Mathematics

Higher Prelim Examination 2008/2009

Paper 1

Assessing Units 1, 2 & 3

NATIONAL
QUALIFICATIONS

Time allowed - 1 hour 30 minutes

Read carefully

Calculators may NOT be used in this paper.

Section A - Questions 1 - 20 (40 marks)

Instructions for the completion of **Section A** are given on the next page.

For this section of the examination you should use an **HB pencil**.

Section B (30 marks)

1. Full credit will be given only where the solution contains appropriate working.
2. Answers obtained by readings from scale drawings will not receive any credit.

Read carefully

1 Check that the answer sheet provided is for **Mathematics Higher Prelim 2008/2009 (Section A)**.
2 For this section of the examination you must use an **HB pencil** and, where necessary, an eraser.
3 Make sure you write your **name**, **class** and **teacher** on the answer sheet provided.
4 The answer to each question is **either** A, B, C or D. Decide what your answer is, then, using your pencil, put a horizontal line in the space below your chosen letter (see the sample question below).
5 There is **only one correct** answer to each question.
6 Rough working should **not** be done on your answer sheet.
7 Make sure at the end of the exam that you hand in your answer sheet for Section A with the rest of your written answers.

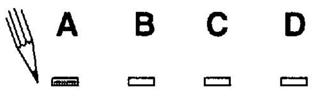
Sample Question

A line has equation $y = 4x - 1$.

If the point $(k,7)$ lies on this line, the value of k is

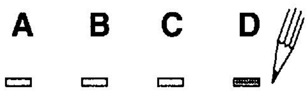
A 2
B 27
C 1·5
D -2

The correct answer is **A** $\rightarrow$ 2. The answer **A** should then be clearly marked in pencil with a horizontal line (see below).



Changing an answer

If you decide to change an answer, carefully erase your first answer and using your pencil, fill in the answer you want. The answer below has been changed to **D**.



FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Trigonometric formulae:

$$\begin{aligned}
\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\sin 2A &= 2 \sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

Scalar Product: $\boldsymbol{a}\cdot\boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}|\cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$.

or

$$\boldsymbol{a}\cdot\boldsymbol{b} = \boldsymbol{a}_1\boldsymbol{b}_1 + \boldsymbol{a}_2\boldsymbol{b}_2 + \boldsymbol{a}_3\boldsymbol{b}_3 \text{ where } \boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \text{ and } \boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

SECTION A

ALL questions should be attempted

1. If $f(x) = 2x^{\frac{3}{2}}$ then $f'(4)$ equals

A 16

B 4

C $25\frac{3}{5}$

D 6

2. Triangle ABC has vertices A$(-3, -3)$, B$(12, -1)$ and C(6, 11).

The gradient of the **altitude** through B is

A $-\frac{9}{14}$

B $\frac{14}{9}$

C $-\frac{3}{8}$

D $\frac{8}{3}$

3. The remainder when $x^3 - 11x + 10$ is divided by $(x + 3)$ is

A 52

B 16

C 4

D -24

4. The point P(8, y) lies on the circle with equation $x^2 + y^2 - 12x + 4y + 20 = 0$.

The value(s) of y is/are

A 2 only

B -6 only

C -6 and 2

D 6 and -2

5. A sequence is defined by the recurrence relation $U_{n+1} = aU_n - 5$ with $U_0 = 10$.

An expression in terms of a for U_2 is

A $10a - 5$

B $10a^2 - 5$

C $10a^2 - 5a - 5$

D $10a^2$

6. $\int (2x+1)^3 \, dx$ equals

A. $\dfrac{(2x+1)^4}{8} + C$

B. $\dfrac{(2x+1)^4}{4} + C$

C. $\dfrac{(2x+1)^2}{2} + C$

D. $\dfrac{(2x+1)^2}{4} + C$

7. The equation $2x^2 + 8 = kx$ has **no real roots**. k must take the values

A ± 8

B $-8 < k < 8$

C $k < -8$ or $k > 8$

D undefined

8. The vectors ***a*** and ***b*** have components $\boldsymbol{a} = \begin{pmatrix} 3 \\ -2 \\ 5 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} -1 \\ -2 \\ z \end{pmatrix}$.

If ***a*** and ***b*** are perpendicular, the value of z is

A. $-\dfrac{2}{5}$

B. $\dfrac{7}{5}$

C. $-\dfrac{1}{5}$

D. $\dfrac{1}{5}$

9. The maximum value of $5 + 3\sin x - 4\cos x$ is

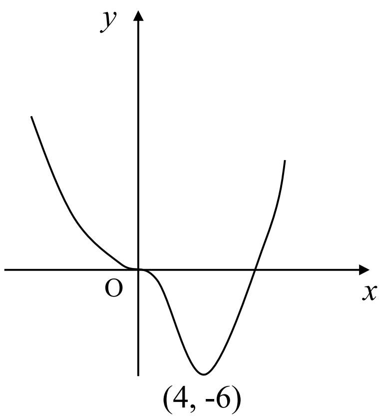
A. 0

B. 10

C. 4

D. -5

10.

