Mini Prelim Examination 2005 / 2006
(Assessing Unit 3 + revision from Units 1 & 2)

MATHEMATICS
Higher Grade

Time allowed - 1 hour 15 minutes

Read Carefully

1. **Calculators may be used in this paper.**
2. Full credit will be given only where the solution contains appropriate working.
3. Answers obtained by readings from scale drawings will not receive any credit.
4. **This examination paper contains questions graded at all levels.**

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Trigonometric formulae:

$$\begin{aligned}
\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\sin 2A &= 2 \sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

Scalar Product: $\boldsymbol{a} \cdot \boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}| \cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$.

or

$\boldsymbol{a} \cdot \boldsymbol{b} = \boldsymbol{a}_1\boldsymbol{b}_1 + \boldsymbol{a}_2\boldsymbol{b}_2 + \boldsymbol{a}_3\boldsymbol{b}_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

Table of standard derivatives:

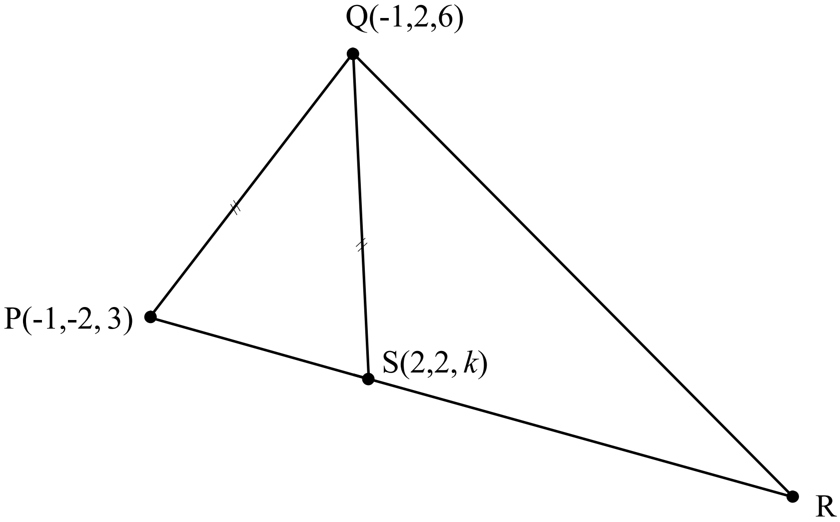
$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

All questions should be attempted

1. Consider the diagram below.



(a) Triangle PQS is isosceles with $|\overrightarrow{QS}|^2 = |\overrightarrow{QP}|^2$.
Use this information to **show algebraically** that $k = 2$ where $k < 5$. **6**

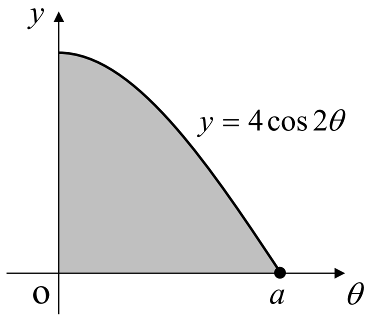
(b) Hence establish the coordinates of R if $\overrightarrow{PR} = 3\overrightarrow{PS}$. **3**

2. Show by solving the following logarithmic equation that the **exact value** of x can be written in the form $4p\sqrt{p}$ and write down the value of p.

$$\log_x(2x) + 2\log_x(8) = 3$$ **6**

3. A funtion is defined as $f(\theta) = 4\cos 2\theta$ where $0 \le \theta \le \pi$.

Part of the graph of $y = f(\theta)$ is shown.



(a) State the value of a in radians. **2**

(b) Calculate the shaded area in square units. **4**

4. Find the value of c if $x + 2$ is a factor of the expression

$$x^3 + (c+2)x^2 + x - 3c$$ **4**

5. The rate of decomposition of an isotope follows the exponential decay $M = M_0\, e^{-Pt}$, where M is the mass remaining after t years and M_0 is the initial mass. P is a constant.

