Prelim Examination 2004 / 2005
(Assessing Units 1 & 2)

MATHEMATICS

Higher Grade - Paper I (Non~calculator)

Time allowed - 1 hour 10 minutes

Read Carefully

1. **Calculators may not be used in this paper.**
2. Full credit will be given only where the solution contains appropriate working.
3. Answers obtained by readings from scale drawings will not receive any credit.
4. **This examination paper contains questions graded at all levels.**

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

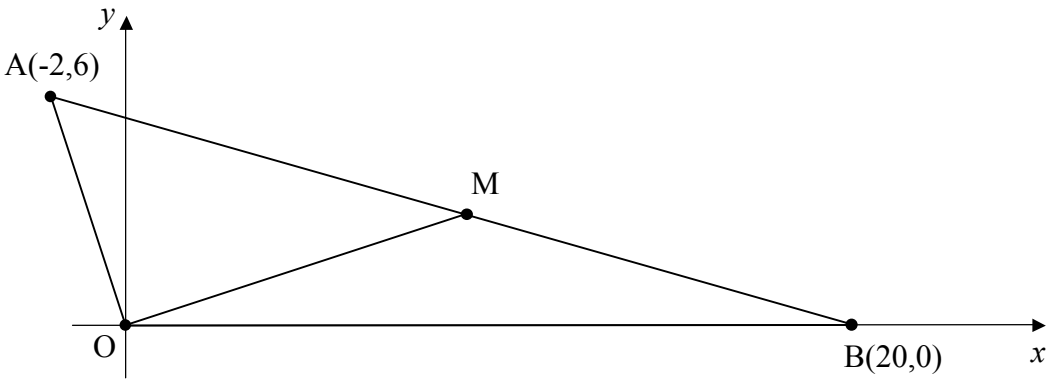
The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Trigonometric formulae:

$$\begin{aligned}
\sin(A \pm B) &= \sin A\cos B \pm \cos A\sin B \\
\cos(A \pm B) &= \cos A\cos B \mp \sin A\sin B \\
\sin 2A &= 2\sin A\cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

All questions should be attempted

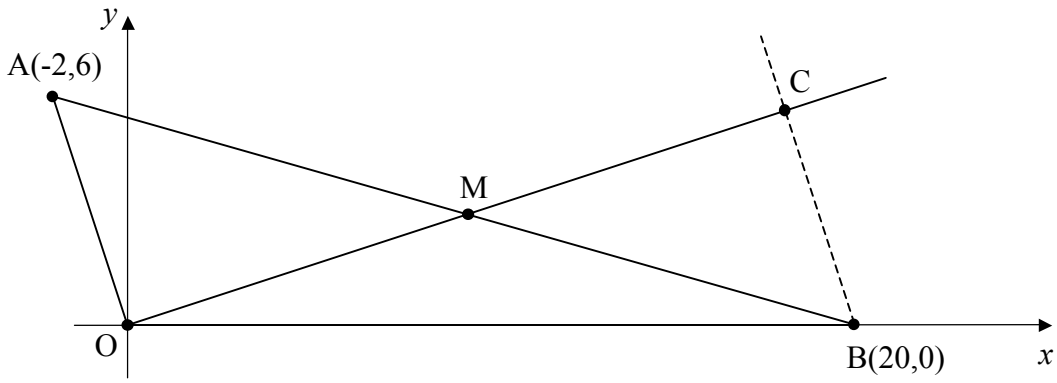
1. The diagram shows triangle OAB with M being the mid-point of AB.
The coordinates of A and B are (-2,6) and (20,0) respectively.



(a) Establish the coordinates of M. **1**

(b) Hence find the equation of the median OM. **2**

(c) A line through B, perpendicular to OM meets OM produced at C.



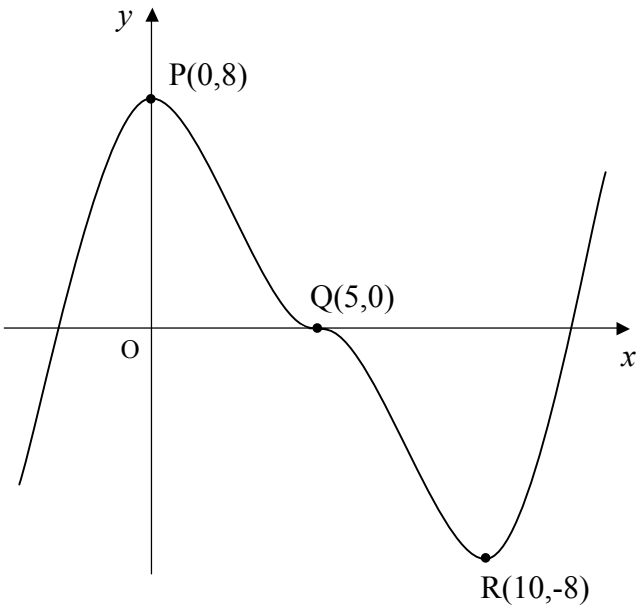
(i) Find the equation of the line BC and hence establish the coordinates of C. **4**