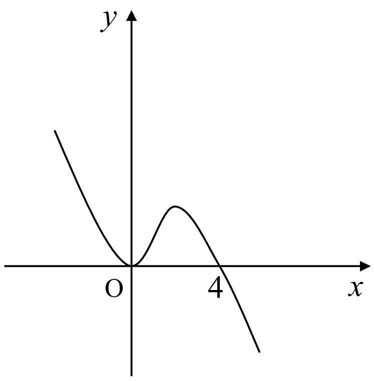


The diagram shows part of the graph of $y = f(x)$.

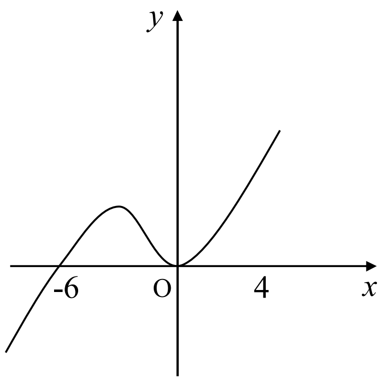
It has stationary points at (0, 0) and $(4, -6)$.

Which of the following could be part of the graph of the derived function $y = f'(x)$?

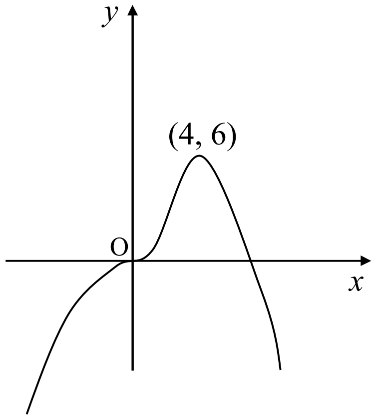
A



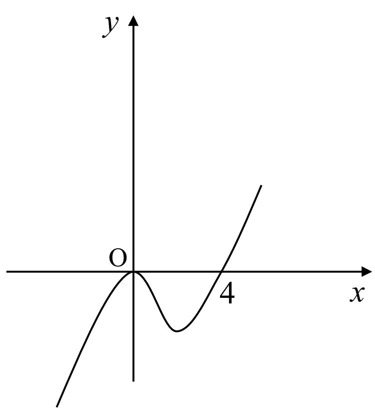
B



C



D



11. The two sequences defined by the recurrence relations $U_{n+1} = 0 \cdot 5U_n + 20$ and $V_{n+1} = 0 \cdot 2V_n + k$ have the same limit. The value of k is

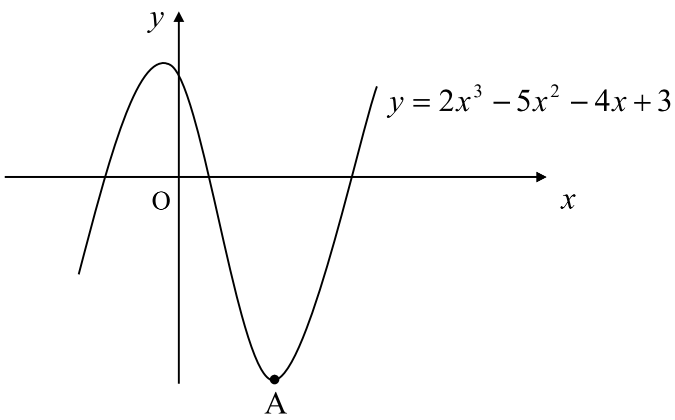
A 8

B 20

C 40

D 32

12. The diagram shows part of the curve with equation $y = 2x^3 - 5x^2 - 4x + 3$.



The x- coordinate of the point A is

A $\frac{1}{3}$

B 2

C 3

D -9

13. The function f is defined as $f(x) = \dfrac{x-6}{x}$, $x \neq 0$. The value of $f(f(3))$ equals

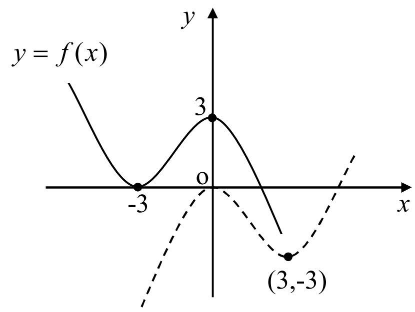
A 7

B -7

C -5

D -1

14. The diagram shows the graph of $y = f(x)$ as a full line and the graph of a related function as a broken line. The equation of the related function is



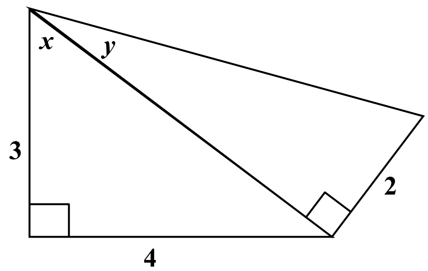
A $y = -f(x) - 3$

B $y = f(x-6) - 3$

C $y = f(-x) - 3$

D $y = f'(x)$

15. The diagram shows two right-angled triangles with lengths as shown.



The exact value of $\cos(x + y)$ is

A $\dfrac{8}{\sqrt{29}}$

B $\dfrac{7}{5\sqrt{29}}$

C $\dfrac{23}{5\sqrt{29}}$

D $\dfrac{3}{\sqrt{29}}$

16. A circle has centre A(1, 3) and radius $\sqrt{5}$. Another circle has centre B(9, 7) and radius $3\sqrt{5}$. Which of the following is true for these two circles?

