sqrt{17}$ ●[6] 14° ●[7] 194° For $k \cos a = 4$, $k \sin a = 1$ ●[4] $\tan a = \frac{1}{4}$, $a = 14°$ ●[5] max and min still $\sqrt{17}$ and $-\sqrt{17}$ ●[6] max at 346° ●[7] min at 166°
14 **5 AB**	●[1] Substitute into equation ●[2] Evaluate exponential exp ●[3] interpret info and substitute ●[4] Convert exp eq to log eq ●[5] process	●[1] $H = 63\, e^{0.014 \times 15}$ ●[2] 78 million, or equiv 77.7 million etc ●[3] $10.4 = 5.2\, e^{0.0041t}$ ●[4] $\ln 2 = 0.0041t$ ●[5] 169 years

15 **5 AB**	•[1] function into differentiable form •[2] know to differentiate •[3] complete chain rule •[4] gradient via differentiation •[5] state equation and complete	•[1] $y = (2x - 3)^{1/2}$ •[2] $\frac{1}{2}(2x - 3)^{-1/2}$ •[3] $\ldots \times 2 = (2x - 3)^{-1/2}$ •[4] $m = \frac{1}{3}$ •[5] $y - 3 = \frac{1}{3}(x - 6)$, complete to $3y = x + 3$

TOTAL 54

MC	18
C marks	26
AB marks	10
Total	54

Paper 2	Source
10	2009 P1 Q22 centre amended
11	2008 P1 Q23 (b) centre amended
12	2007 P2 Q3 centre amended
13	2010 P2 Q2 centre amended
14	2009 P2 Q6 centre amended
15	2010 P2 Q6 centre amended