Prelim Examination 2006 / 2007
(Assessing Units 1 & 2)

MATHEMATICS

Higher Grade - Paper I (Non~calculator)

Time allowed - 1 hour 10 minutes

Read Carefully

1. **Calculators may not be used in this paper.**
2. Full credit will be given only where the solution contains appropriate working.
3. Answers obtained by readings from scale drawings will not receive any credit.
4. **This examination paper contains questions graded at all levels.**

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Trigonometric formulae:

$$\begin{aligned}
\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\sin 2A &= 2 \sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

All questions should be attempted

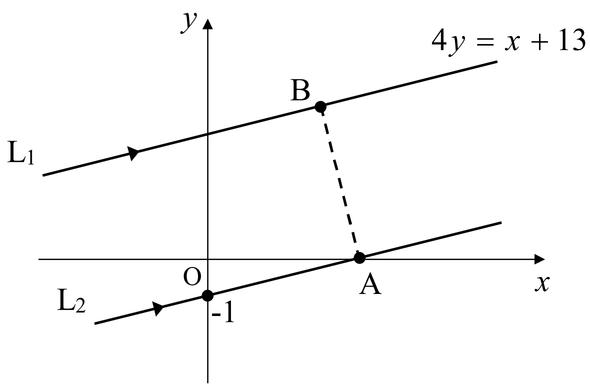
1. Two functions, defined on suitable domains, are given as

$$f(x) = x(x^2 - 1) \qquad \text{and} \qquad g(x) = x - 1.$$

(a) Show that the composite function, $h(x) = f\left(g(x)\right)$, can be written in the form $h(x) = ax^3 + bx^2 + cx$, where a, b and c are constants, and state the value(s) of a, b and c. **4**

(b) Hence solve the equation $h(x) = 6$, for x, showing clearly that there is only one solution. **4**

2. Part of the line, L_1 , with equation $4y = x + 13$, is shown in the diagram.
The line L_2 is parallel to L_1 and passes through the point (0,-1). Point A lies on the x-axis.



(a) Establish the equation of line L_2 and write down the coordinates of the point A. **3**

(b) Given that the line AB is perpendicular to both lines, find, algebraically, the coordinates of point B. **5**

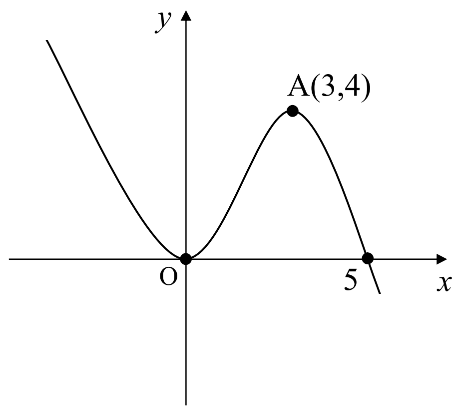
(c) Hence calculate the **exact** shortest distance between the lines L_1 and L_2. **2**

3. For what value of p, where $p > 0$, does the equation $(p^2 + 11)x^2 - 12px + p^2 = 0$ have equal roots? **6**

4. Given that $\sin A = \frac{2}{\sqrt{6}}$ and $\cos B = \frac{\sqrt{2}}{\sqrt{3}}$, with angles A and B both being acute, show clearly that

$$3\cos(A - B) = 2\sqrt{2}\,.$$ **6**

5. The diagram shows part of the graph of $y = f(x)$.



Sketch the graph of $y = -\left[f(x+3)\right]$ marking clearly the **new** positions of the highlighted points and stating their new coordinates. **3**

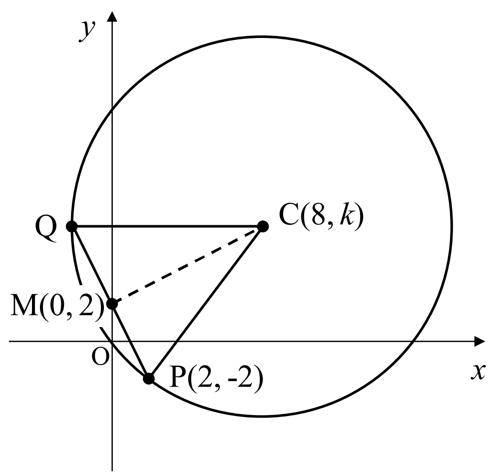
6. A function, *f*, is defined on a suitable domain as $f(x) = \frac{1}{x}\left(x^2 - \sqrt{x}\right)$.