A they intersect at two points

B they touch externally

C they touch internally

D they do not intersect or touch

17. Given that $|\boldsymbol{a}| = 3$, $|\boldsymbol{b}| = 5$ and $\boldsymbol{a}.\boldsymbol{b} = 7$, the value of $\boldsymbol{a}.(\boldsymbol{a}+\boldsymbol{b})$ is

A. 15

B. 10

C. 16

D. 24

18. A ball is thrown upwards reaching a height of 'h' metres after 't' seconds where $h(t) = 2 + 12t - 3t^2$. The time taken, in seconds, to reach its maximum height is

A 2

B 3

C 4

D 5

19. If $\log_a 2x + \log_a x = \log_a 18$, where $x > 0$, the value of x is

A. $4 \cdot 5$

B. $\sqrt{18}$

C. 3

D. 6

20. (x, y) is a solution for the system of equations

$$x^2 + 7y^2 = 16$$

$$x - 3y = 0.$$

Possible values for $x + y$ are

(1) 0 (2) 4 (3) -4

A (1) only

B (2) only

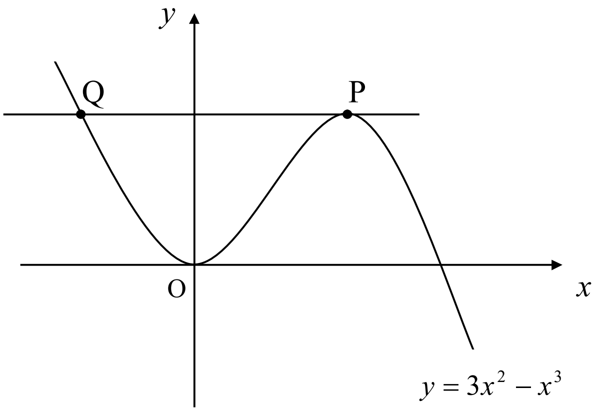
C (2) and (3) only

D (1), (2) and (3)

[**END OF SECTION A**]

SECTION B
ALL questions should be attempted

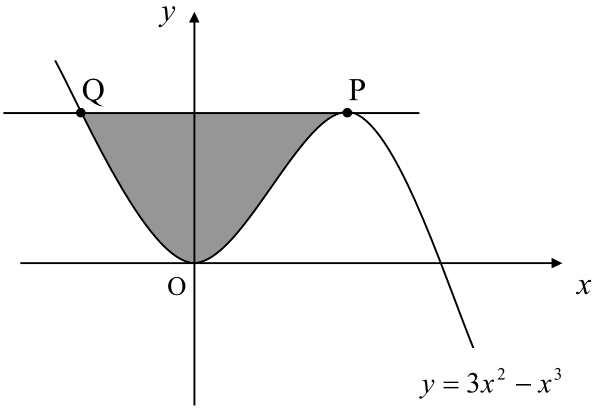
21. Part of the graph of the curve with equation $y = 3x^2 - x^3$ is shown below. The diagram is not drawn to scale.



(a) Establish the coordinates of the stationary point P. **4**

(b) The horizontal line through P meets the curve again at Q.
Find the coordinates of Q. **3**

(c) Hence calculate the shaded area shown in the diagram below.

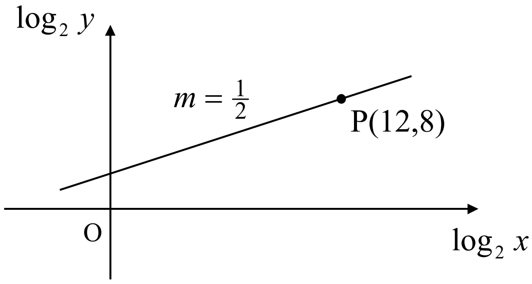


6

22. Two functions, defined on suitable domains, are given as $f(x) = x^2 - 1$ and $g(x) = 2 - x$.

Show that $f(g(a))$ can be expressed in the form $pa^2 + qa + r$ and write down the values of p, q and r. **3**

23.

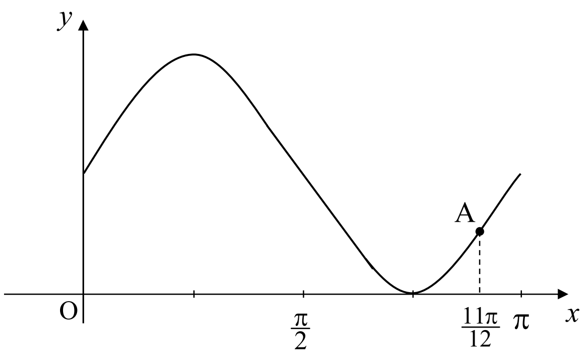


The equation of the line in the diagram above can be written as

$$\log_2 y = m\log_2 x + \log_2 k\ .$$

Given that the gradient of the line is $\frac{1}{2}$ and the point P(12,8) lies on the line, find the value of *k*. **3**

24. The diagram below shows part of the graph of $y = \sin 2x + 1$, for $0 \le x \le \pi$.
The point A has $\frac{11\pi}{12}$ as its *x*-coordinate.