(ii) What can you say about triangles OAM and BMC? Explain your answer. **2**

2. A curve has as its equation $y = \dfrac{x^2 - 4x}{\sqrt{x}}$, where $x \in R$ and $x > 0$.

Find the gradient of the tangent to this curve at the point where $x = 4$. **6**

3. The diagram shows part of the graph of $y = f(x)$.



The function has stationary points at P(0,8), Q(5,0) and R(10,-8) as shown.

Sketch a possible graph for $y = f'(x)$, where $f'(x)$ is the derivative of $f(x)$. **4**

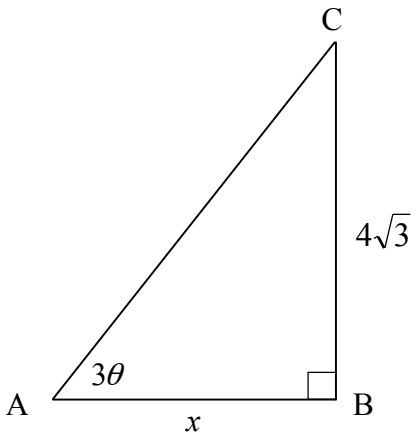
4. Two functions, defined on suitable domains, are given as

$$g(x) = x^2 - 3x \quad \text{and} \quad h(x) = 2x + 1.$$

Show that the composite function $g(h(x))$ can be written in the form $a(ax+b)(x-b)$, where a and b are constants, and state the value(s) of a and b. **4**

5. Consider the triangle opposite.

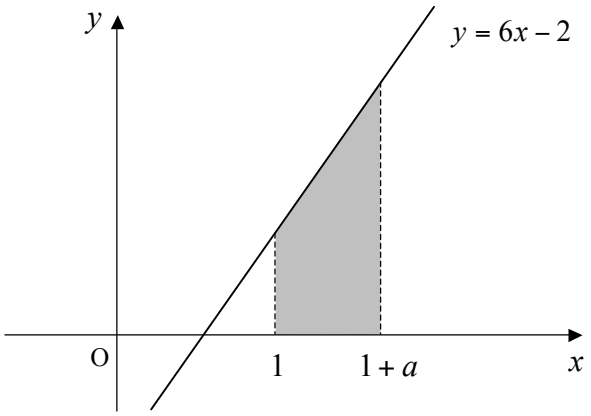
AB is x units long, BC $= 4\sqrt{3}$ units long and angle BAC = 3θ radians.



(a) Given that the exact area of the triangle is $8\sqrt{3}$ units^2, **show clearly** that $x = 4$. **3**

(b) Hence find the value of θ, **in radians**, given that 3θ is acute. **3**

6. The diagram below, which is not to scale, shows part of the graph of the line with equation $y = 6x - 2$. Also shown are ordinates at $x = 1$ and at $x = 1 + a$.



Find a given that the shaded part of the diagram has an area of 4 square units. **7**

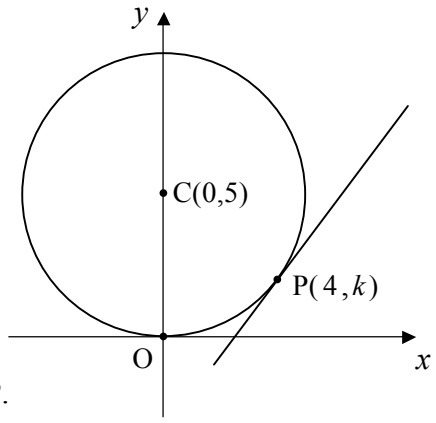
7. Two sequences are defined by the following recurrence relationships

$U_{n+1} = 0 \cdot 6U_n + 20$ and $U_{n+1} = 0 \cdot 9U_n + b$, where b is a constant.