Given that the isotope loses approximately 20% of its mass in 32 years, calculate the value of P , giving your answer correct to one significant figure. **5**

6. A function defined on a suitable domain is given as $f(x) = (x^2 - x)^4$.

Find the x coordinates of the three stationary points of this function. **6**

7. The power , E , emitting from a pulse generator is given by the formula

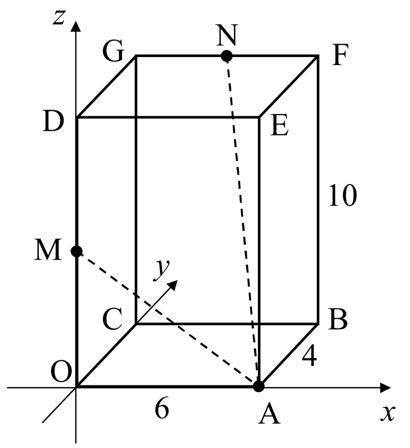
$$E = \sqrt{7}\sin 30t^\circ + 3\cos 30t^\circ + 4 ,$$ where t is the time elapsed, in seconds, from switch on.

(a) Express E in the form $k\sin(30t+\alpha)^\circ + 4$, where $k > 0$ and $0 \le \alpha \le 360$. **4**

(b) Hence find t when $E = 4$, where t lies in the interval $0 < t < 6$. **3**

8. A cuboid measuring 6 by 4 by 10 units is placed on a rectangular grid as shown.

M is the mid-point of side OD and N is the mid-point of side GF.



(a) Write down the coordinates of M and N. **1**

(b) Calculate the size of angle MAN. **6**

[END OF QUESTION PAPER]

	Give 1 mark for each •	Illustration(s) for awarding each mark
1.	**(a) ans:** proof **6 marks** •1 for components of QS •2 for magnitude squared of QS •3 for componentsof QP and mag.squ. •4 for equating together •5 for solving and discard •6 for answer **(b) ans:** R(8,10,0) **3 marks** •1 for strategy (i.e. vector algebra) •2 for ***r*** the subject •3 answer	(a) •1 $\overrightarrow{QS} = \begin{pmatrix} 2 \\ 2 \\ k \end{pmatrix} - \begin{pmatrix} -1 \\ 2 \\ 6 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ k-6 \end{pmatrix}$ •2 $\left\lvert\overrightarrow{QS}\right\rvert^2 = 9 + 0 + (k-6)^2 = k^2 - 12k + 45$ •3 $\overrightarrow{QP} = \begin{pmatrix} 0 \\ -4 \\ -3 \end{pmatrix}$, $\left\lvert\overrightarrow{QP}\right\rvert^2 = 25$ •4 $k^2 - 12k + 45 = 25$ •5 $(k-10)(k-2) = 0 \;\therefore\; \cancel{k = 10}$ *or* 2 •6 $k = 2$ (b) •1 $\boldsymbol{r} - \boldsymbol{p} = 3(\boldsymbol{s} - \boldsymbol{p})$ •2 $\boldsymbol{r} = 3\boldsymbol{s} - 2\boldsymbol{p}$ •3 R(8,10,0)
2.	**ans:** $x = 8\sqrt{2}$, $p = 2$ **6 marks** •1 2 up as a power •2 combining logs •3 converting to index form •4 solving for x •5 simplifying surd •6 answer for p	•1 $\log_x 2x + \log_x 8^2 = 3$ •2 $\log_x 128x = 3$ •3 $x^3 = 128x$ •4 $x^2 = 128 \;\therefore\; x = \sqrt{128}$ •5 $\sqrt{128} = 8\sqrt{2}$ •6 $p = 2$
3.	**(a) ans:** $a = \frac{\pi}{4}$ **2 marks** •1 solving $\cos 2\theta$ to zero •2 answer **(b) ans:** 2 units^2 **4 marks** •1 setting up integral •2 integrating •3 substituting limits •4 answer	(a) •1 $\cos 2\theta = 0$ •2 $2\theta = \frac{\pi}{2}$, $\theta = \frac{\pi}{4}$ (b) •1 $A = \int_0^{\frac{\pi}{4}} 4\cos 2\theta \; d\theta$ •2 $\ldots \; 4 \times \frac{1}{2}\sin 2\theta \;\; \therefore [2\sin 2\theta]_0^{\frac{\pi}{4}}$ •3 $A = (2\sin 2(\frac{\pi}{4})) - (2\sin 0)$ •4 $A = 2$ units^2