Show clearly that the **rate of change** of *y*, with respect to *x*, at the point A can be expressed in the form $\sqrt{k}$, and state the value of *k*. **4**

25. A recurrence relation is defined by the formula $U_{n+1} = 0\cdot 6U_n + 24$.

(a) Establish the limit of this sequence. **2**

(b) Given now that U_1 is exactly half of this limit, find U_0, the initial value of the sequence. **2**

(c) A second recurrence relation in the form $U_{n+1} = aU_n + b$ has the **same limit** as the sequence above and is such that $b = 90a$.

Find the values of *a* and *b* in this second sequence. **3**

[END OF SECTION B]

[END OF QUESTION PAPER]

NATIONAL QUALIFICATIONS

Mathematics

Higher Prelim Examination 2008/2009

Paper 1 - Section A - Answer Sheet

NAME :	CLASS :

TEACHER :

You should use an HB pencil.
Erase all incorrect answers thoroughly.

Indicate your choice of answer with a single mark as in this example →

A	B	C	D
▭	▬	▭	▭

	A	B	C	D
1	▭	▭	▭	▭
2	▭	▭	▭	▭
3	▭	▭	▭	▭
4	▭	▭	▭	▭
5	▭	▭	▭	▭
6	▭	▭	▭	▭
7	▭	▭	▭	▭
8	▭	▭	▭	▭
9	▭	▭	▭	▭
10	▭	▭	▭	▭
11	▭	▭	▭	▭
12	▭	▭	▭	▭
13	▭	▭	▭	▭
14	▭	▭	▭	▭
15	▭	▭	▭	▭
16	▭	▭	▭	▭
17	▭	▭	▭	▭
18	▭	▭	▭	▭
19	▭	▭	▭	▭
20	▭	▭	▭	▭

Section A	/ **40**	
Section B	/ **30**	
Total (P1)	/ **70**	
Total (P2)	/ **60**	
Overall Total	/ **130**	**%**

Please make sure you have filled in all your details above before handing in this answer sheet.

Higher Grade - Paper 1 2008/2009 ANSWERS - Section A

1	D
2	A
3	B
4	C
5	C
6	A
7	B
8	C
9	B
10	D
11	D
12	B
13	A
14	C
15	B
16	B
17	C
18	A
19	C
20	A

	A	B	C	D
1	□	□	□	■
2	■	□	□	□
3	□	■	□	□
4	□	□	■	□
5	□	□	■	□
6	■	□	□	□
7	□	■	□	□
8	□	□	■	□
9	□	■	□	□
10	□	□	□	■
11	□	□	□	■
12	□	■	□	□
13	■	□	□	□
14	□	□	■	□
15	□	■	□	□
16	□	■	□	□
17	□	□	■	□
18	■	□	□	□
19	□	□	■	□
20	■	□	□	□

	Give 1 mark for each •	Illustration(s) for awarding each mark
21(a)	**ans: P(2, 4)** **(4 marks)**	
	•[1] knows to make derivative equal to 0	•[1] $\frac{dy}{dx} = 0$
	•[2] finds derivative	•[2] $\frac{dy}{dx} = 6x - 3x^2 = 0$
	•[3] solves for x	•[3] $3x(2-x) = 0;\ x = 0,\ 2$
	•[4] states coordinates of P	•[4] P(2, 4)
(b)	**ans: Q(– 1, 4)** **(3 marks)**	
	•[1] knows to equate functions	•[1] $3x^2 - x^3 = 4$
	•[2] uses app method to factorise expression	•[2] evidence leading to $(x-2)(x-2)(x+1)$
	•[3] solves and states coordinates of Q	•[3] Q(– 1, 4)
(c)	**ans: $6\frac{3}{4}$ units²** **(6 marks)**	
	•[1] knows to use integration	•[1] $\int$........
	•[2] uses correct integration	•[2] $\int_{-1}^{2} 3x^2 - x^3 dx$
	•[3] integrates	•[3] $\left[x^3 - \frac{x^4}{4}\right]_{-1}^{2}$
	•[4] subs values	•[4] $\left[(2)^3 - \frac{(2)^4}{4}\right] - \left[(-1)^3 - \frac{(-1)^4}{4}\right]$
	•[5] evaluates	•[5] $(8-4) - (-1-\frac{1}{4}) = 4 + 1\frac{1}{4} = 5\frac{1}{4}$
	•[6] subtracts from 12 to answer	•[6] $12 - 5\frac{1}{4} = 6\frac{3}{4}$ units^2
22	**ans: $p = 1, q = -4, r = 3$** **(3 marks)**	
	•[1] substitutes	•[1] $f(g(a)) = (2-a)^2 - 1$
	•[2] multiplies out and reorganises	•[2] $4 - 4a + a^2 - 1 = a^2 - 4a + 3$
	•[3] states values of p, q and r	•[3] $p = 1,\ q = -4,\ r = 3$
23	**ans: $k = 4$** **(3 marks)**	
	•[1] substituting values	•[1] $8 = \frac{1}{2}(12) + \log_2 k$
	•[2] simplifying	•[2] $\log_2 k = 2$
	•[3] to index form and answer	•[3] $k = 2^2 = 4$