(a) Explain why both sequences have a limit as $n \to \infty$. **1**

(b) Find the value of b if both these sequences have the same limit. **4**

8. A circle passes through the origin and has the point C(0,5) as its centre.



(a) Establish the equation of this circle giving your answer in **expanded form**. **4**

(b) The point P$(4,k)$ lies on the circumference of this circle as shown.
Find **algebraically** the value of k. **5**

(c) Find the equation of the tangent to the circle at P. **3**

9. A curve has as its equation $y = (p+1)x^3 - 3px^2 + 4x + 1$, where p **is a positive integer**.

(a) Find $\frac{dy}{dx}$. **2**

(b) Hence establish the value of p given that this curve has only **one stationary point**. **5**

[END OF QUESTION PAPER]

Prelim Examination 2004 / 2005
(Assessing Units 1 & 2)

MATHEMATICS
Higher Grade - Paper II

Time allowed - 1 hour 30 minutes

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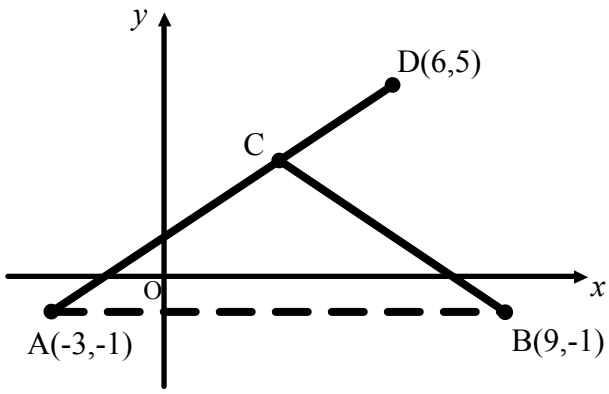
All questions should be attempted

1. The diagram shows a line joining the points A(-3,-1) and D(6,5).

B has coordinates (9,-1) and C is a point on AD.



(a) Find the equation of the line AD. **2**

(b) Hence establish the coordinates of C given that triangle ABC is isosceles. **3**

(c) Use gradient theory to calculate the size of angle BCD, giving your answer correct to the nearest degree. **3**

2. A lead shot is discharged from a gun at a clay pigeon.

The height, h feet, of the shot after t seconds is given by the function

$$h(t) = 288t - 48t^2 .$$

(a) What is the maximum height the shot can reach ? **4**

(b) For the shot to actually break the clay pigeon it must strike the pigeon at a speed greater than or equal to 48 feet per second.

The speed, s , of the shot after t seconds can be found from $s = h'(t)$, where $0 < t \leq 3$.

Will the shot break the clay pigeon after a flight of $2 \cdot 7$ seconds ? Explain. **2**

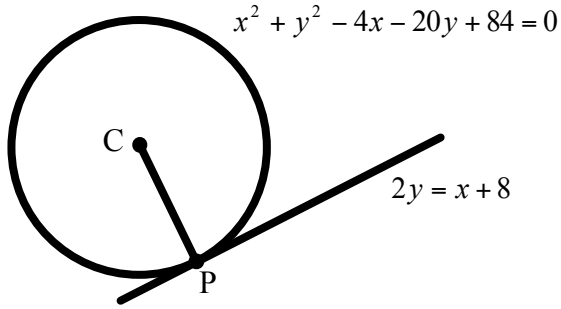
(c) Calculate the maximum **height** the shot can reach **and** still break the clay pigeon. **3**

3. Solve algebraically the equation

$$9\sin x^\circ + 4 = 2\cos 2x^\circ \qquad \text{where } 0 \leq x < 360 \qquad \textbf{6}$$

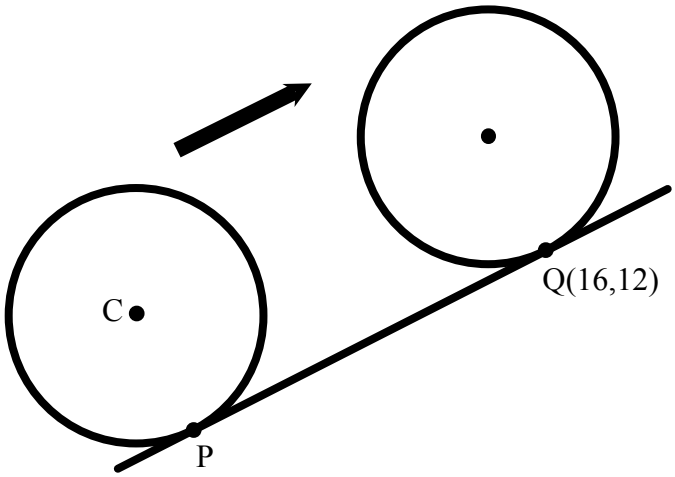
4. A circle, centre C, has as its equation $x^2 + y^2 - 4x - 20y + 84 = 0$.