(a) Differentiate f with respect to x, expressing your answer with positive indices. **4**

(b) Hence find x when $f'(x) = 5$. **3**

7. A circle, centre C(8, *k*), has the points P(2,-2) and Q on its circumference as shown.

M(0,2) is the mid-point of the chord PQ.



(a) Find the coordinates of Q. **1**

(b) Given that radius CQ is horizontal, write down the value of *k*, the *y*-coordinate of C. **1**

(c) Hence establish the equation of the circle. **3**

8. A sequence is defined by the recurrence relation $U_{n+1} = aU_n + 20$, where a is a constant.

(a) Given that $U_0 = 10$ and $U_1 = 26$, find a. **2**

(b) Find the value of S_2, if $S_2 = U_1 + U_2$. **2**

9. A curve has as its derivative $\frac{dy}{dx} = 3x^2 - 4x$.

Given that the point $(3, -7)$ lies on this curve, express y in terms of x. **4**

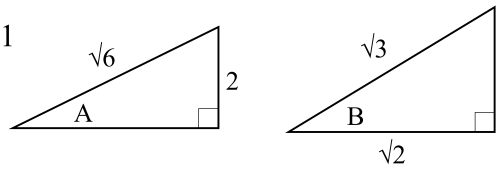
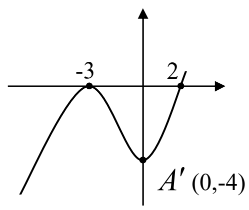
10. A function is given as $f(\theta) = 4\cos^2 2\theta + 8\cos 2\theta + 6$ for $0 \leq \theta \leq \pi$.

(a) Express the function in the form $f(\theta) = a(\cos 2\theta + b)^2 + c$ and write down the values of a, b and c. **4**

(b) Hence state the minimum value of this function and the corresponding replacement for θ. **3**

[END OF QUESTION PAPER]

	Give 1 mark for each •	Illustration(s) for awarding each mark
1.	**(a) ans:** $a = 1,\ b = -3,\ c = 2$ **4 marks** •1 for knowing g through f •2 for correct substitution (algebra) •3 expanding and simplifying •4 for a, b and c **(b) ans:** $x = 3$ **4 marks** •1 solving to zero •2 strategy - synthetic division •3 finding root •4 showing only one solution	(a) •1 strategy •2 $f(g(x)) = (x-1)\left((x-1)^2 - 1\right)$ (or equivalent) •3 $f(g(x)) = x^3 - 3x^2 + 2x$ •4 $a = 1,\ b = -3,\ c = 2$ (b) •1 $x^3 - 3x^2 + 2x - 6 = 0$ •2 $\begin{array}{l\|rrrr} & 1 & -3 & 2 & -6 \\ \hline & & & & \end{array}$ •3 $\begin{array}{r\|rrrr} 3 & 1 & -3 & 2 & -6 \\ & & 3 & 0 & 6 \\ \hline & 1 & 0 & 2 & 0 \end{array}$ $x = 3$ •4 $x^2 + 2 = 0\ \therefore\ x^2 = -2$ .. no solution (or equivalent)
2.	**(a) ans:** $y = \frac{1}{4}x - 1$; A(4,0) **3 marks** •1 for gradient •2 writing down equation of line •3 establishing the coordinates of A **(b) ans:** B(3,4) **5 marks** •1 for perpendicular gradient •2 equation of line AB •3 for strategy of a system •4 for first coordinate •5 second coordinate **(c) ans:** $\sqrt{17}$ units **2 marks** •1 strategy + lengths to use in Pyth. •2 calculation to answer	(a) •1 $y = \frac{1}{4}x + \frac{13}{4}\ \therefore\ m = \frac{1}{4}$ •2 $y = \frac{1}{4}x - 1$ •3 $0 = \frac{1}{4}x - 1 \Rightarrow 0 = x - 4\ \therefore\ x = 4$ (b) •1 $m = -4$ •2 $y - 0 = -4(x-4) \Rightarrow y = -4x + 16$ •3 $y + 4x = 16$ $4y - x = 13$ ….. or equiv. •4 $y = 4$ •5 $x = 3$ *Pupils may attempt to step out using -4 gradient and then check if point satisfies equation - award marks on your discretion.* (c) •1 Pyth + using 1 and 4 •2 $d^2 = 1^2 + 4^2 = 17\ \therefore\ d = \sqrt{17}$
3.	**ans:** $p = 5$ **6 marks** •1 for discriminant statement •2 for a, b and c •3 substituting correctly •4 simplifying •5 factorising and answer(s) •6 discarding	•1 for equal roots $b^2 - 4ac = 0$ •2 $a = p^2 + 11$, $b = -12p$, $c = p^2$ •3 $(-12p)^2 - 4(p^2 + 11)p^2 = 0$ (or equivalent) •4 $100p^2 - 4p^4 = 0$ •5 $4p^2(25 - p^2) = 0\ \therefore\ p = 0$ *or* ± 5 •6 $p = 5$ discard 0 and -5