	Give 1 mark for each •	Illustration(s) for awarding each mark
24	**ans:** $\sqrt{3}$, $k = 3$ **(4 marks)**	
	•[1] differentiates correctly	•[1] $\frac{dy}{dx} = 2\cos 2x$
	•[2] substitutes in angle	•[2] $\frac{dy}{dx} = 2\cos 2\left(\frac{11\pi}{12}\right) = 2\cos\left(\frac{11\pi}{6}\right)$
	•[3] establishes correct exact value	•[3] $\frac{dy}{dx} = 2 \times \frac{\sqrt{3}}{2}$
	•[4] answer	•[4] $\frac{dy}{dx} = \sqrt{3} \quad \therefore \quad k = 3$
25(a)	**ans: 60** **(2 marks)**	
	•[1] knows how to find limit	•[1] $L = \frac{24}{1 - 0 \cdot 6}$
	•[2] moves term to LHS and divides	•[2] 60
(b)	**ans: 10** **(2 marks)**	
	•[1] makes RR equal to 30	•[1] $0 \cdot 6U_0 + 24 = 30$ [stated or implied]
	•[2] solves for U_0	•[2] $0 \cdot 6U_0 = 6;\ U_0 = 10$
(c)	**ans:** $\boldsymbol{a = \frac{2}{5};\ b = 36}$ **(3 marks)**	
	•[1] subs for b and finds expression for limit	•[1] $U_{n+1} = aU_n + 90a;\ L = \frac{90a}{1-a}$
	•[2] equates limit to 60 and solves for a	•[2] $\frac{90a}{1-a} = 60;\ 60 - 60a = 90a;\ a = \frac{2}{5};$
	•[3] finds value of b	•[3] $b = 90 \times \frac{2}{5} = 36$

NATIONAL QUALIFICATIONS

Mathematics
Higher Prelim Examination 2008/2009
Assessing Units 1, 2 & 3
Paper 2

Time allowed - 1 hour 10 minutes

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2. Full credit will be given only where the solution contains appropriate working.
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\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
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\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

Scalar Product:

$\boldsymbol{a} \cdot \boldsymbol{b} = |\boldsymbol{a}||\boldsymbol{b}| \cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$.

or

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Table of standard derivatives:

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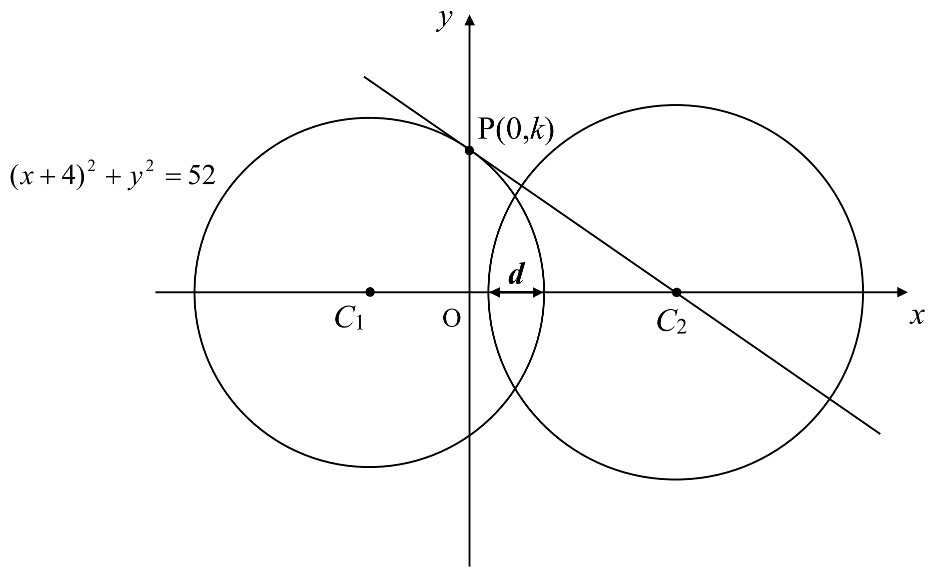
$\cos ax$	$\frac{1}{a} \sin ax + C$

ALL questions should be attempted

1. Consider the diagram below.

The circle centre C_1 has as its equation $(x+4)^2 + y^2 = 52$.
The point P(0, k) lies on the circumference of this circle and the tangent to this circle through P has been drawn.

A second circle with centre C_2 is also shown.



(a) What is the value of k? **2**

(b) Hence find the equation of the tangent through P. **4**

(c) The tangent through P passes through C_2 the centre of the second circle.
State the coordinates of C_2. **1**

(d) Given that the second circle has a radius of 8 units, calculate the distance marked $\boldsymbol{d}$ on the diagram, giving your answer correct to 1 decimal place. **3**

2. Solve algebraically the equation

$$\sqrt{3}\sin x^\circ + \sqrt{6}\cos x^\circ = 1 \quad \text{for} \quad 0 \le x < 180 .$$ **5**