It touches the line with equation $2y = x + 8$ at point P, as shown.



(a) Find **algebraically** the coordinates of P. **4**

(b) The circle is rolled up the line until Q(16,12) becomes the new point of tangency.



Establish the equation of the circle in this new position. **5**

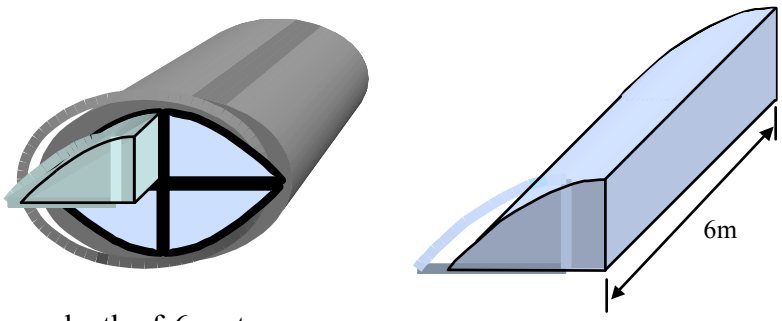
5. A sequence is defined by the recurrence relation $U_{n+1} = aU_n + b$, where a and b are constants.

(a) Given that $U_0 = a - 2$ and $b = 1$, show clearly that $U_1 = a^2 - 2a + 1$. **2**

(b) Hence find an expression for U_2 in terms of a. **2**

(c) Given now that $U_2 = 37$, form an equation and solve it to find a.
Explain why there is only one possible answer for a. **4**

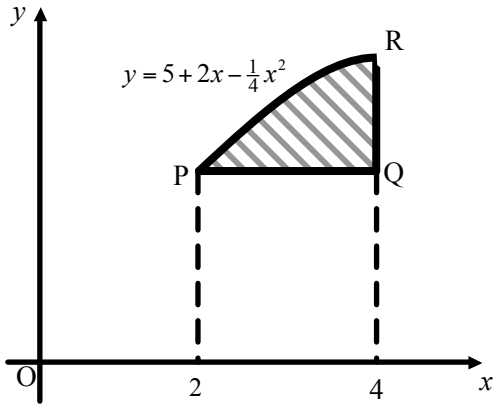
6. A titanium rod from a nuclear reactor is a solid prism which slots into an elliptical chamber along with three other identical rods. It has a cross-sectional shape made up of two straight lines and a curved edge.



Each rod has a depth of 6 metres.

The cross section of a rod is shown geometrically in the coordinate diagram below where the **units are in metres**. The diagram is not drawn to scale.

The curved section is part of the graph of the curve with equation $y = 5 + 2x - \frac{1}{4}x^2$.
PQ is horizontal and QR is vertical.



(a) Calculate the shaded area in square metres. **7**

(b) Hence calculate the **total volume** of titanium contained in **all four rods**. **2**

7. The angle θ is such that $\tan\theta = \frac{2}{\sqrt{2}}$ where $0 < \theta < \frac{\pi}{2}$.

(a) Find the exact values of $\sin\theta$ and $\cos\theta$. **3**

(b) Hence show clearly that the exact value of $\sin(\theta + \frac{\pi}{3})$ can be expressed as

$$\sin(\theta + \tfrac{\pi}{3}) = \tfrac{1}{6}(\sqrt{6} + 3).$$ **5**

8. Three functions are defined on suitable domains as

$$f(x) = x - 1, \quad g(x) = 3x^2 - 3 \text{ and } h(x) = x^3 - 6x.$$

(a) Given that $y = g(f(x)) - h(x)$, find a formula for y in its simplest form. **3**

(b) Hence find the coordinates of the maximum turning point of the graph of $y = g(f(x)) - h(x)$, **justifying your answer**. **4**

9. An equation is given as $ax(x-1) = c(x-1)$, where $a \neq 0$, $c \neq 0$, and a and c are constants.

(a) Show clearly that this equation can be written in the form

$$ax^2 - (a+c)x + c = 0. \quad \textbf{2}$$

(b) What condition needs to be met for this quadratic equation to have equal roots? **4**

[END OF QUESTION PAPER]