	Give 1 mark for each •	Illustration(s) for awarding each mark
4.	**ans:** Proof **6 marks** •1 strategy of drawing R.A. triangles •2 calculating missing sides by Pyth. •3 correct expansion •4 substitution •5 calculation and simplifying surd •6 rationalising denom. to answer	•1  •2 $\sqrt{2}$ and 1 •3 $\cos(A-B)=\cos A\cos B+\sin A\sin B$ •4 $....=\frac{\sqrt{2}}{\sqrt{6}}\times\frac{\sqrt{2}}{\sqrt{3}}+\frac{2}{\sqrt{6}}\times\frac{1}{\sqrt{3}}$ •5 $....=3\times\frac{4}{\sqrt{18}}\Rightarrow\frac{12}{\sqrt{18}}=\frac{12}{3\sqrt{2}}=\frac{4}{\sqrt{2}}$ •6 $...=\frac{4}{\sqrt{2}}\times\frac{\sqrt{2}}{\sqrt{2}}=\frac{4\sqrt{2}}{2}=2\sqrt{2}$ (or equivalent)
5.	**ans:** sketch as opposite **3 marks** •1 for 3 units to the left •2 for reflection in the x-axis •3 new positions of S.P.'s (-3,0) and (0,-4) and for new position of root (2,0)	•1 •2 •3 
6.	**(a) ans:** $f'(x)=1+\frac{1}{2x^{\frac{3}{2}}}$ **4 marks** •1 for preparing to differentiate •2 expanding brackets •3 differentiating •4 express with positive indices **(b) ans:** $x=\frac{1}{4}$ **3 marks** •1 for forming equation •2 for x to subject •3 answer *various ways to solve use own discretion*	(a) •1 $f(x)=x^{-1}(x^2-x^{\frac{1}{2}})$ •2 $f(x)=x-x^{-\frac{1}{2}}$ •3 $f'(x)=1+\frac{1}{2}x^{-\frac{3}{2}}$ •4 $f'(x)=1+\frac{1}{2x^{\frac{3}{2}}}$ (b) •1 $1+\frac{1}{2x^{\frac{3}{2}}}=5$ •2 mult. by 2 then $2+\frac{1}{x^{\frac{3}{2}}}=10$ $\Rightarrow\frac{1}{x^{\frac{3}{2}}}=8$ *then* $\frac{1}{8}=x^{\frac{3}{2}}$ •3 square both sides $x^3=\frac{1}{64}$ $\therefore$ $x=\frac{1}{4}$ *alternative ……* •2 realise that $\frac{1}{2x^{\frac{3}{2}}}=4$ *then* $\frac{1}{8}=x^{\frac{3}{2}}$