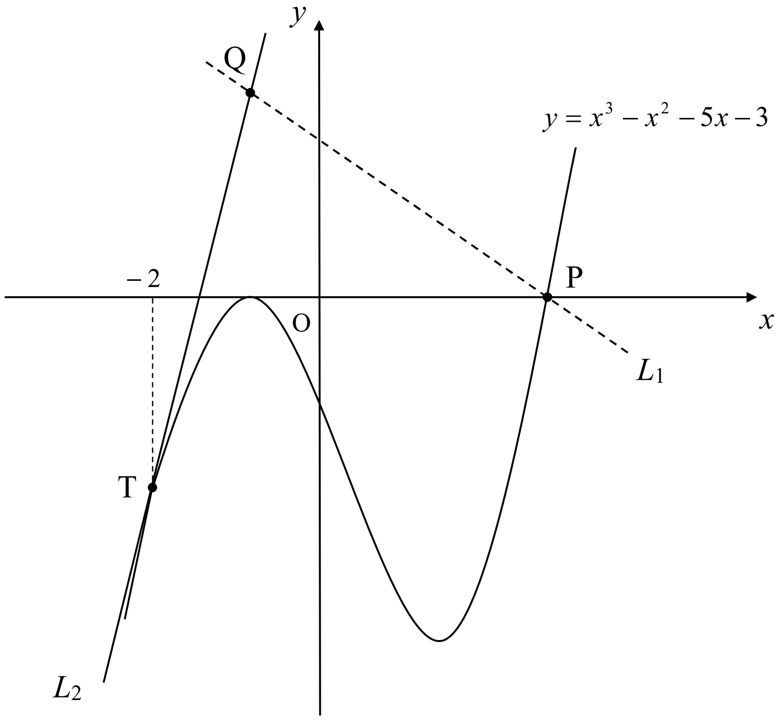
3. A curve has as its derivative $\dfrac{dy}{dx} = 2x - \dfrac{6}{x^2}$.

(a) Given that the point (2, 3) lies on this curve, express y in terms of x. **4**

(b) Hence find p if the point $(3,\ p)$ also lies on this curve. **1**

4. The diagram below, **which is not drawn to scale**, shows part of the graph of the curve with equation $y = x^3 - x^2 - 5x - 3$.

Two straight lines are also shown, L_1 and L_2.



(a) Find the coordinates of P. **2**

(b) Line L_1 has a gradient of $-\frac{3}{2}$ and passes through the point P.
Find the equation of L_1. **1**

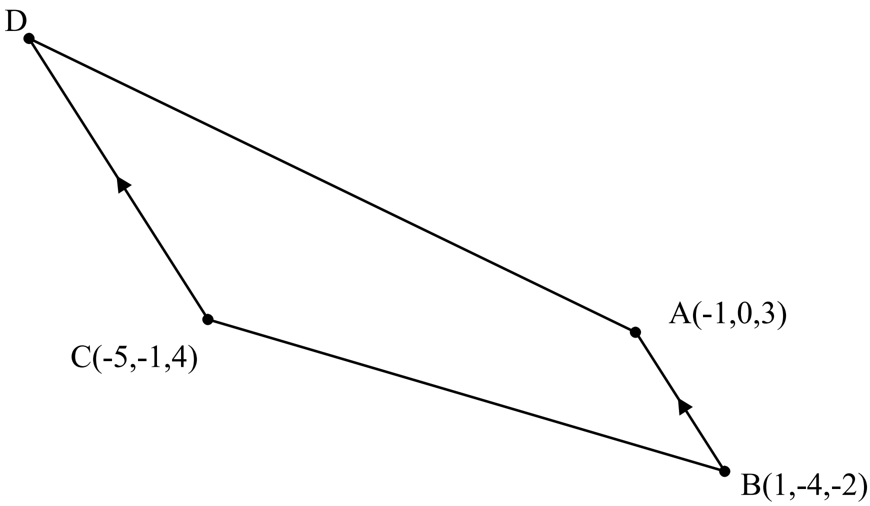
(c) Line L_2 is a tangent to the curve at the point T where $x = -2$.
Find the equation of L_2. **4**

(d) Hence find the coordinates of Q, the point of intersection of the two lines. **3**

5. ABCD is a quadrilateral with three of its vertices as A(-1,0,3), B(1,-4,-2) and C(-5,-1,4) as shown in the diagram below.

The diagram is not drawn to scale.

BA is parallel to CD.



(a) Given that CD = 2BA, establish the coordinates of D. **3**

(b) Hence calculate the size of angle ADC. **5**

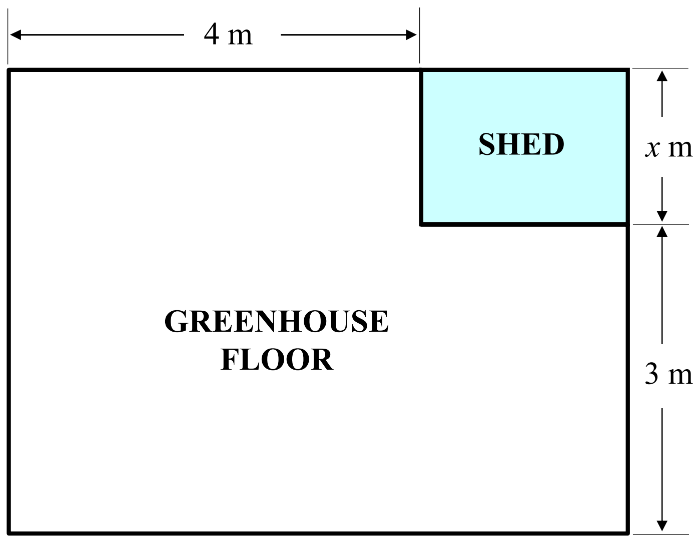
6. (a) If $k = \dfrac{(x-1)^2}{x^2+4}$, where k is a real number, show clearly that

$(k-1)x^2 + 2x + (4k-1) = 0$. **3**

(b) Hence find the value of k given that the equation $(k-1)x^2 + 2x + (4k-1) = 0$ has equal roots and $k > 0$. **5**

7. The floor plan of a rectangular greenhouse is shown below. All dimensions are in metres.

The gardener places a rectangular wooden storage shed, of width x metres, in one corner.