	Give 1 mark for each •	Illustration(s) for awarding each mark
4.	**ans:** $c = 2$ **4 marks** •1 for setting up synthetic division •2 for -2 •3 for performing division •4 solving to answer	•1 $\begin{array}{l\|cccc} & 1 & c+2 & 1 & -3c \\ \hline & & & & \end{array}$ •2 $\begin{array}{r\|cccc} -2 & 1 & c+2 & 1 & -3c \\ \hline & & & & \end{array}$ •3 $\begin{array}{r\|cccc} -2 & 1 & c+2 & 1 & -3c \\ & & -2 & -2c & 4c-2 \\ \hline & 1 & c & 1-2c & 0 \end{array}$ •4 $-3c + 4c - 2 = 0 \quad \therefore \quad c = 2$
5.	**ans:** $P = 0 \cdot 007$ **5 marks** •1 for realising 80% left •2 solving exponential to $0 \cdot 8$ •3 taking logs of both sides •4 releasing power •5 solving to answer (ignore rounding, instructions on rounding given to allow students to settle on an answer)	•1 $0 \cdot 8$ appearing •2 $e^{-32P} = 0 \cdot 8$ •3 $\log_e e^{-32P} = \log_e 0 \cdot 8$ (pupils may take logs to the base 10) •4 $-32P \log_e e = \log_e 0 \cdot 8$ (or equiv) •5 $P = \dfrac{\log_e 0 \cdot 8}{-32} = 0 \cdot 00697 = 0 \cdot 007$
6.	**ans:** $x = 0\ ,\ \frac{1}{2}\ ,\ 1$ **6 marks** •1 knowing to differentiate •2 differentiating power •3 differentiating inside bracket •4 solving to zero •5 for first value of x •6 for remaining two values (if only two values -1 mark)	•1 $f'(x) = ...$ •2 $f'(x) = 4(x^2 - x)^3$ ...... •3 $f'(x) = ... \times (2x - 1)$ •4 $4(x^2 - x)^3\,(2x - 1) = 0$ •5 $2x - 1 = 0 \quad \therefore x = \frac{1}{2}$ •6 $x^2 - x = 0$ $x(x-1) = 0 \quad \therefore \quad x = 0 \ \ or \ \ 1$

	Give 1 mark for each •	Illustration(s) for awarding each mark
7.	**(a) ans:** $E = 4\sin(30t + 48\cdot6)^\circ + 4$ **4 marks** •1 for replacement •2 equating coefficients •3 for k •4 for alpha **(b) ans:** $t = 4\cdot38$ seconds **3 marks** •1 forming equation and dealing with numbers •2 finding 3 solution for $30t + 48\cdot6$ •3 correct answer	(a) •1 $= k\sin 30t\cos\alpha + k\cos 30t\sin\alpha$ •2 $k\sin\alpha = 3$; $k\cos\alpha = \sqrt{7}$ •3 $k = \sqrt{3^3 + (\sqrt{7})^2} = \sqrt{16} = 4$ •4 $\tan\alpha = \frac{3}{\sqrt{7}}$ $\therefore$ $\alpha = 48\cdot6^\circ$ (b) •1 $4\sin(30t + 48\cdot6)^\circ + 4 = 4$ $4\sin(30t + 48\cdot6)^\circ = 0$ $\sin(30t + 48\cdot6)^\circ = 0$ •2 $30t + 48\cdot6 = 0$, 180 , 360 •3 $t = 4\cdot38$ (off the 180)
8.	**(a) ans:** M(0,0,5) , N(3,4,10) **1 mark** •1 answers **(b) ans:** $\angle\text{MAN} = 38\cdot9^\circ$ **6 marks** •1 choosing correct displacements •2 finding components (both displacements) •3 calculating magnitudes •4 finding scalar product •5 substitution in formula •6 answer	(a) •1 M(0,0,5) , N(3,4,10) (b) •1 $\overrightarrow{AM} = \ldots$; $\overrightarrow{AN} = \ldots.$ •2 $\overrightarrow{AM} = \begin{pmatrix}-6\\0\\5\end{pmatrix}$; $\overrightarrow{AN} = \begin{pmatrix}-3\\4\\10\end{pmatrix}$ •3 $\lvert\overrightarrow{AM}\rvert = \sqrt{61}$; $\lvert\overrightarrow{AN}\rvert = \sqrt{125}$ •4 $\overrightarrow{AM}\cdot\overrightarrow{AN} = \begin{pmatrix}-6\\0\\5\end{pmatrix}\cdot\begin{pmatrix}-3\\4\\10\end{pmatrix} = 18 + 50 = 68$ •5 $\cos\theta = \frac{68}{\sqrt{61}.\sqrt{125}}$ •6 $\theta = 38\cdot9^\circ$
		Total 50 marks