	Give 1 mark for each •	Illustration(s) for awarding each mark
7.	**(a) ans:** Q(-2,6) **1 mark** •1 answer **(b) ans:** $k=6$ **1 mark** •1 answer **(c) ans:** $(x-8)^2+(y-6)^2=100$ **3 marks** •1 for strategy …. radius and centre •2 finding radius •3 answer	(a) •1 stepping out to answer (b) •1 answer (c) •1 strategy •2 r can be found from horiz. line but some pupils will use points P and C. $r^2=6^2+8^2=100$ •3 $(x-8)^2+(y-6)^2=100$
8.	**(a) ans:** $a=0\cdot6$ **2 marks** •1 forming equation •2 solving **(b) ans:** $S_2=61\cdot6$ **2 marks** •1 finding U_2 •2 finding S_2	(a) •1 $26=a(10)+20$ •2 $10a=6 \quad \therefore\ a=6$ (b) •1 $U_2=0\cdot6(26)+20=35\cdot6$ •2 $S_2=26+35\cdot6=61\cdot6$
9.	**ans:** $y=x^3-2x^2-16$ **4 marks** •1 knows to integrate •2 integrates correctly (with c) •3 substitutes and finds c •4 writes down answer	•1 $y=\int(3x^2-4x)\ dx$ •2 $y=x^3-2x^2+c$ •3 $-7=3^3-2(3^2)+c \quad \therefore\ c=-16$ •4 $y=x^3-2x^2-16$
10.	**(a) ans:** $f(\theta)=4(\cos 2\theta+1)^2+2$ $a=4$, $b=1$, $c=2$ **4 marks** •1 common factor •2 completing the square •3 multiplying back through •4 answer **(b) ans:** min of 2 at $\theta=\frac{\pi}{2}$ **3 marks** •1 knowing minimum of 2 •2 knowing to solve to zero •3 establishing answer *no marks off if given in degrees*	(a) •1 $f(\theta)=4(\cos^2 2\theta+2\cos 2\theta)+6$ •2 $\ldots=4\left((\cos 2\theta+1)^2-1\right)+6$ •3 $\ldots=4(\cos 2\theta+1)^2-4+6$ •4 $f(\theta)=4(\cos 2\theta+1)^2+2$ $a=4$, $b=1$, $c=2$ (b) •1 min $=2$ •2 @ $\cos 2\theta+1=0$ •3 $\cos 2\theta=-1 \quad 2\theta=\pi \quad (180)$ $\theta=\frac{\pi}{2} \quad (90)$

Total 60 marks

Prelim Examination 2006 / 2007
(Assessing Units 1 & 2)

MATHEMATICS

Higher Grade - Paper II

Time allowed - 1 hour 30 minutes

Read Carefully

1. **Calculators may be used in this paper.**
2. Full credit will be given only where the solution contains appropriate working.
3. Answers obtained by readings from scale drawings will not receive any credit.
4. **This examination paper contains questions graded at all levels.**

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

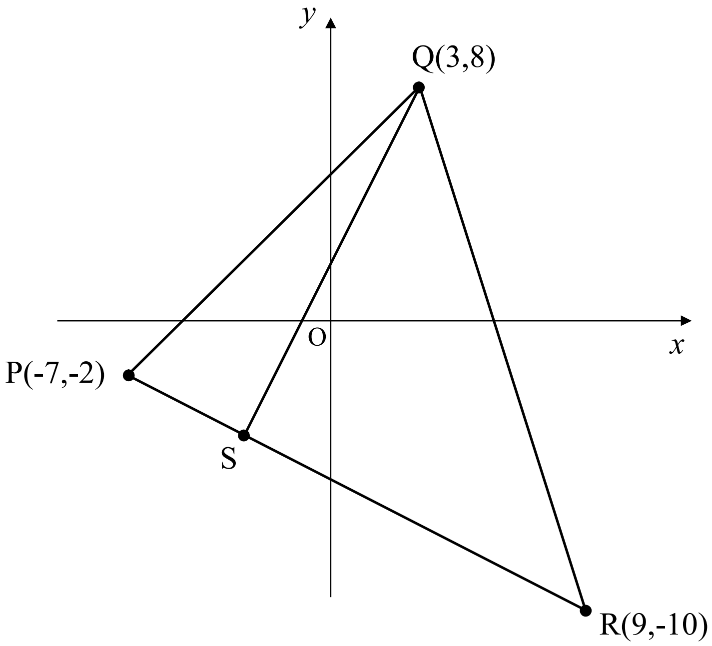
The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Trigonometric formulae:

$$
\begin{aligned}
\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\sin 2A &= 2 \sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}
$$

All questions should be attempted

1. Triangle PQR has as its vertices P(-7,-2), Q(3,8) and R(9,-10) as shown.



(a) Find the equation of side PR. **2**

(b) Find the equation of the **altitude** QS. **3**

(c) Hence find the coordinates of S, the point where the altitude QS meets side PR. **4**

(d) Establish the equation of the circle which passes through the points Q, S and R. **4**

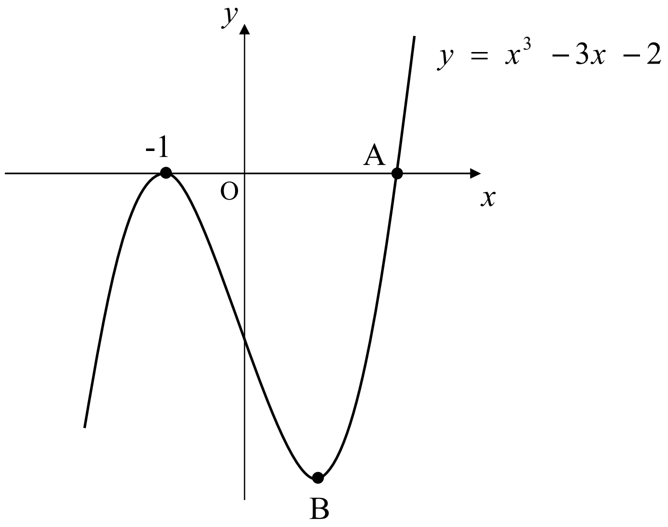
2. A recurrence relation is defined as $u_{n+1} = 0 \cdot 75 u_n + 12$.

Given that $U_0 = 32$, find the **difference** between the limit of the sequence and the third term, U_3. **5**

3. A curve has as its equation $y = (x-6)^2 + 8$.

Given that the line with equation $y = 2x - 5$ is a tangent to this curve, establish the coordinates of the point T, the point of contact between the curve and the line. **4**

4. Part of the graph of the curve with equation $y = x^3 - 3x - 2$ is shown below.
The curve passes throught the point (-1,0).



Find, algebraically, the coordinates of the points A and B. **7**

5. A circle has as its equation $(x-9)^2 + (y+1)^2 = 117$.

(a) Given that the point A(3, *k*) lies on this circle, find *k* where $k > 0$. **4**

(b) Find the equation of the tangent to this circle at the point A. **4**

(c) Show clearly that this tangent passes through the centre of the circle with equation $x^2 + y^2 + 6x - 8y + 12 = 0$. **2**

6. The functions f and g, defined on suitable domains, are given as

$$f(x) = \frac{x^2}{2} - \frac{3}{4} \quad \text{and} \quad g(x) = \frac{5ax}{4} - a, \text{ where } a \text{ is a constant.}$$