(a) Given that the **area of the shed** is 3 square metres, show clearly that the area of greenhouse floor remaining, A square metres, is given in terms of x as

$$A(x) = 12 + 4x + \frac{9}{x}.$$ **3**

(b) Hence find the value of x which **minimises** the area of the greenhouse floor remaining, **justifying your answer**. **5**

8. Angle A is acute and such that $\tan A = \frac{\sqrt{6}}{3}$.

(a) Show clearly that the exact value of $\sin A$ can be written in the form $\frac{1}{5}\sqrt{k}$, and state the value of k. **3**

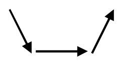
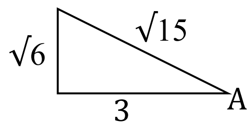
(b) Hence, or otherwise, show that the value of $\cos 2A$ is exactly $\frac{1}{5}$ **3**

[END OF QUESTION PAPER]

	Give 1 mark for each •	Illustration(s) for awarding each mark
1(a)	**ans: $k = 6$** **(2 marks)**	
	•[1] knows to substitute point	•[1] $(0+4)^2 + k^2 = 52$
	•[2] establishes value of k	•[2] $k = 6$
(b)	**ans: $y = -\frac{2}{3}x + 6$** **(4 marks)**	
	•[1] finds coordinates of C_1	•[1] $C(-4, 0)$
	•[2] finds gradient of radius	•[2] $m_{C_1P} = \frac{6}{4} = \frac{3}{2}$
	•[3] finds gradient of tangent	•[3] $m_{\tan} - \frac{2}{3}$
	•[4] substitutes into formula	•[4] $y = -\frac{2}{3}x + 6$
(c)	**ans: $C_2(9, 0)$** **(1 mark)**	
	•[1] subs point, solves for x and states point	•[1] $0 = -\frac{2}{3}x + 6;\ x = 9;\ (9,0)$
(d)	**ans: 2·2 units** **(3 marks)**	
	•[1] finds radius C_1 circle	•[1] radius $C_1 = 7{\cdot}2$
	•[2] finds distance between centres	•[2] $C_1C_2 = 13$
	•[3] establishes d	•[3] $d = (7{\cdot}2 + 8) - 13 = 2{\cdot}2$
2	**ans: 105·8°** **(5 marks)**	
	•[1] for expansion and equating coeff.s	•[1] $k\sin\alpha = \sqrt{3}$, $k\cos\alpha = \sqrt{6}$ (or equiv.)
	•[2] for α	•[2] $\tan\alpha = \frac{\sqrt{3}}{\sqrt{6}} \quad \therefore \ \alpha = 35\cdot3^\circ$
	•[3] for k	•[3] $k^2 = (\sqrt{3})^2 + (\sqrt{6})^2 \ \therefore \ k = 3$
	•[4] solving to one-third	•[4] $\cos(x - 35\cdot3)^\circ = \frac{1}{3}$
	•[5] for answer (first angle, no marks off for 2nd angle given)	•[5] $x - 35\cdot3 = 70\cdot5 \quad \therefore \ x = 105\cdot8^\circ$

	Give 1 mark for each •	Illustration(s) for awarding each mark
3(a)	**ans:** $y = x^2 + \frac{6}{x} - 4$ **(4 marks)**	
	•[1] knows to integrate	•[1] $y = \int 2x - \frac{6}{x^2}\,dx$
	•[2] integrates	•[2] $y = x^2 + \frac{6}{x} + C$
	•[3] subs point	•[3] $3 = 2^2 + \frac{6}{2} + C$
	•[4] solves for C and states function	•[4] $y = x^2 + \frac{6}{x} - 4$
(b)	**ans:** $p = 7$ **(1 mark)**	
	•[1] subs point and solves for p	•[1] $p = 3^2 + \frac{6}{3} - 4 = 7$
4(a)	**ans:** **P(3, 0)** **(2 marks)**	
	•[1] knows to make function equal to 0	•[1] $x^3 - x^2 - 5x - 3 = 0$
	•[2] solves for x and states cords of P	•[2] $x = 3$; P(3, 0)
(b)	**ans:** $2y + 3x = 9$ **(1 mark)**	
	•[1] subs info into formula for straight line	•[1] $y = -\frac{3}{2}(x - 3)$
(c)	**ans:** $y - 11x = 17$ **(4 marks)**	
	•[1] knows to take derivative	•[1] $\frac{dy}{dx} = 3x^2 - 2x - 5$
	•[2] subs to find gradient	•[2] $3(-2)^2 - 2(-2) - 5 = 11$
	•[3] subs to find point of contact	•[3] $y = (-2)^3 - (-2)^2 - 5(-2) - 3 = -5$
	•[4] subs into straight line formula	•[4] $y + 5 = 11(x + 2)$; $y - 11x = 17$
(d)	**ans:** **Q(–1, 6)** **(3 marks)**	
	•[1] knows to use sim. eqs.	•[1] evidence
	•[2] solves for x and y	•[2] $x = -1$ and $y = 6$
	•[3] states coordinates of Q	•[3] Q(–1, 6)