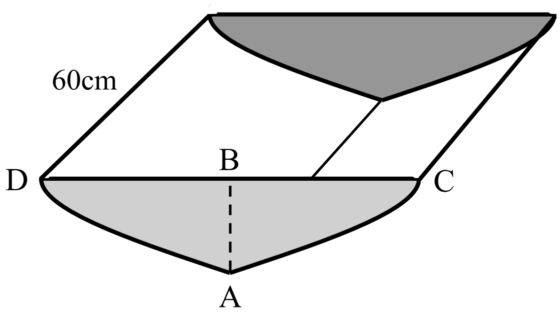
(a) Given that $f(a) = g(1)$, find the value of a, where $a < 0$. **4**

(b) With a taking this value, find the **rate of change** of g. **2**

7. A small feeding trough is shown opposite.

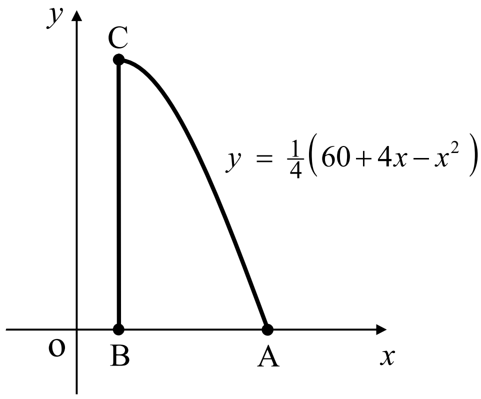
The end face has an axis of symmetry AB.

Edge CD is perpendicular to the axis of symmetry.



When the end face is rotated through 90° and then halved along the axis of symmetry, shape ABC can be placed on a coordinate diagram as shown below.

AB lies along the x-axis with the curved edge CA being part of the curve with equation $y = \frac{1}{4}\left(60 + 4x - x^2\right)$.



(a) Establish the coordinates of A and B. **4**

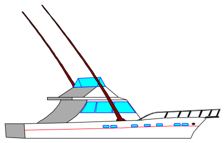
(b) Hence calculate the **area** of shape ABC given that all the units are in centimetres. **4**

(c) Given that the trough is a prism and measures 60cm from back to front, calculate the volume of feed the trough can hold when full, giving your answer correct to the nearest litre. **3**

8. Solve algebraically the equation

$$2\sin x^\circ - 3 = 5\cos 2x^\circ, \text{ where } 0 < x < 360. \qquad \textbf{6}$$

9. The captain of a small pleasure boat wishes to take a group of passengers from one island to the next, a journey of 100 kilometres.



The amount of fuel used is dependent upon the speed, v kilometres per hour, of the boat.

(a) Given that the rate of fuel used is $(1 + 0\cdot0000625v^3)$ gallons per hour, **show clearly** that the total fuel used, F, for this 100 kilometre journey is given by

$$F = \frac{100}{v} + 0\cdot00625v^2 \quad \text{gallons.}$$ **3**

(b) Hence find the speed which keeps the amount of fuel used to a minimum and the amount of fuel needed, at this speed, for the voyage. **5**

[END OF QUESTION PAPER]

	Give 1 mark for each •	Illustration(s) for awarding each mark
1.	**(a) ans:** $2y = -x - 11$ (or equiv.) **2 marks** •1 for gradient •2 for equation of line	(a) •1 $m = \frac{-10+2}{9+7} = -\frac{1}{2}$ •2 $y + 10 = -\frac{1}{2}(x - 9)$ (or equivalent)
	(b) ans: $y = 2x + 2$ **3 marks** •1 knowing gradients mult. to -1 •2 for gradient •3 eaquation of altitude	(b) •1 *if perpen.* $m_1 \times m_2 = -1$ stated or implied •2 $m = 2$ •3 $y - 8 = 2(x - 3)$ (or equiv.)
	(c) ans: S(-3,-4) **4 marks** •1 knowing to solve as a system •2 system strategy …. (subst. or elimin.) •3 first coordinate •4 second coordinate	(c) •1 $2y = -x - 11$ $y = 2x + 2$ •2 attempts to substitute or eliminate •3 $5y = -20 \therefore y = -4$ •4 $-4 = 2x + 2 \therefore x = -3$
	(d) ans: $(x-6)^2 + (y+1)^2 = 90$ **4 marks** •1 realising strategy of R.A. ∴ QR = diam. •2 finding centre •3 calculating value of r^2 •4 equation of circle	(d) •1 strategy •2 $C(\frac{3+9}{2}, \frac{8+(-10)}{2}) \rightarrow C(6,-1)$ •3 $r^2 = 9^2 + 3^2 = 90$ •4 $(x-6)^2 + (y+1)^2 = 90$
2.	**ans:** $6 \cdot 75$ **5 marks** •1 for finding U_1 •2 for U_2 and U_3 •3 knowing how to find limit •4 finding limit •5 calculating difference	•1 $U_1 = 0 \cdot 75(32) + 12 = 36$ •2 $U_2 = 0 \cdot 75(36) + 12 = 39$ $U_3 = 0 \cdot 75(39) + 12 = 41 \cdot 25$ •3 $L = \frac{b}{1-a}$ (or equivalent) •4 $L = \frac{12}{1 - 0 \cdot 75} = 48$ •5 diff. $= 48 - 41 \cdot 25 = 6 \cdot 75$
3.	**ans:** T(7,9) **4 marks** •1 knows to solve a system and sub. for y •2 simplifies •3 first coordinate •4 second coordinate	•1 $2x - 5 = (x-6)^2 + 8$ •2 $2x - 5 = x^2 - 12x + 36 + 8$ $0 = x^2 - 14x + 49$ •3 $(x-7)(x-7) = 0 \therefore x = 7$ •4 $y = 2(7) - 5 = 9$

	Give 1 mark for each •	Illustration(s) for awarding each mark
4.	**ans:** A(2,0) , B(1,-4) **7 marks** •1 **to find A** … set up synth. division •2 use -1 ….. or other •3 find x coordinate of A and hence A •4 **for B** … know to diff. and solve to 0 •5 differentiate correctly •6 find x coordinate of B •7 find y coordinate of B	•1 set up synth. division for root •2 $\begin{array}{r\|rrrr} -1 & 1 & 0 & -3 & -2 \\ & & -1 & 1 & 2 \\ \hline & 1 & -1 & -2 & 0 \end{array}$ •3 $x^2 - x + 2 = 0 \;\therefore\; (x-2)(x+1) = 0$ $x = 2$, $\cancel{x = -1}$ $\therefore$ A(2,0) •4 know S.P. $\therefore \dfrac{dy}{dx} = 0$ •5 $\dfrac{dy}{dx} = 3x^2 - 3 = 0$ •6 $3(x^2 - 1) = 0 \;\therefore\; x = 1$ $(discard\ -1)$ •7 $y = 1^3 - 3(1) - 2 = -4 \;\therefore$ B(1,–4)
5.	**(a) ans:** $k = 8$ **4 marks** •1 for subst. point in equation •2 for simplifying to quadratic •3 factorise and solve for k •4 discarding (or implied) and answer **(b) ans:** $3y = 2x + 18$ **4 marks** •1 for coordinates of centre •2 gradient of radius •3 gradient of tangent •4 equation of tangent **(a) ans:** proof **2 marks** •1 drawing out centre •2 show point satisfies equation of tangent	(a) •1 $(3-9)^2 + (k+1)^2 = 117$ •2 $k^2 + 2k - 80 = 0$ •3 $(k+10)(k-8) = 0$, $k = -10$ *or* 8 •4 $k = 8$ (b) •1 C(9,-1) •2 $m_r = \dfrac{-1-8}{9-3} = -\frac{3}{2}$ •3 $m_T = \frac{2}{3}$ •4 $y - 8 = \frac{2}{3}(x-3)$ (or equivalent) (c) •1 C(-3,4) •2 $3(4) = 2(-3) + 18$ $12 = -6 + 18$ proved
6.	**(a) ans:** $a = -1$ **4 marks** •1 for writing down $f(a)$ •2 for $g(1)$ •3 for equating and factorising •4 answer **(b) ans:** $-\frac{5}{4}$ **2 marks** •1 for g when $a = -1$ •2 for differentiation to answer	(a) •1 $f(a) = \dfrac{a^2}{2} - \dfrac{3}{4}$ •2 $g(1) = \dfrac{5a}{4} - a$ •3 $2a^2 - 3 = 5a - 4a$ $2a^2 - a - 3 = 0$ $(2a-3)(a+1) = 0$ •4 $a = -1$ (other root discard, implied) (b) •1 $g(x) = \dfrac{-5x}{4} - (-1)$ (or equiv.) •2 $g'(x) = -\dfrac{5}{4}$