	Give 1 mark for each •	Illustration(s) for awarding each mark
5(a)	**ans: D(-9,7,14)** **(3 marks)** •1 for $\vec{BA}$ and $2\vec{BA}$ •2 for equating •3 solving to answer Pupils may use 'vector algebra' before subs. components <u>or</u> by simply "stepping out". Attach marks accordingly.	•1 $\vec{BA} = \begin{pmatrix} -2 \\ 4 \\ 5 \end{pmatrix}$ .... $2\vec{BA} = \begin{pmatrix} -4 \\ 8 \\ 10 \end{pmatrix}$ •2 $d - c = 2\vec{BA}$ •3 $d = \begin{pmatrix} -4 \\ 8 \\ 10 \end{pmatrix} + \begin{pmatrix} -5 \\ -1 \\ 4 \end{pmatrix} = \begin{pmatrix} -9 \\ 7 \\ 14 \end{pmatrix}$ $\therefore$ $D(-9,7,14)$
(b)	**ans: 15·3°** **(5 marks)** •1 for scalar product strategy •2 correct vectors and their components •3 scalar product •4 magnitudes •5 substitution and answer	•1 $\cos\theta = \dfrac{a.b}{\lvert a \rvert . \lvert b \rvert}$, stated or implied. •2 $\vec{DA} = \begin{pmatrix} 8 \\ -7 \\ -11 \end{pmatrix}$ and $\vec{DC} = \begin{pmatrix} 4 \\ -8 \\ -10 \end{pmatrix}$ •3 $\vec{DA} \,.\, \vec{DC} = 32 + 56 + 110 = 198$ •4 $\lvert\vec{DA}\rvert = \sqrt{234}$, $\lvert\vec{DC}\rvert = \sqrt{180}$ •5 $\cos D = \dfrac{198}{\sqrt{234} \times \sqrt{180}}$, $\therefore$ $\angle D = 15 \cdot 3^{\circ}$
6(a)	**ans: proof** **(3 marks)** •1 cross multiplies and multiplies out •2 brings to LHS •3 rearranges as required	 •1 $k(x^2 + 4) = x^2 - 2x + 1$ •2 $kx^2 - x^2 + 2x + 4k - 1$ •3 $(k-1)x^2 + 2x + (4k-1) = 0$
(b)	**ans: $k = \frac{5}{4}$** **(5 marks)** •1 states condition for equal roots •2 states values of a, b and c •3 substitutes into $b^2 - 4ac$ •4 multiplies out and simplifies •5 solves for k	 •1 $b^2 - 4ac = 0$ for equal roots [stated/implied] •2 $a = (k-1)$; $b = 2$; $c = (4k-1)$ •3 $2^2 - 4(k-1)(4k-1) = 0$ •4 $20k - 16k^2 = 0$ •5 $k = \frac{5}{4}$

	Give 1 mark for each •	Illustration(s) for awarding each mark
7(a)	**ans: proof** **(3 marks)**	
	•[1] finds expression for length of shed	•[1] length of shed $= \frac{3}{x}$
	•[2] finds expression for area of g'house	•[2] $A = (x+3)(4+\frac{3}{x}) - 3$
	•[3] simplifies to correct form	•[3] $A = 4x + 3 + 12 + \frac{9}{x} - 3 \rightarrow$ answer
(b)	**ans: 15** **(5 marks)**	
	•[1] knows to equate derivative to 0	•[1] $\frac{dy}{dx} = 0$
	•[2] prepares to differentiate	•[2] $A(x) = 12 + 4x + 9x^{-1}$
	•[3] differentiates	•[3] $A'(x) = 4 - \frac{9}{x^2} = 0$
	•[4] solves for x	•[4] $4 - \frac{9}{x^2} = 0;\ x^2 = \frac{9}{4};\ x = \frac{3}{2}$
	•[5] justifies answer	•[5]  or other acceptable method
8(a)	**ans: proof; $k = 10$** **(3 marks)**	
	•[1] assembles facts in rt. triangle	•[1]
	•[2] finds $\sin A$	•[2] $\sin A = \frac{\sqrt{6}}{\sqrt{15}}$
	•[3] rationalises denominator & states k	•[3] $\frac{\sqrt{6}}{\sqrt{15}} \times \frac{\sqrt{15}}{\sqrt{15}} = \frac{\sqrt{10}}{5}$; $k = 10$
(b)	**ans: proof** **(3 marks)**	
	•[1] knows to change $\cos 2A$	•[1] $\cos 2A = 1 - 2\sin^2 A$
	•[2] substitutes	•[2] $1 - 2(\frac{\sqrt{10}}{5})^2$
	•[3] simplifies to required answer	•[3] $1 - 2 \times \frac{10}{25} = 1 - \frac{4}{5} = \frac{1}{5}$
		Total: 60 marks