	Give 1 mark for each •	Illustration(s) for awarding each mark
7.	**(a) ans:** A(10,0) , B(2,0) **4 marks** •1 for solving to zero •2 factorising and roots •3 stating A •4 finding B **(b) ans:** $85\frac{1}{3}$ cm^2 **4 marks** •1 for setting up integral •2 for integration •3 substitution •4 correct calculation to answer **(c) ans:** 10 litres **3 marks** •1 knows to double area •2 finds volume •3 answers to nearest litre	(a) •1 $\frac{1}{4}(60+4x-x^2)=0$ •2 $\frac{1}{4}(10-x)(6+x)=0$ $x=10$ *or* $x=-6$ •3 A(10,0) •4 B half way between roots …. $(10+(-6))\div 2=2 \quad \therefore$ B(2,0) (for B .. *pupils may diff. find x-coordinate of S.P.*) (b) •1 $A = \int_2^{10}(15+x-\frac{1}{4}x^2)\ dx$ •2 $=\left[15x+\frac{x^2}{2}-\frac{x^3}{12}\right]_2^{10}$ •3 $=(150+50-\frac{1000}{12})-(30+2-\frac{8}{12})$ •4 $=(116\frac{2}{3})-(31\frac{1}{3})=85\frac{1}{3}$ (or equiv.) (c) •1 $A_{\text{face}} = 85\frac{1}{3}\times 2=170\frac{2}{3}$ cm^2 •2 $V=170\frac{2}{3}\times 60=10240$ cm^3 •3 $V=10$ litres (to nearest litre)
8.	**ans:** $\{53\cdot 1, 126\cdot 9, 270\}$ **6 marks** •1 correct substitution •2 simplifying and rearranging to zero •3 factorising •4 correct roots •5 for first angle •6 for remaining two	•1 $2\sin x-3=5(1-2\sin^2 x)$ •2 $2\sin x-3=5-10\sin^2 x$ $10\sin^2 x+2\sin x-8=0$ •3 $2(5\sin x-4)(\sin x+1)=0$ (or equiv.) •4 $\sin x=\frac{5}{4}$ *or* $\sin x=-1$ •5 $x=53\cdot 1^\circ$ •6 $x=126\cdot 9^\circ$, $x=270^\circ$
9.	**(a) ans:** proof **3 marks** •1 for expression for time of journey •2 realising multiplication to find fuel used •3 simplifying to answer **(b) ans:** $v=20$ km/h, $7\cdot 5$ gallons **5 marks** •1 knows to diff and solve to zero •2 differentiates correctly •3 strategy for solving equation •4 solves equation •5 substitutes 20 into F and calc. fuel used	(a) •1 $T=\dfrac{D}{S}=\dfrac{100}{v}$ •2 $F=T\times \textit{rate of fuel used}$ $=\dfrac{100}{v}\left(1+0\cdot 0000625v^3\right)$ •3 $=\dfrac{100}{v}+0\cdot 00625v^2$ (b) •1 at min $F'(v)=0$ (stated or implied) •2 $F'(v)=\dfrac{-100}{v^2}+0\cdot 0125v$ •3 $\dfrac{-100}{v^2}+0\cdot 0125v=0 \quad (\times v^2)$ •4 $-100+0\cdot 0125v^3=0 \quad \therefore \ v=\sqrt[3]{8000}=20$ •5 $F=\dfrac{100}{20}+0\cdot 00625(20^2)=7\cdot 5$

Total 70 marks